$P \to Q$

Converse: $Q \to P$

inverse: $\neg P \to \neg Q$

Contrapositive: $\neg Q \to \neg P$

Let P be the prop: it is raining

Let Q be the prop: I'm sleeping

$P \to Q$: If it is raining then I'm sleeping.

$Q \to P$: If I'm sleeping then it is raining.

$\neg P \to \neg Q$: If it's not raining then I'm not sleeping

$\neg Q \to \neg P$: If I'm not sleeping then it's not raining.

$$P \to Q$$

Converse: $Q \to P$

inverse: $\neg P \to \neg Q$

Contrapositive: $\neg Q \to \neg P$

Let P be the prop: it is raining

Let Q be the prop: I'm sleeping

$P \to Q$: If it is raining then I'm sleeping.

$Q \to P$: If I'm sleeping then it is raining.

$\neg P \to \neg Q$: If it's not raining then I'm not sleeping

$\neg Q \to \neg P$: If I'm not sleeping then it's not raining.

It is raining therefore I'm sleeping. $\neq P \to Q$

Prove: $\overline{A \cap B} = \bar{A} \cup \bar{B}$

A	B	$A \cap B$	$\overline{A \cap B}$	$\bar{A}$	$\bar{B}$	$\bar{A} \cup \bar{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

$p \to q$
$r \wedge \neg s$
$p \vee \neg r$

---

q

proof:

1.	$r \wedge \neg s$	hyp
2.	r	Conj. simpl. 1.
3.	$p \vee \neg r$	hyp
4.	p	disj. simpl. 2.3.
5.	$p \to q$	hyp
6.	q	Dir. reas, 4,5

QED

proof:

1.	$\neg q$	Neg. of conc.
2.	$p \to q$	hyp
3.	$\neg p$	Ind. Reas, 1,2
4.	$p \vee \neg r$	hyp
5.	$\neg r$	Disj. simpl. 3.4.
6.	$r \wedge \neg s$	hyp
7.	r	Conj. simp. 6
	⨳	5,7

$\therefore q$

QED

$p \to q$

$r \wedge \neg s$

$p \vee \neg r$

---

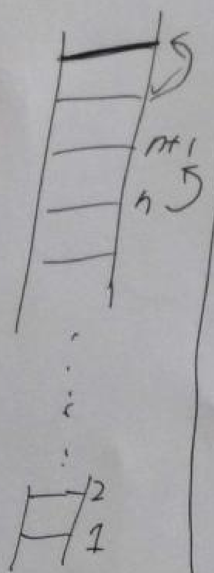
q

proof:

1. $p \vee \neg r$ hyp
2. $r \to p$ Cond. equiv. 1.
3. $p \to q$ hyp
4. $r \to q$ Chain rule, 2, 3
5. $r \wedge \neg s$ hyp
6. r Conj. simp. 5
7. q dir. reas, 4, 6

QED

Induction & Recursion



Let $p(n)$ be a family of propositions, $n \geq 1$

Given the following:

(1) $p(1)$ is true

(2) $p(n) \to p(n+1)$ $\forall n \geq 1$

Then we know:

$p(n)$ is true $\forall n \geq 1$

$$\frac{\begin{array}{l} p(1) \\ p(n) \to p(n+1) \ \forall n \geq 1 \end{array}}{p(n) \ \forall n \geq 1}$$

buildSet (input)

output = ϕ

for each char in input:

    if char is not in output:

        add char to output

---

input: hello

output: helo

char = ~~h~~~~e~~~~l~~~~l~~o

```
Union (A, B)
--------------------------------
output = A
for each char in B:
    if char is not in output:
        add char to output
--------------------------------
intersect (A, B)
output = φ
for each char in A:
    if char is in B:
        add char to output
```

```
join(array1, array2)
    len1, len2
```

```
outloc=0
output= empty
for i=0 to len1-1:                or output[outloc++]

    output[outloc]= array1[i]
    outloc=outloc+1
```

```
for i=0 to len2 -1
    output[outloc]= array2[i]
    outloc++ //outloc=outloc+1
```

Union (A, B)

output = A

for each char in B:

    if char is not in ~~B~~ output:

        add char to output

intersect (A, B)

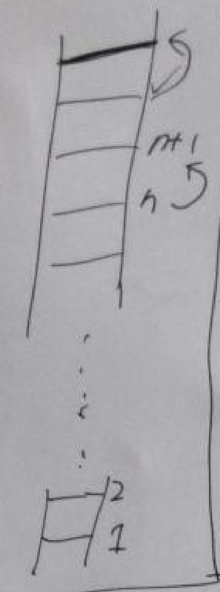
output = ϕ

for each char in A:

    if char is in B:

        add char to output

Induction & Recursion



Let $p(n)$ be a family of propositions, $n \geq 1$

Given the following:

