prove $n(n+1)(n+2)$ is a mult. of 3 $(n \geq 1)$

(Can't use modular arith for this HW)

Let n be any integer

I. $n \equiv 0 \pmod 3$

II. $n \equiv 1$

III. $n \equiv 2$

I. $n(n+1)(n+2) \equiv 0(0+1)(0+2) \equiv 0$

II. $n(n+1)(n+2) \equiv 1(1+1)(1+2) \equiv 1 \cdot 2 \cdot 3 \equiv 6 \equiv 0$

III. $n(n+1)(n+2) \equiv 2(2+1)(2+2) \equiv 2 \cdot 3 \cdot 4 \equiv 24 \equiv 0$

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$6\ \cancel{5}! = 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1$

$2^6\ \cancel{2^5} = 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2$

3.4.1 Prove $1^3 + 2^3 + \dots + n^3 = \dfrac{n^2(n+1)^2}{4}$

Let $P(n)$ be the proposition: $1^3 + 2^3 + \dots + n^3 = \dfrac{n^2(n+1)^2}{4}$

$P(\boxed{n+1})$ is the prop: $1^3 + 2^3 + \dots + \boxed{n+1}^3 = \dfrac{\boxed{n+1}^2(\boxed{n+1}+1)^2}{4}$

$P(n+1)$ says: $\boxed{1^3 + 2^3 + \dots + (n-1)^3 + n^3 + (n+1)^3 = \dfrac{(n+1)^2(n+2)^2}{4}}$

Prove n^3-n is divisible by 3 $\forall n \geq 1$ using M.I.

Let $P(n)$ be the prop: n^3-n is div by 3

$$\exists i \in \mathbb{Z} : n^3-n = 3 \cdot i$$

BASE CASE:

$P(1)$ says 1^3-1 is div by 3. True

Inductive Step: prove $P(n) \rightarrow P(n+1)$

Inductive Step

proof:

1. $p(n)$ Hypothesis

2. $\exists i \in \mathbb{Z}: n^3 - n = 3i$ Def of $p(n)$

Want to Show

$p(n+1)$

$p(n+1)$ says $\exists j \in \mathbb{Z}: (n+1)^3 - (n+1) = 3j$

Inductive Step

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Want to Show

$p(n+1)$

$p(n+1)$ says $\exists j \in \mathbb{Z}: \underbrace{(n+1)^3 - (n+1)} = 3j$

$n^3 + 3n^2 + 3n + 1 - (n+1) =$

~~$n^3 + 3n^2 + 2n$~~ $n^3 + 3n^2 + 3n - n =$

$3n^2 + 3n + n^3 - n$

Inductive Step

proof:

1. $p(n)$ Hypothesis

2. $\exists i \in \mathbb{Z}: n^3 - n = 3i$ Def of $p(n)$

3. $(n+1)^3 - (n+1) =$
$n^3 + 3n^2 + 3n + 1 - n - 1 =$
$(3n^2 + 3n) + (n^3 - n) =$
$3(n^2+n) + 3i = 3(n^2+n+i)$ Alg, 2.

4. $n^2+n+i \in \mathbb{Z}$ 2, $n \in \mathbb{Z}$, closure of ints.

5. $3 \mid (n+1)^3 - (n+1)$ Def of $\mid$, 3, 4.

6. $p(n+1)$ Def of $p(n+1)$, 5.

$\therefore p(n) \rightarrow p(n+1)$

Prove n^3-n is divisible by 3 $\forall n \geq 1$ using M.I.

Let $P(n)$ be the prop: n^3-n is div by 3

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BASE CASE:

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Inductive Step: prove $P(n) \rightarrow P(n+1)$ True

$\therefore P(n)\ \forall n \geq 1$ Math. Ind.

QED

Prove $\sum_{i=1}^{n} 1 = n$ $\forall n \geq 1$

Let $p(n)$ be the prop $\sum_{i=1}^{n} 1 = n$ $p(1)$ True

$$\underbrace{1+1+\cdots+1}_{n \text{ of these}} = n$$

Prove $p(n) \to p(n+1)$

1. $p(n)$ Hyp

2. $\underbrace{1+1+\cdots+1}_{n \text{ of these}} = n$ Def of $p(n)$

3. $\underbrace{1+1+\cdots+1}_{n \text{ of these}} + 1 = n+1$

4. $p(n+1)$

Prove $\sum_{i=1}^{n} 1 = n$ $\quad \forall n \geq 1$

Let $p(n)$ be the prop $\sum_{i=1}^{n} 1 = n$ $\quad$ $P(1)$ True

$$\underbrace{1+1+\cdots+1}_{n \text{ of these}} = n$$

Prove $p(n) \to p(n+1)$

1. $p(n)$ $\quad$ Hyp
2. $\underbrace{1+1+\cdots+1}_{n \text{ of these}} = n$ $\quad$ Def of $p(n)$
3. $\underbrace{1+1+\cdots+1}_{n \text{ of these}} + 1 = n+1$ $\quad$ alg
4. $P(n+1)$

