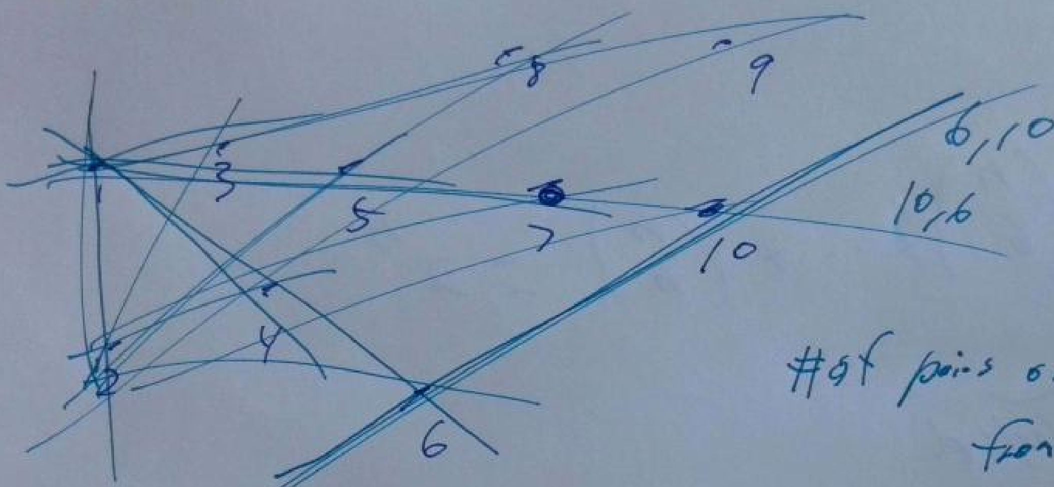
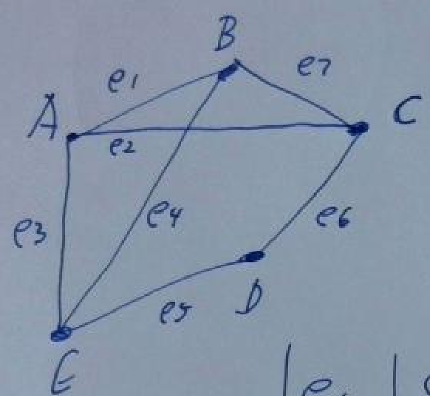


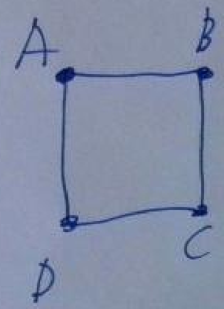
# of pairs of points,
from $\{1, 2, \dots, 10\}$

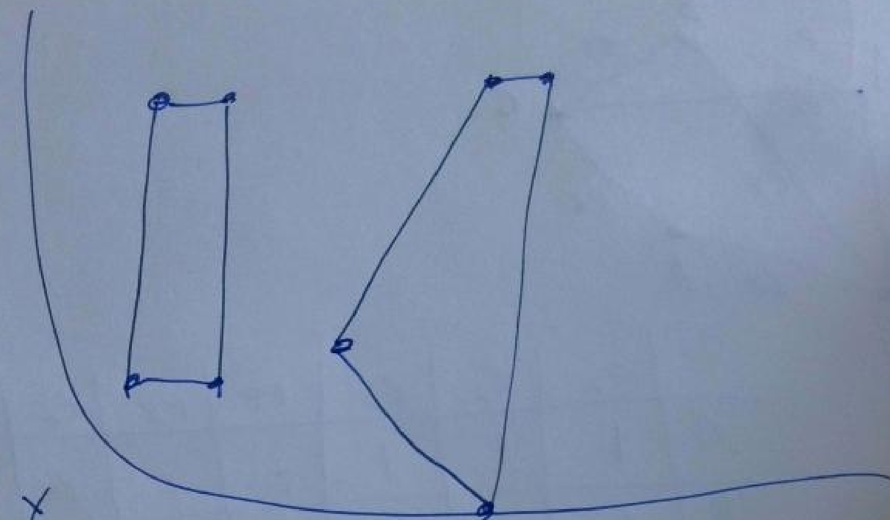
1,2	2,1	3,1
1,3	2,3	3,2
1,4	2,4	3,4
⋮	⋮	⋮
1,9		
1,10	2,10	3,10