(1) $p(1)$ is true

(2) $p(n) \rightarrow p(n+1)$ $\forall n \geq 1$

Then we know:

$p(n)$ is true $\forall n \geq 1$

$$\boxed{\frac{p(1) \qquad p(n) \rightarrow p(n+1)\ \forall n \geq 1}{p(n)\ \forall n \geq 1}}$$

"BASE CASE"

"INDUCTIVE STEP"

Theorem: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad (n \geq 1)$

1
2
3
4
5
6
7
8
9
10

Let $P(n)$ be the proposition

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

Prove $P(n)$ $\forall n \geq 1$

Proof (by mathematical induction)

I. $P(1)$ is the proposition: $1 = \frac{1(1+1)}{2} = \frac{1 \cdot 2}{2} = \frac{2}{2} = 1$

$P(1)$ is true ✓

II (Inductive Step)

Prove $p(n) \to p(n+1) \quad \forall n \geq 1$

Proof:

1. $p(n)$ Hypothesis

2. $1+2+3+\cdots+n = \frac{n(n+1)}{2}$ Def of $p(n)$

$= \frac{n(n+1)}{2}$

3. $\underbrace{1+2+3+\cdots+n}_{} + (n+1) =$ Subs, 2.

$\frac{n(n+1)}{2} + (n+1) =$

$\frac{n^2+n}{2} + (n+1) = \frac{n^2+n+2(n+1)}{2}$ Algebra

$= \frac{n^2+3n+2}{2} = \frac{(n+2)(n+1)}{2}$

IGNORE

want to prove

$p(n+1)$

$p(n+1)$ says;

$1+2+3+\cdots+n+(n+1) =$

$\frac{(n+1)(n+1+1)}{2}$

$\frac{\quad}{2}$ def of $p(n)$

$= \frac{n(n+1)}{2}$

3. $\underbrace{1+2+3+\ldots+n}_{}+(n+1) =$ Subs, 2.

$\frac{n(n+1)}{2} + (n+1) =$

$\frac{n^2+n}{2} + (n+1) = \frac{n^2+n+2(n+1)}{2}$ Algebra

$= \frac{n^2+3n+2}{2} = \frac{(n+2)(n+1)}{2}$

$1+2+3+\ldots+n+(n+1) =$

$\frac{(n+1)(n+1+1)}{2}$

$\therefore p(n+1)$

QED

$\therefore p(n) \; \forall n \geq 1$ Mathematical Induction

QED

Prove $1+2+4+8+\dots+2^n = 2^{n+1}-1 \quad \forall n \geq 1$

Let $P(n)$ be the prop: $1+2+\dots+2^n = 2^{n+1}-1$
$2^0+2^1+\dots+2^n$

Base Case: Prove $P(1)$

$P(1)$ is the prop: $1+2 = 2^{1+1}-1$ TRUE ✓

Inductive Step: prove $P(n) \to P(n+1), \forall n \geq 1$

1. $P(n)$ Hyp
2. $1+2+4+\dots+2^n = 2^{n+1}-1$ Def of $P(n)$

Prove $1+2+4+8+\cdots+2^n = 2^{n+1}-1 \quad \forall n \geq 1$

Let $p(n)$ be the prop: $1+2+\cdots+2^n = 2^{n+1}-1$

$2^0+2^1+\cdots+2^n$

Base Case: Prove $p(1)$

$p(1)$ is the prop: $1+2 =$

Inductive Step: prove $p(n) \to p(n+$

1. $p(n)$ Hyp
2. $1+2+4+\cdots+2^n = 2^{n+1}-$

IGNORE

Show

$1+2+\cdots+2^n+2^{n+1}$

$=$

$2^{(n+1)+1}-1$

Prove $1+2+4+8+\cdots+2^n = 2^{n+1}-1 \quad \forall n \geq 1$

Let $p(n)$ be the prop: $1+2+\cdots+2^n = 2^{n+1}-1$

$2^0+2^1+\cdots+2^n$

Base Case: Prove $P(1)$

$P(1)$ is the prop: $1+2 = 2^{1+1}-1$ TRUE ✓

Induction Step: prove $p(n) \to p(n+1), \forall n \geq 1$

1. $P(n)$ Hyp

2. $1+2+4+\cdots+2^n = 2^{n+1}-1$ Def of $p(n)$

3. $1+2+4+\cdots+2^n+\boxed{2^{n+1}} = 2^{n+1}-1+\boxed{2^{n+1}}$ add 2^{n+1} to both sides of 2.