$P(1)$ $P(n) \to P(n+1)$

$P(1)$

$P(1) \to P(2)$

$P(2)$

$P(2) \to P(3)$

$P(3)$

ODD PIE FIGHTS

n people, n pies,

each person throws their pie at the* person closest to them.

*Distances must be unique

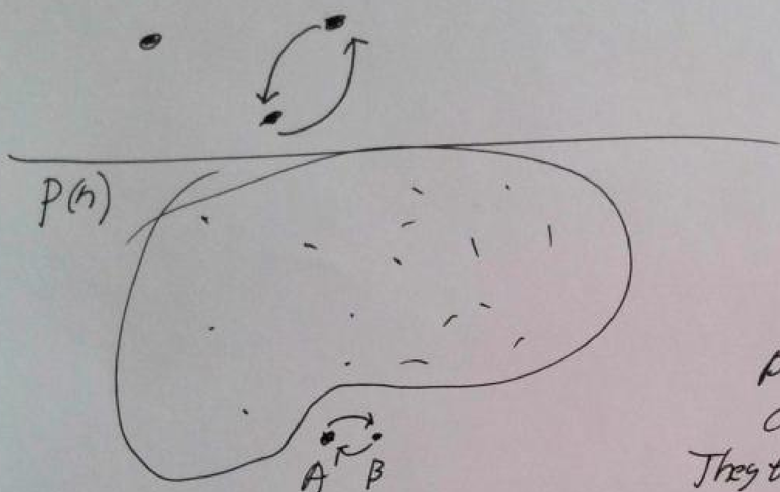
Theorem: If n is odd, ≥ 3, there is at least one survivor.

Let $P(n)$ be the prop: there is a survivor in an n-person pie fight.

Prove $P(n)$ $\forall n \geq 3$, n odd.

BASE CASE: Prove $P(3)$

INDUCTIVE STEP: Prove $P(n) \rightarrow P(n+2)$



$n+2$ people

pick the 2 people closest to each other, They throw at each other.

Possibility 1: a survivor throws at A or B

n survivors
n pies

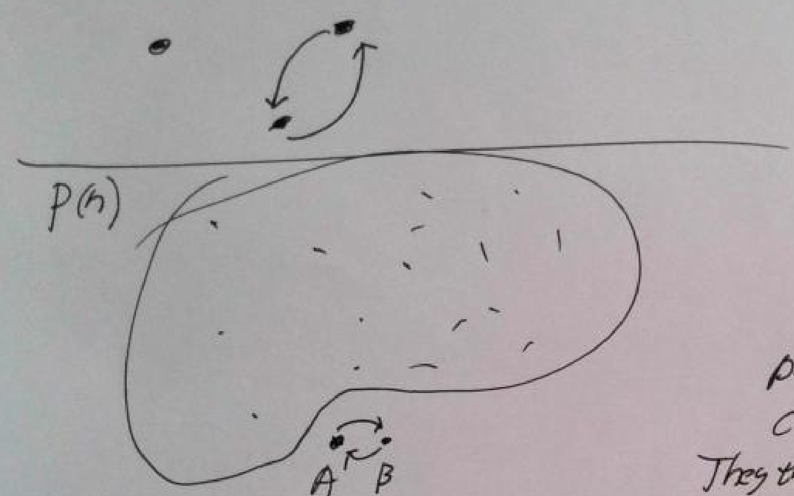
n survivors
$n-1$ pies $\therefore$ Someone survives

Possibility 2: No survivor throws at A or B

n people
n pies

by $P(n)$, Someone survives

$\therefore P(n+2)$



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n people
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by $p(n)$, Someone survives

$\therefore p(n+2)$

Prove all horses are the same color.

Let $p(n)$ be the proposition: in any group of n horses, they are all the same color.