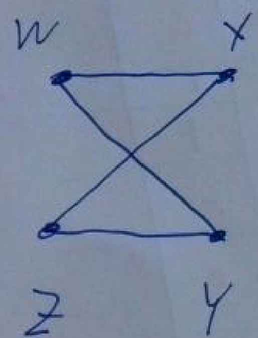
$$\binom{10}{2}$$



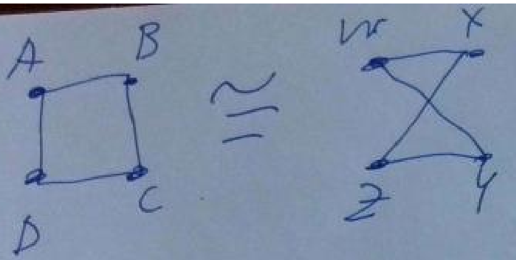
	e_1	e_2	e_3	e_4	e_5	e_6	e_7
A	1	1	1	0	0	0	0
B	1	0	0	1	0	0	1
C	0	1	0	0	0	1	1
D	0	0	0	0	1	1	0
E	0	0	1	1	1	0	0







DEF: Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic $G_1 \cong G_2$ iff (iff and only if) there exists a bijective (one-to-one/onto) map between V_1 & V_2 which preserves adjacencies.



Here's a bijective mapping from

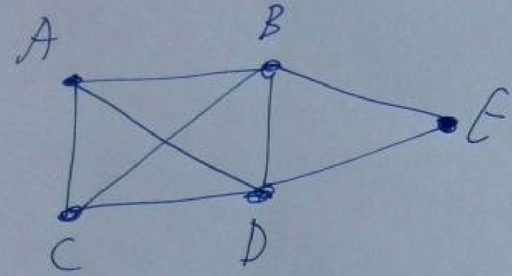
$\{A, B, C, D\} \to \{W, X, Y, Z\}$

$A \to W$ $A \to X$
$B \to X$ or $B \to Z$ or ... etc.
$C \to Z$ $C \to Y$
$D \to Y$ $D \to W$

An invariant is a property that's preserved under isomorphism, e.g. something true for any $\cong$ graph.

e.g. #vertices, #edges, adjacencies, collection of degrees of each vertex

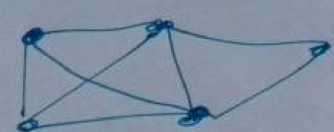
PATHS



Can you find a path that travels along each edge exactly once? (YES)

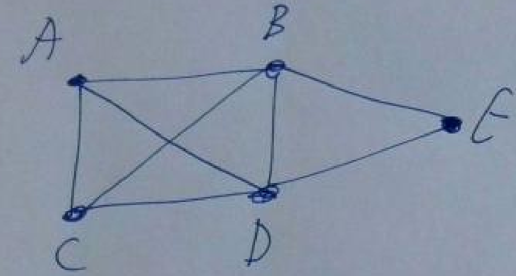
ABCDBEDAC

Such a path is called an Euler path.



If # of vertices with odd degree is 2 or 0, then there is an Euler path.

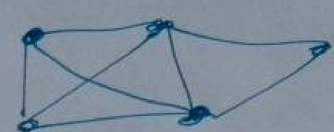
PATHS



Can you find a path that travels along each edge exactly once? (YES)

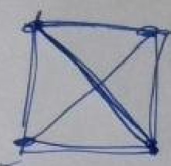
ABCDBEDAC

Such a path is called an Euler path.