$= 2\cdot 2^{n+1}-1 = 2^{n+2}-1$ alg.

4. $p(n+1)$ def of $p(n)$, 3.

$\therefore p(n) \to p(n+1)$

$\therefore p(n) \; \forall n \geq 1$ M.I.

QED

Prove $1+2+4+8+\cdots+2^n = 2^{n+1}-1 \quad \forall n \geq 1$

Let $P(n)$ be the prop: $1+2+\cdots+2^n = 2^{n+1}-1$

$2^0+2^1+\cdots+2^n$

Base Case: Prove $P(1)$

$P(1)$ is the prop: $1+2 = 2^{1+1}-1$ TRUE ✓

Inductive Step: prove $P(n) \to P(n+1), \forall n \geq 1$

1. $P(n)$ Hyp

2. $1+2+4+\cdots+2^n = 2^{n+1}-1$ Def of $P(n)$

3. $1+2+4+\cdots+2^n + \boxed{2^{n+1}} = 2^{n+1}-1+\boxed{2^{n+1}}$ add 2^{n+1} to both sides of 2.

$= 2\cdot 2^{n+1}-1 = 2^{n+2}-1$ alg.

4. $P(n+1)$ def of $P(n)$, 3.

$\therefore P(n) \to P(n+1)$

$\therefore P(n) \; \forall n \geq 1$ M.I.

QED

know: $\overline{A \cup B} = \bar{A} \cap \bar{B}$

also: $\underbrace{\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C}}$ and $\overline{A \cup B \cup C \cup D} = \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$

$\overline{A \cup B \cup C} =$

$$\overline{(A \cup B) \cup C} = \overline{A \cup B} \cap \bar{C}$$

$$= \bar{A} \cap \bar{B} \cap \bar{C}$$

$\overline{A \cup B \cup C \cup D} =$

$$\overline{(A \cup B \cup C) \cup (D)} = \overline{A \cup B \cup C} \cap \bar{D}$$

$$= \bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$$

Prove $\overline{A_1 \cup A_2 \cup \ldots \cup A_n} = \overline{A_1} \cap \overline{A_2} \cap \ldots \overline{A_n}$ $\forall n \geq 2$

Let $P(n)$ be the prop: $\overline{A_1 \cup A_2 \cup \ldots \cup A_n} = \overline{A_1} \cap \overline{A_2} \cap \ldots \cap \overline{A_n}$

BASECASE: $P(2)$ is the prop $\overline{A_1 \cup A_2} = \overline{A_1} \cap \overline{A_2}$

True (de morgan's law)

Inductive step: prove $P(n) \to P(n+1)$ $n \geq 2$

Proof:

1. $P(n)$ hyp

2. $\overline{A_1 \cup A_2 \cup \ldots \cup A_n} = \overline{A_1} \cap \overline{A_2} \cap \ldots \cap \overline{A_n}$ Def of $P(n)$

3. $\overline{A_1 \cup A_2 \cup \ldots \cup A_n \cup A_{n+1}} = \overline{(A_1 \cup A_2 \cup \ldots \cup A_n) \cup A_{n+1}}$

$= \overline{A_1 \cup A_2 \cup \ldots \cup A_n} \cap \overline{A_{n+1}}$ DeMorgan

$= \overline{A_1} \cap \overline{A_2} \cap \ldots \cap \overline{A_n} \cap \overline{A_{n+1}}$ 2. Subs.

4. $P(n+1)$ def of $P(n+1)$, & 3.

$\therefore P(n) \rightarrow P(n+1)$

Prove $\overline{A_1 \cup A_2 \cup \ldots \cup A_n} = \overline{A_1} \cap \overline{A_2} \cap \ldots \overline{A_n}$ $\forall n \geq 2$

Let $p(n)$ be the prop: $\overline{A_1 \cup A_2 \cup \ldots \cup A_n} = \overline{A_1} \cap \overline{A_2} \cap \ldots \cap \overline{A_n}$

BASECASE: $p(2)$ is the prop $\overline{A_1 \cup A_2} = \overline{A_1} \cap \overline{A_2}$

True (de morgan's law)

Inductive step: prove $p(n) \rightarrow p(n+1)$ $n \geq 2$

$\therefore$ $p(n)$ $\forall n \geq 2$ M.I.

QED