prove $p(n) \to p(n+1)$

1. $p(n)$

2. ~

~~3. $p(n+1)$~~

~~$1+2+\dots+n+(n+1) = \frac{(n+1)(n+2)}{2}$ $p(n+1)$~~

~~//Does not work~~

4. ~

5 ~

6 ~

7 $n^2 = n^2$

Let $p(n)$ be the prop

$1+2+\dots+n = \frac{n(n+1)}{2}$

prove $1+3+5+\dots+(2n-1)=n^2$ $\quad n \geq 1$

Let $P(n)$ be the prop $1+3+5+\dots+(2n-1)=n^2$

BASE CASE: $P(1)$ says $1=1^2$ TRUE

prove $P(n) \rightarrow P(n+1)$

1. $P(n)$ Hyp

2. $1+3+5+\dots+(2n-1)=n^2$ Def of $P(n)$

3. $1+3+5+\dots+(2n-1)+(2n+1)=\underbrace{n^2+(2n+1)}$ Alg, add $(2n+1)$ to both sides of 2.

4. $1+3+5+\dots+(2n-1)+(2n+1)=(n+1)^2$ alg

$\therefore P(n+1)$

Let $P(n)$ be the prop $3 \mid n(n+1)(n+2)$ $\quad n \geq 1$

Prove $P(n)$ $\forall n \geq 1$

BASE CASE: $P(1)$ says $3 \mid 1 \cdot 2 \cdot 3$ True

Prove $P(n) \to P(n+1)$

1. $P(n)$ Hyp

2. $3 \mid n(n+1)(n+2)$ Def of $P(n)$

2.5 $\exists k \in \mathbb{Z}: n(n+1)(n+2) = 3k$

3. $(n+1)(n+2)(n+3) = (n+1)(n+2)n + (n+1)(n+2)3$

$= 3k + 3(n+1)(n+2)$

$= 3(k + (n+1)(n+2))$ alg, sub, 2.5

4. $k + (n+1)(n+2) \in \mathbb{Z}$ closure, 2.5

5. $3 \mid (n+1)(n+2)(n+3)$ Def of "$\mid$"

$\therefore$ $P(n+1)$

Show $6 \mid n^3+5n$

$$(n+1)^3 + 5(n+1) =$$

$$n^3 + \boxed{3n^3 + 3n + 1} + 5n + \boxed{5} =$$

$$n^3 + 5n + \boxed{\underbrace{3n^2 + 3n}+6}$$

$$= 3(n^2+n) + 6$$

$$= 3(2j) + 6$$

$$= 6j + 6$$

$$= 6(j+1)$$

NOT A PROOF!

prove $n! > 2^n$ IGNORE $n \geq 4$

$n! > 2^n$

$(n+1)! = (n+1)n! \geq 5 \cdot n! > 2 \cdot n! > 2 \cdot 2^n = 2^{n+1}$

prove $n^2 > 4n+1$ IGNORE $n \geq 5$

$n^2 > 4n+1$

$(n+1)^2 = (n^2) + 2n + 1 > (4n+1) + 2n + 1 = 6n+2$

$= 4n + 2n + 2$

$\geq 4n + 12$

$> 4n + 5$

$= 4(n+1) + 1$

IG

$4(n+1) + 1 =$

$4n + 5$

How many n bit strings are there with no ~~[illegible]~~ consecutive 0's

$n=1$ 1, 0 $f(1)=2$

$n=2$ 11, 10, 01 $f(2)=3$

$n=3$ 010, 011, 101, 110, 111 $f(3)=5$

Let $f(n)$ = # of n-bit strings w/out consecutive 0's

$$\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_\ \cdots\ \_}_{f(n-1)}\ \underline{1}$$