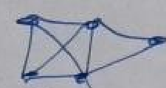
If # of vertices with odd degree is 2 or 0, then there is an Euler path.

A circuit is a path that starts & ends at the same vertex.



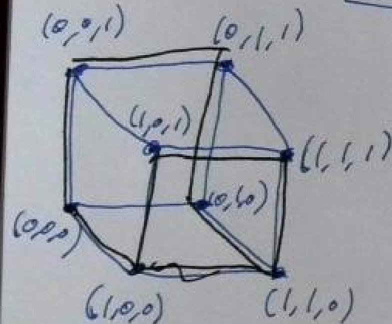
A Hamilton Path is a path that visits each vertex exactly once.

yes



yes

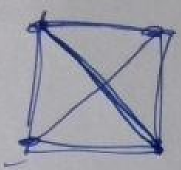
no



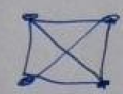
Hamilton path?

(0,0,0)
(0,0,1)
(0,1,1)
(0,1,0)
(1,1,0)
~~(1,0,0)~~ (1,1,1)
(1,0,1)
(1,0,0)
(0,0,0)

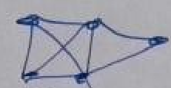
Gray Code



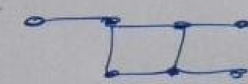
A Hamilton Path is a path that visits each vertex exactly once.



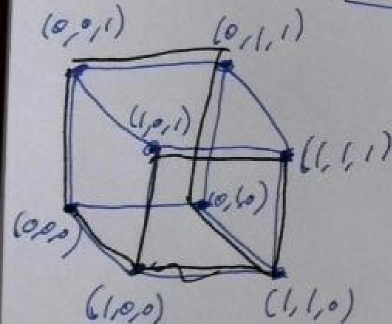
yes



yes



no



Hamilton path?

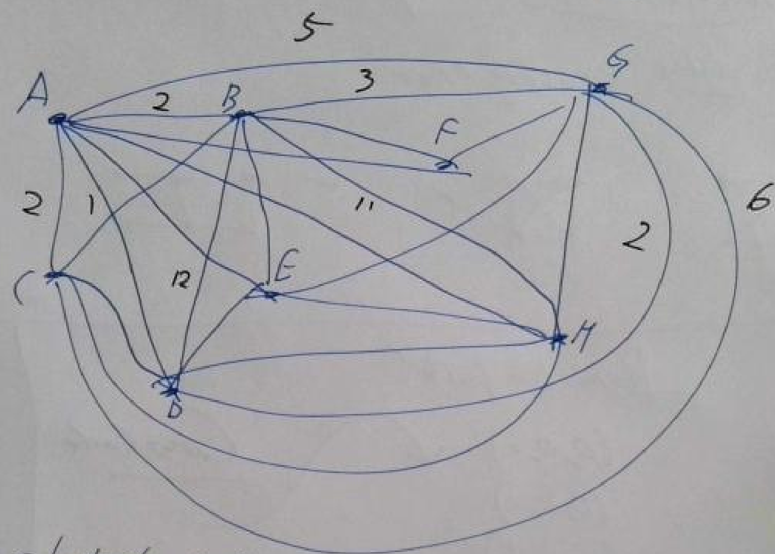
(0,0,0)
(0,0,1)
(0,1,1)
(0,1,0)
(1,1,0)
~~(1,0,0)~~ (1,1,1)
(1,0,1)
(1,0,0)
(0,0,0)

Gray Code

000
001
010
011
100
101
110
111
000

Not GRAY CODE

TRAVELING SALESPERSON



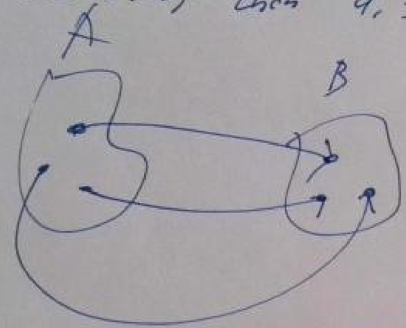
find a shortest path from (say) A to A
that visits every vertex at least once.

