Prove the following

If you send me an email, then I'll do my homework.

If you don't send me an email, then I'll go to sleep early.

If I go to sleep early, then I'll wake up happy.

Therefore, if I don't do my homework then I'll wake up happy.

Let E be the prop "you send me an email"
Let W be the prop "I'll do my Homework"
Let S be the prop "I'll go to sleep early"
Let H be the prop "I'll wake up happy"

$$\begin{array}{l} E \to W \\ \neg E \to S \\ S \to H \\ \hline \neg W \to H \end{array}$$

Prove: $E \to W$

$\cancel{\neg S \to}$

$\neg E \to S$

$S \to H$

$\neg W \to H$

Proof:

1.	$E \to W$	Hypothesis
2.	$\neg W \to \neg E$	Contrapositive 1.
3.	$\neg E \to S$	Hypothesis
4.	$\neg W \to S$	Chain rule, 2, 3.
5.	$S \to H$	Hypothesis
6.	$\neg W \to H$	Chain rule, 4. 5.

Q.E.D.

IGNORE

Fallacies

$$\begin{array}{l} P \to Q \\ Q \\ \hline P \end{array}$$

"fallacy of affirming the conclusion"

If I win the lottery then I'll eat dinner tonight.

I'll eat dinner tonight.

$$\begin{array}{l} P \to Q \\ \neg P \\ \hline \neg Q \end{array}$$

"fallacy of denying the hypothesis"

PROOFS

$$\begin{array}{c} A \\ B \\ C \\ \hline X \end{array}$$

Direct Proof: Assume A, B, C

try to prove X.

Indirect proof: Assume A, B, C

Assume $\neg X$

Try to prove something ridiculous.

"proof by contradiction"

Def: An integer n is <u>even</u> if $\exists k \in \mathbb{Z} : n = 2k$

n is <u>odd</u> if $\exists k \in \mathbb{Z} : n = 2k+1$

Note: n can't be even and odd

every int is <u>either</u> even or odd

Prove: If n is even, ~~so is~~ then $7*n$ is even.

Proof:

1. n is even — Hypothesis
2. $\exists k \in \mathbb{Z} : n = 2k$ — Def. of even, 1.
3. $7n = 7(2k) = 14k = 2(7k)$ — 2., algebra
4. $7k \in \mathbb{Z}$ — 2., Closure of integers
5. $7n$ is even — 3., 4., Def of even.

Q.E.D.

IGNORE

want to show $7*n$ is even

Show $\exists j \in \mathbb{Z} : 7n = 2j$

Prop: If n is even then n^2 is even.

Proof:

1. n is even — Hypothesis
2. $\exists i \in \mathbb{Z}: n = 2 \cdot i$ — 1., def of even
3. $n^2 = n \cdot n = (2i)(2i)$ — 2, algebra
   $= 4i^2 = 2(2i^2)$
4. $2i^2 \in \mathbb{Z}$ — 2, closure of integers
5. n^2 is even — 3., 4., def of even

QED

Prove: If n^2 is even then n is even

1. n^2 is even — Hyp.

2. $\exists k \in \mathbb{Z} : n^2 = 2k$ — 1., def of even

3. $n^2 = n \cdot n$

4. $n = \sqrt{2k} = \sqrt{2}\sqrt{k}$

5. $n^2 = \sqrt{2}\sqrt{k}\sqrt{2}\sqrt{k}$

?

Prop: If n^2 is even then n is even

Proof (indirect):

1.	n^2 is even	Hypothesis
2.	n is not even	Negation of conclusion
3.	n is odd	Properties of even & odd
4.	$\exists k \in \mathbb{Z} : n = 2k+1$	3. Def of odd
5.	$n^2 = (2k+1)^2 = 4k^2+4k+1 = 2(2k^2+2k)+1$	Algebra
6.	$2k^2+2k \in \mathbb{Z}$	Closure of ints, 4.
7.	n^2 is odd	6, 5, Def of odd
8.	$\times$	1, 7

$\therefore$ n is even

Q.E.D.

IGNORE

WTS

$4k^2+4k+1 =$

$2(\text{int})+1$

Prove: If n^2 is even then n is even

Proof (indirect):

1.	n^2 is even	Hypothesis
2.	n is not even	Negation of conclusion
3.	n is odd	Properties of even & odd
4.	$\exists k \in \mathbb{Z} : n = 2k+1$	3. Def of odd
5.	$n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2+2k)+1$	Algebra, 4.
6.	$2k^2+2k \in \mathbb{Z}$	Closure of ints, 4.
7.	n^2 is odd	6, 5, Def of odd
8.		1, 7

$\therefore$ n is even

Q.E.D.