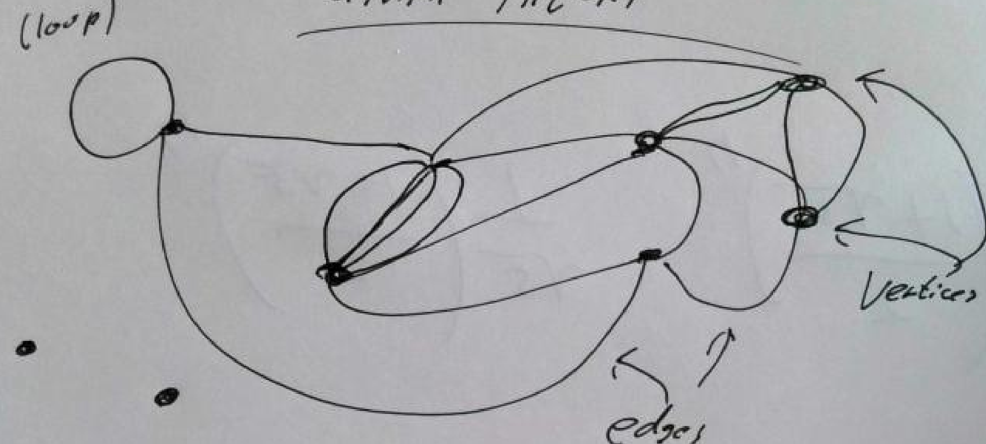
n bits

$$\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \cdots}_{\substack{n-2 \\ f(n-2)}}\ \underline{1}\ \underline{0}$$