$\sim n^n$

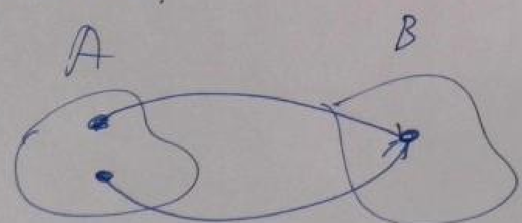
One-to-one map: $f: A \to B$

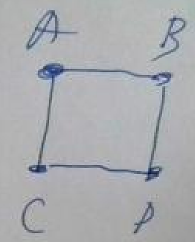
different a's map to different b's

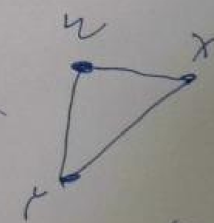
if $f(a_1) = f(a_2)$ then $a_1 = a_2$



not one-to one:





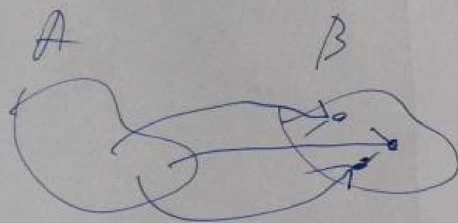


not one-to-one

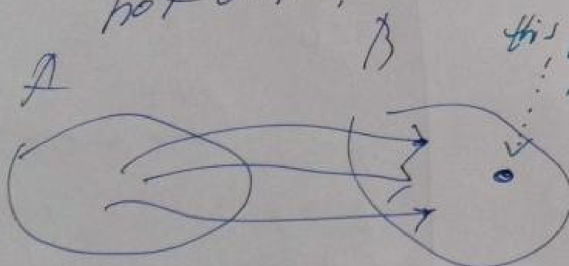
$A \to w$
$B \to x$
$C \to y$
$D \to ?\, w$

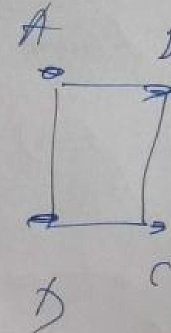
$f: A \to B$ is onto

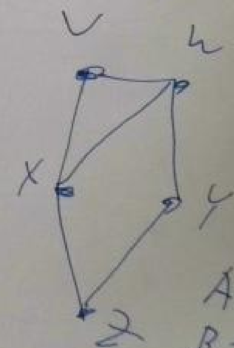
iff every $b \in B$ is mapped from something in A



not onto:







$A \to W$

$B \to W$

$C \to Y$

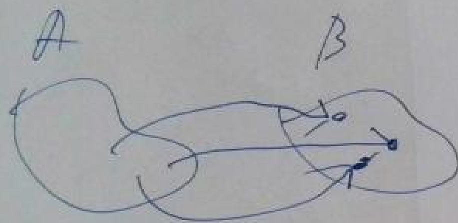
$D \to X$

$? \to Z$

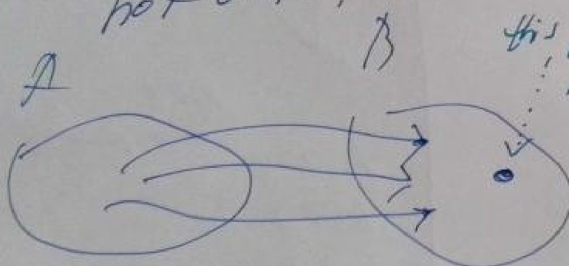
Not onto

$f: A \to B$ is <u>onto</u>

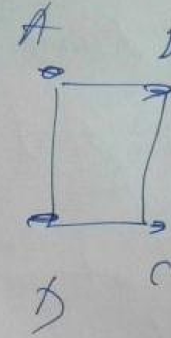
iff every $b \in B$ is mapped from something in A

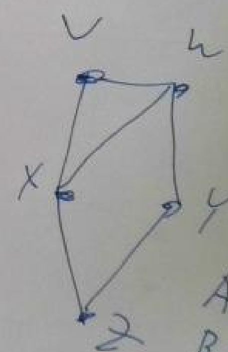


not onto:



this point has nothing mapping to it.