IGNORE

WTS

$4k^2+4k+1 =$

$2(\text{int})+1$

Prove: if n^2 is odd then n is odd.

Proof (indirect)

1.	n^2 is odd	Hypothesis
2.	n is even	Negation of conclusion
3.	n^2 is even	Previously proven
	⨳	1, 3

∴ n is odd

QED

⨳ "contradiction"

∴ "therefore"

Prove: if n^2 is odd then n is odd.

Proof (indirect)

1. n^2 is odd T Hypothesis
2. n is even T Negation of conclusion
3. n^2 is even F Previously proven

1, 3

$\neg(n^2 \text{ is odd})$

$\therefore$ n is odd

QED

$T \wedge F$

F

"contradiction"

$\therefore$ "therefore"

Why does an indirect proof make Sense?

Prove: $H \to C$

1.	H	Hypothesis
2.	$\neg C$	Neg. of conclusion

$\ldots$

$F \equiv T$

$\#$

$$\neg C \to F \equiv \neg F \to C$$

$$T \to C$$

$$\frac{T}{C}$$

Proof (indirect):

1. n^2 is even — Hypothesis
2. n is not even — Negation of conclusion
3. n is odd — Properties of even & odd
4. $\exists k \in \mathbb{Z} : n = 2k+1$ — 3. Def of odd
5. $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2+2k)+1$ — Algebra, 4.
6. $2k^2+2k \in \mathbb{Z}$ — Closure of ints, 4.
7. n^2 is odd — 6, 5, Def of odd
8. $\times$ — 1, 7

$\therefore$ n is even

Q.E.D.

IGNORE

WTS

$4k^2+4k+1 =$

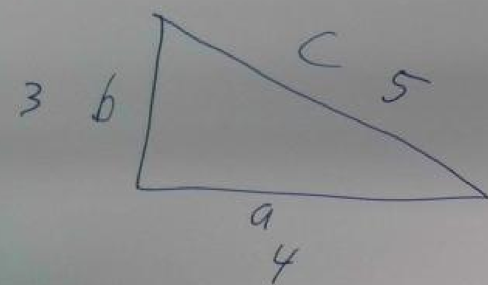
$2(\text{int})+1$

Prove: If a, b are even, then $a+b$ is even.

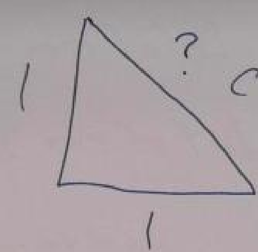
Proof:

1.	a is even	Hyp.
2.	b is even	Hyp.
3.	$\exists k \in \mathbb{Z} : a = 2k$	Def of even, 1
4.	$\exists j \in \mathbb{Z} : b = 2j$	Def of even, 2
5.	$a+b = 2k+2j$	Substitution, 3, 4
6.	$a+b = 2(k+j)$	algebra, 5
7.	$k+j \in \mathbb{Z}$	3., 4., closure of ints.
8.	$a+b$ is even	7, 6, Def of even

Q.E.D.



$$a^2+b^2=c^2$$



$$c = \frac{m}{n}$$

$$c^2 = 1^2+1^2 = 2$$

$$\approx 1.414\ldots$$

$\sqrt{2}$ is irrational

Prove $\sqrt{2}$ is not a rational number (not a fraction of ints)

Proof (indirect proof)

1. $\sqrt{2} = \frac{a}{b}$ $\quad a,b \in \mathbb{Z}$ $\quad$ Neg. of conclusion
(a/b is in simplest form)

2. $2 = \frac{a^2}{b^2}$ $\quad$ alg, 1.

3. $2b^2 = a^2$ $\quad$ alg, 2

4. a^2 is even $\quad$ 3, def of even

5. a is even $\quad$ previously proven, 4.

6. $\exists n \in \mathbb{Z} : a = 2n$ $\quad$ def of even, 5.

7. $a^2 = (2n)^2 = 4n^2$ $\quad$ alg, 6

8. $4n^2 = a^2 = 2b^2$ $\quad$ 3.

9. $2n^2 = b^2$ $\quad$ alg, 8

(a/b is in simplest form)

2. $2 = \frac{a^2}{b^2}$ alg, 1.

3. $2b^2 = a^2$ alg, 2

4. a^2 is even 3, def of even

5. a is even previously proven, 4.

6. $\exists n \in \mathbb{Z} : a = 2n$ def of even, 5.

7. $a^2 = (2n)^2 = 4n^2$ alg, 6

8. $4n^2 = a^2 = 2b^2$ 3.

9. $2n^2 = b^2$ alg, 8

10. b^2 is even 9, def of even

11. b is even prov. shown, 10

⨳ 5, 11, a/b is in simplest form

$\therefore \sqrt{2} \neq \frac{a}{b}$

QED