$$f(n) = f(n-1) + f(n-2)$$

$$\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

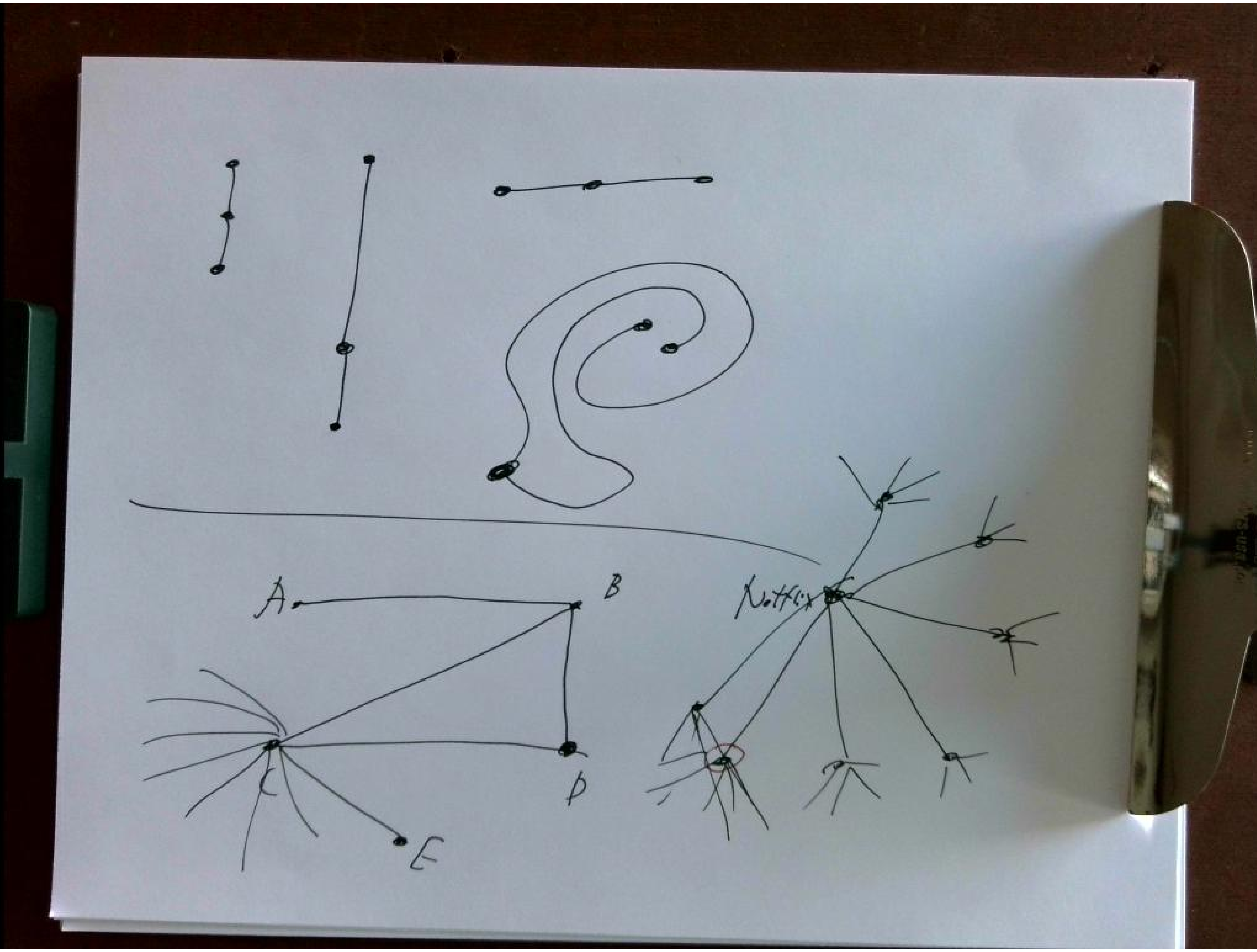
GRAPH THEORY

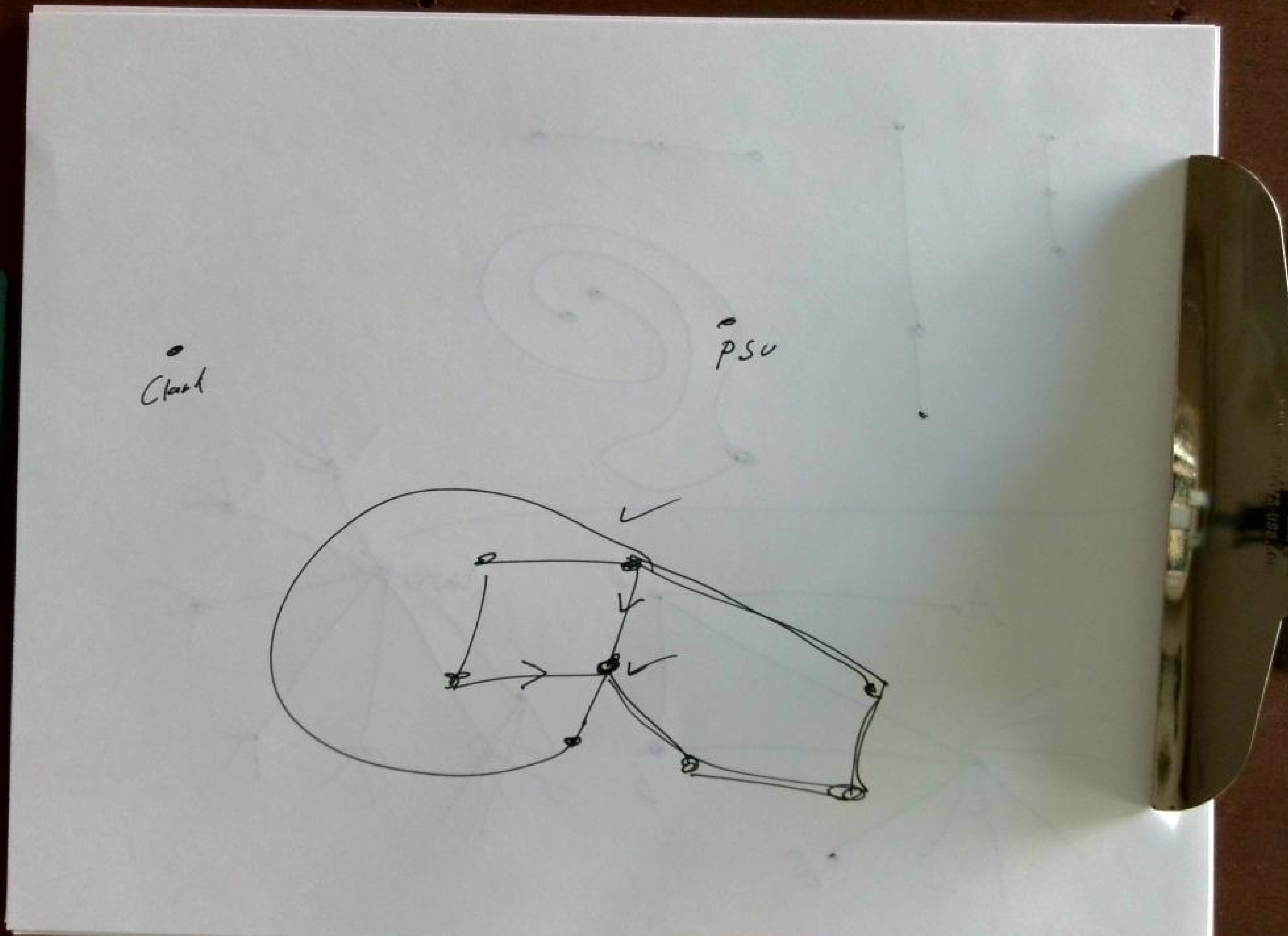


$G = (V, E)$

V = set of vertices $\neq \emptyset$

E = set of edges


A
B
C
D
E
Netflix


Clark
PSU

Graph $G = (V, E)$

vertices $\uparrow V$ edges $\uparrow E$

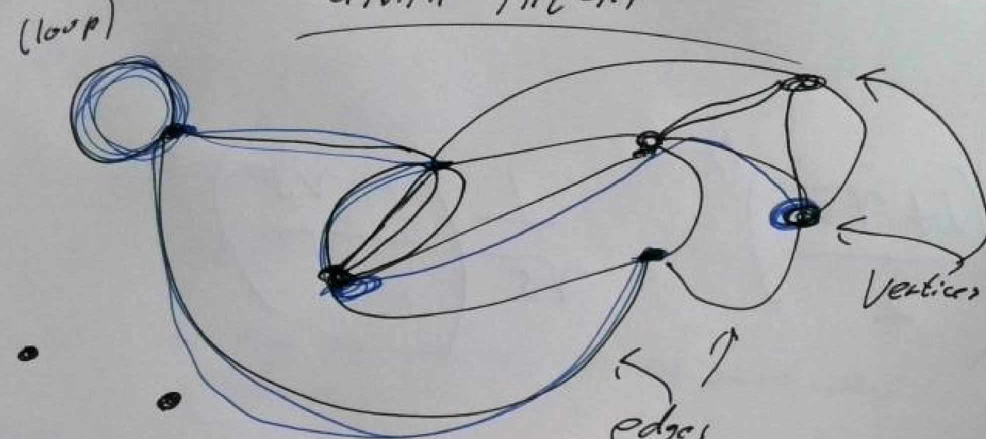
$V \neq \phi$

Simple: no more than one edge for any pair of vertices
also no loops

Path from $\underbrace{v_1 \text{ to } v_2}_{\text{vertices}}$ is a set of edges from v_1 to v_2

$\nexists$ Simple path: visits each vertex at most once

GRAPH THEORY



$G = (V, E)$

V = set of vertices $\neq \emptyset$
E = set of edges

Graph $G = (V, E)$

vertices $\uparrow V$, edges $\uparrow E$

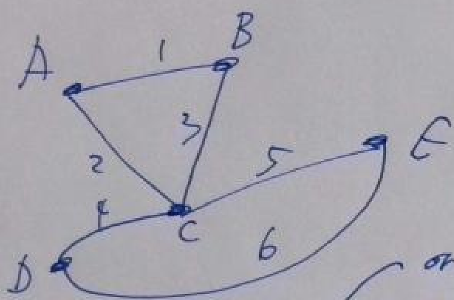
$V \neq \phi$

Simple: no more than one edge for any pair of vertices
also no loops

Path from $\underbrace{V_1 \text{ to } V_2}_{\text{vertices}}$ is a set of edges from V_1 to V_2

A Simple path visits each vertex at most once

A connected graph has a path from any vertex
to any other



or "node"

The degree of a vertex = # of edges connected to it.