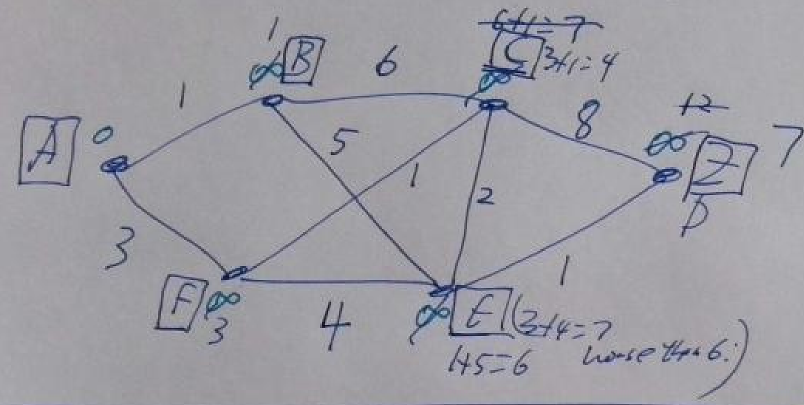




$A \to V$
$B \to W$
$C \to Y$
$D \to X$
$? \to Z$

Not onto

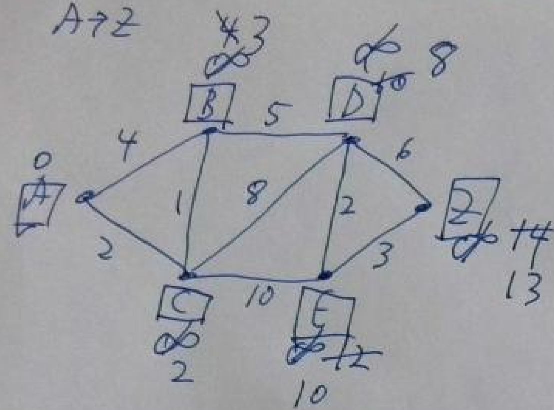
Dijkstra's Algorithm



- Write down tentative distances of ∞ at each vertex (except starting one: 0)
- "Visit" first node
- "relax" distance to all connected (unvisited) nodes
- Mark that node as done
- Visit an unvisited node with smallest tentative distance