$P(1)$ True

Prove $p(n) \to p(n+1)$

1. $p(n)$ Hyp

2. any group of n horses are all the same color Def $p(n)$

1 2 3 4 . . . n-1 n n+1

Prove all horses are the same color.

Let $p(n)$ be the proposition: in any group of n horses, they are all the same color.

$P(1)$ True

Prove $p(n) \to p(n+1)$

1. $p(n)$ Hyp

2. any group of n horses are all the same color Def $p(n)$

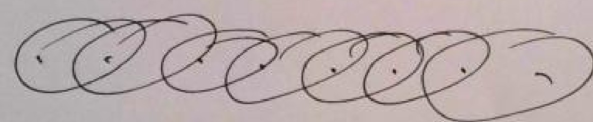
3. $p(n+1)$

$\therefore p(n)\ \forall n \geq 1$

1 2 3 4 . . . $n-1$ n $n+1$ $n>1$

$p(n) \to p(n+1)$ $n>1$

$\boxed{1}$ $\boxed{2}$



Need to use $p(2)$ as a base case

Prove $n = n+1$

Let $p(n)$ be the prop: $n = n+1$

prove $p(n) \Rightarrow p(n+1)$

1. $p(n)$ hyp

2. $n = n+1$ Def of $p(n)$

3. $n+1 = n+2$ add 1 to both sides of prop 2.

4. $p(n+1)$

$\therefore p(n) \Rightarrow p(n+1)$

But: don't have a base case

COUNTING

Product Rule: Given a procedure P that can be accomplished by first doing a procedure P_1 and then a procedure P_2, if P_1 can be done n_1 different ways, and P_2 can be done n_2 different ways, then

P can be done $n_1 \cdot n_2$ different ways.

Ex: 2 people, 30 seats; how many ways can they be seated?

P: both students choose their seat. $n = 30 \cdot 29 = 870$

P_1: Student 1 picks a seat $\quad n_1 = 30$

P_2: Student 2 picks a seat. $\quad n_2 = 29$

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A student ID = 1 letter (A-Z) + 1 digit (0-9)

How many student IDs are there?

P is the procedure "Create a student ID" $n = 26 \cdot 10 = \underline{260}$

P_1: Pick a letter (A-Z) $n_1 : 26$

P_2: pick a digit (0-9) $n_2 : 10$

LUNCH: Soup + Salad 35

5 7

P P_1 P_2 P_3

$\uparrow$ n_1 n_2 n_3

$n_1 n_2 n_3$

Soap, salad, drink

5 7 50

P_1: order Soap & Salad (5.7)

P_2: order drink (50)

5.7.50

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P P_1 P_2 P_3 P_4

$\uparrow$ n_1 n_2 n_3 n_4

$n_1 n_2 n_3 n_4$

Soap, salad, drink

5 7 50

P_1: order Soap & Salad (5.7)

P_2: order drink (50)

5.7.50

How many n-bit numbers are there?

$$\underbrace{\_\ \ \_\ \ \_\ \ \_\ \ \cdots\ \ \_\_}_{n}$$

$$\underbrace{2 \times 2 \times 2 \times 2 \times \cdots \times 2}_{n} = 2^n$$

Shuffle a deck of 52 cards

How many different orders can we have?

P_1 : pick a card for bottom of the deck $\quad n_1 : 52$

P_2 : pick a card to put in 2^{nd} position from bottom $\quad n_2 : 51$

P_3 : ——— 3^{rd} ——— $\quad n_3 : 50$

$n_4 : 49$

$\vdots$

P_{52} : pick a card for the top $\quad n_{52} : 1$

$$52 \cdot 51 \cdot 50 \cdots 2 \cdot 1 = 52!$$

How many 3-digit #s are there?

050

Let S be a finite set $\qquad S = \{s_1, s_2, s_3, \dots, s_n\}$

What is $|\mathcal{P}(S)|$?

How many subsets of S are there?

P: To create a subset of S:

P_1: Decide whether to include s_1 $\qquad n_1 = 2$

P_2: ______ s_2 $\qquad$ ~~S~~ $n_2 = 2$

$\vdots$ $\qquad \vdots$

P_n: ______ s_n $\qquad n_n = 2$

$\underbrace{2 \cdot 2 \cdot \cdots \cdot 2}_{n} = 2^n$ $\qquad$ so $|\mathcal{P}(S)| = 2^{|S|}$