$\deg(A) = 2$

$\deg(B) = 2$

$\deg(C) = 4$

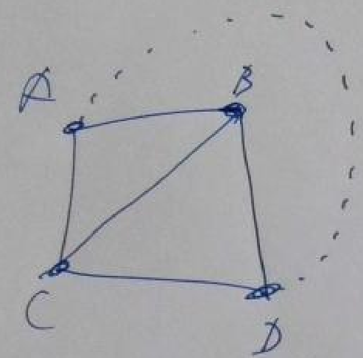
$\deg(D) = 2$

$\deg(E) = 2$

$12 = 2 \cdot 6$

In a graph $G = (V, E)$

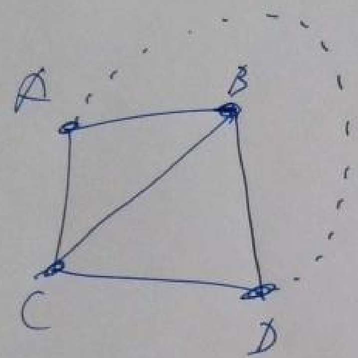
$$\sum_{i \in V} \deg(i) = 2|E|$$



$$\deg(A)+\deg(B)+\deg(C)+\deg(D) = 2\cdot 5 = 10$$

$$1+\deg(A) + \quad + 1+\deg(D)$$

$$2\cdot 6 = 12$$



$$\deg(A) + \deg(B) + \deg(C) + \deg(D) = 2 \cdot 5 = 10$$

$$1 + \deg(A) + \cdots + 1 + \deg(D)$$

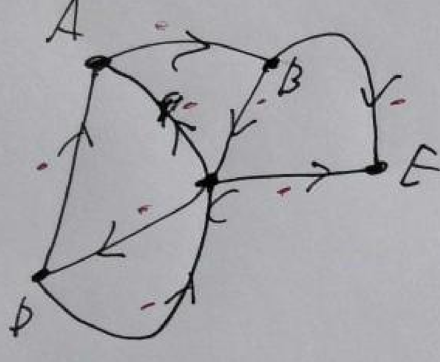
$$2 \cdot 6 = 12$$

Corollary: # of vertices with odd degree is even.

given n vertices with odd degree, $\sum \deg \equiv n \pmod 2$

$\equiv 0$

DIRECTED GRAPH



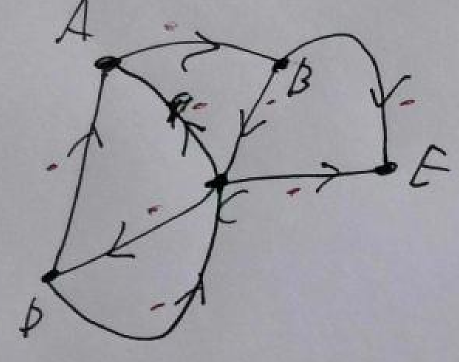
8 edges

indegree = # of edges going into a vertex = $\deg^-$

out degree = ~~~ out of a vertex = $\deg^+$

$\deg^-(A) = 2$	$\deg^+(A) = 1$
$\deg^-(B) = 1$	$\deg^+(B) = 2$
$\deg^-(C) = 2$	$\deg^+(C) = 3$
$\deg^-(D) = 1$	$\deg^+(D) = 2$
$\deg^-(E) = 2$	$\deg^+(E) = 0$
8	8

DIRECTED GRAPH



8 edges

(loops count twice)

indegree = # of edges going into a vertex = $\deg^-$

out degree = ~~~ out of a vertex = $\deg^+$

$\deg^-(A) = 2$ $\quad$ $\deg^+(A) = 1$

$\deg^-(B) = 1$ $\quad$ $\deg^+(B) = 2$

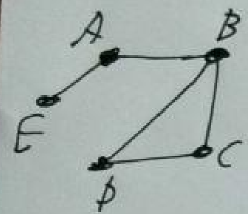
$\deg^-(C) = 2$ $\quad$ $\deg^+(C) = 3$

$\deg^-(D) = 1$ $\quad$ $\deg^+(D) = 2$

$\deg^-(E) = 2$ $\quad$ $\deg^+(E) = 0$

8 $\quad$ 8

STORING GRAPHS IN A COMPUTER



Adjacency Matrix

	A	B	C	D	E
A	0	1	0	0	1
B	1	0	1	1	0
C	0	1	0	1	0
D	0	1	1	0	0
E	1	0	0	0	0

Adj. List

A : B, E

B : A, C, D

C : B, D

D : B, C

E : A