if no nodes left, DONE

A→Z

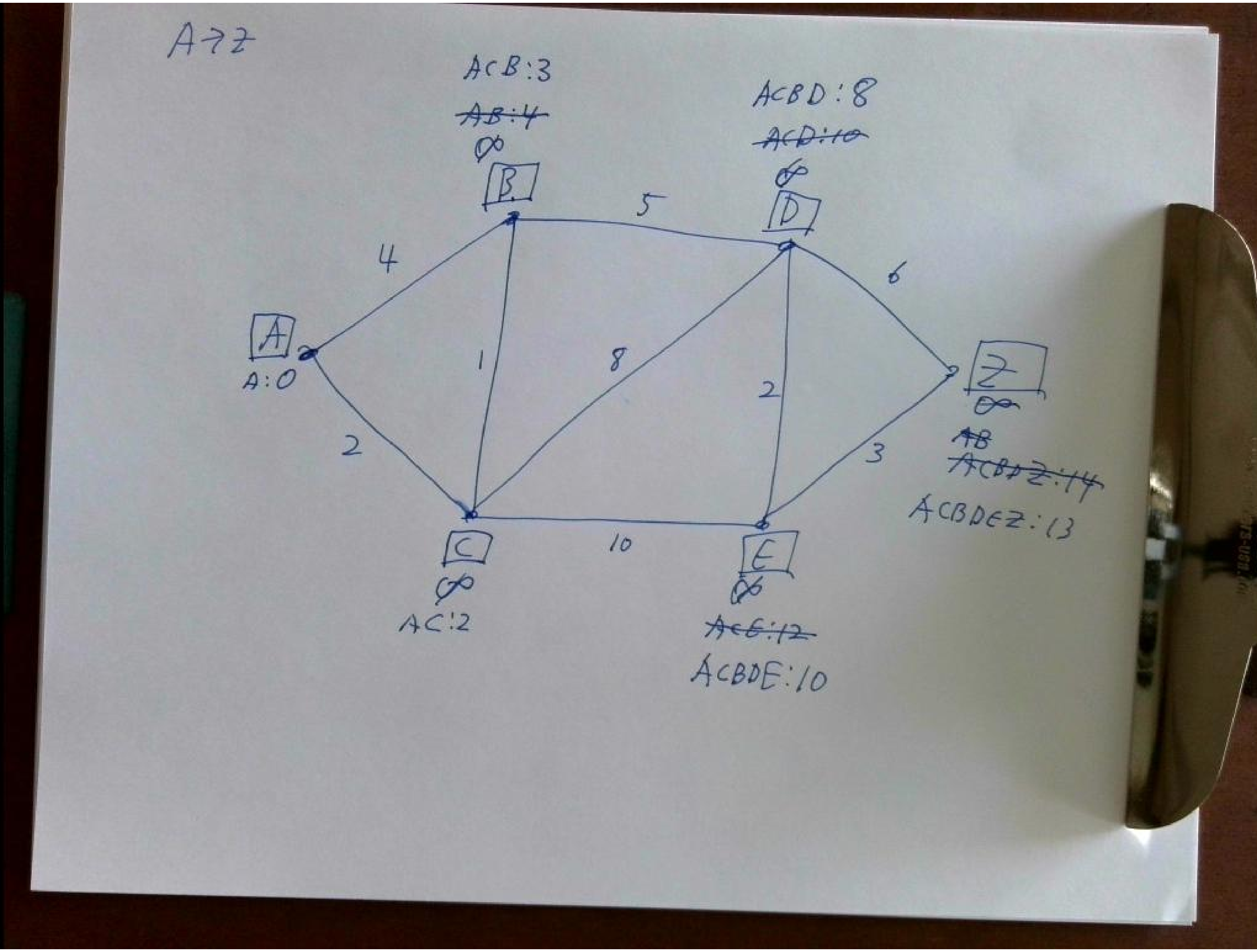


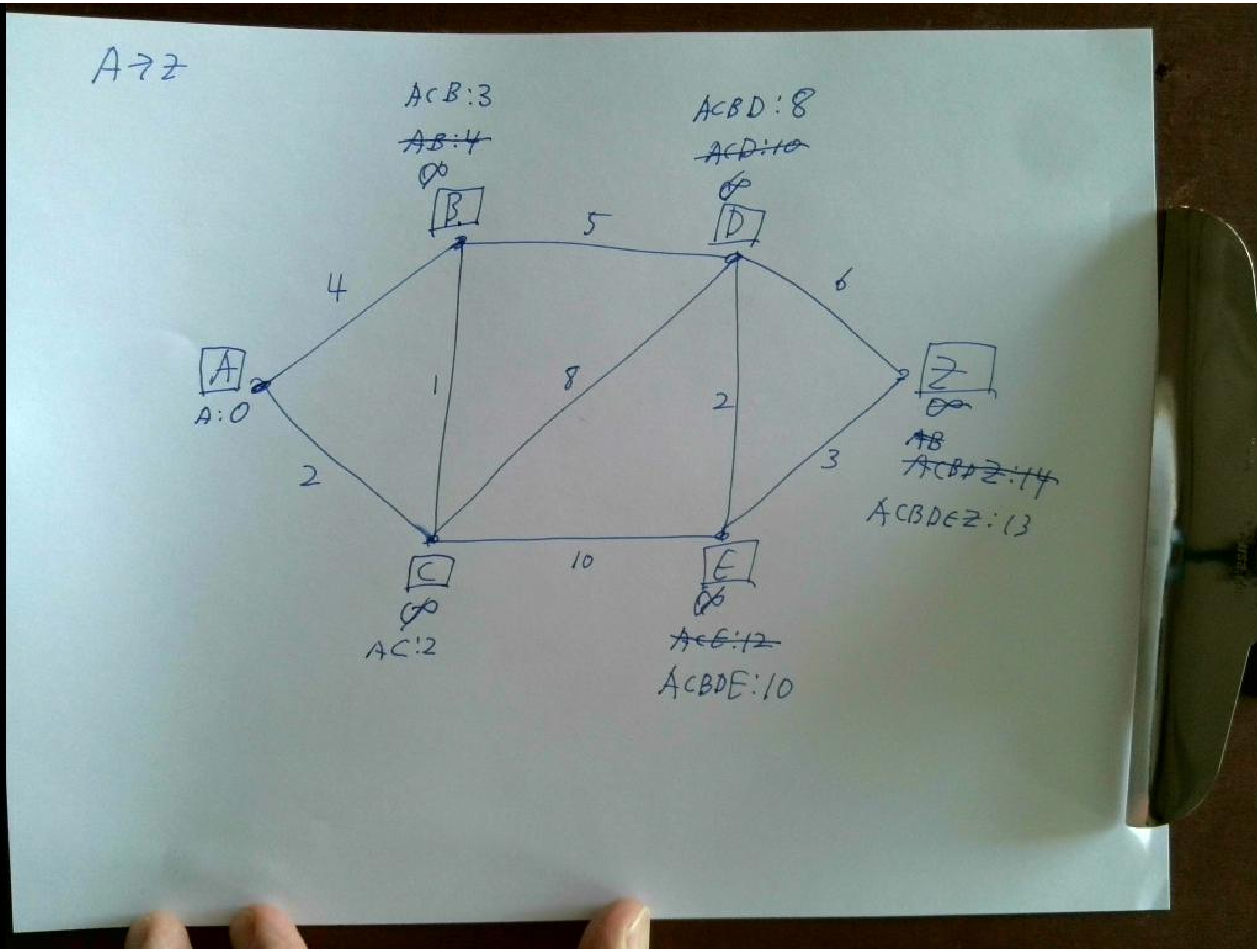
	A	B	C	D	E	Z
A		4	2			
B	4		1	5		
C	2	1		8	10	
D		5	8		2	6
E			10	2		3
Z				6	3	

VISITED?		TENT DIST
YES ~~NO~~	A	~~∞~~ 0
NO	B	~~∞~~ ~~6~~ ~~2~~ ~~4~~ 3
YES ~~NO~~	C	~~∞~~ 2
NO	D	~~∞~~ 10
NO	E	~~∞~~ 12
NO	Z	∞

CURRENT NODE

~~A~~ C


A→Z
ACB:3
~~AB:4~~
~~∞~~
B
ACBD:8
~~ACD:10~~
~~∞~~
D
4
5
6
A
A:0
1
8
2
Z
~~∞~~
~~AB~~
~~ACBDZ:14~~
ACBDEZ:13
2
3
10
C
~~∞~~
AC:2
E
~~∞~~
~~ACE:12~~
ACBDE:10



A→Z
ACB:3
~~AB:4~~
∞
B
5
ACBD:8
~~ACD:10~~
∞
D
6
4
A
A:0
1
8
2
Z
∞
~~AB~~
~~ACBDZ:14~~
ACBDEZ:13
3
2
C
∞
AC:2
10
E
∞
~~ACE:12~~
ACBDE:10