$\{1, 10, 100, 1000, 10000\}$

$\{10^n : n \geq 0 \text{ and } n \leq 4, n \in \mathbb{Z}\}$

	NEGATION
$\forall x \; p(x)$	$\exists x : \neg p(x)$
$\exists x : p(x)$	$\forall x \; \neg p(x)$

$$\neg(\forall x \; p(x)) \equiv \exists x : \neg p(x)$$

$$\forall x \in \mathbb{Z} \; P(x) \equiv P(0) \wedge P(1) \wedge P(-1) \wedge P(2) \wedge P(-2) \wedge \ldots$$

$$\exists x \in \mathbb{Z} : P(x) \equiv P(0) \vee P(1) \vee P(-1) \vee P(2) \vee P(-2) \vee \ldots$$

$\{1, 10, 100, 1000, 10000\}$

$\{10^n : n \geq 0 \text{ and } n \leq 4, n \in \mathbb{Z}\}$

$$\forall x\, (P(x) \wedge Q(x)) \equiv P(1) \wedge Q(1) \wedge P(2) \wedge Q(2) \wedge \ldots$$

$$\equiv P(1) \wedge P(2) \wedge P(3) \wedge \ldots$$

$$\wedge\, Q(1) \wedge Q(2) \wedge Q(3) \wedge \ldots$$

$$\equiv (\forall x\, P(x)) \wedge (\forall x\, Q(x))$$

$$\exists x : (P(x) \vee Q(x)) \equiv (\exists x : P(x)) \vee (\exists x : Q(x))$$

$$\forall x \in \mathbb{Z}\,(P(x) \vee Q(x)) \overset{?}{\not\equiv} \left(\forall x \in \mathbb{Z}\; P(x)\right) \vee \left(\forall x \in \mathbb{Z}\; Q(x)\right)$$

Let $P(x)$ be the prop "$x \geq 0$"

Let $Q(x)$ be the prop "$x \leq 0$"

$$\exists x \in \mathbb{Z} : (P(x) \wedge Q(x)) \not\equiv \left(\exists x \in \mathbb{Z} : P(x)\right) \wedge \left(\exists x \in \mathbb{Z} : Q(x)\right)$$

Let $P(x)$ be the prop "$x > 0$"

Let $Q(x)$ be the prop "$x < 0$"

(over $\mathbb{Z}$)

$\forall x\, (\exists y: x+y=0)$ True

$\exists x: (\forall y\ x+y=0)$ false

$\forall x\ \forall y\ \ xy=yx$ True

$\forall x\ \forall y\ \ x-y=y-x$ false

(over $\mathbb{Z}$)

$\forall x\,(\exists y : x+y=0)$ True

$\exists x : (\forall y\ x+y=0)$ false

$\forall x\ \forall y\ \ xy=yx$ True

$\forall x\ \forall y\ \ x-y=y-x$ false

RULES OF INFERENCE

(See pg. 5a)

Let P be the proposition "Dogs like bones."

Let Q be the proposition "You are a dog."

R

Let P be the proposition "If you are a dog then you like bones"

Let P be the proposition "You are a dog."

Let Q be the proposition "You like bones."

$P \to Q$ is the proposition "If you are a dog then you like bones."

If $P \to Q$ is true

and P is true

"Direct Reasoning"

then we know that Q is true.

$$((P \to Q) \wedge P) \to Q$$

$$\begin{array}{l} P \to Q \\ P \\ \hline Q \end{array}$$

$$P \to Q,\ P \Rightarrow Q$$

$$\begin{array}{l} P \to Q \\ \neg Q \\ \hline \neg P \end{array}$$

Indirect Reasoning

$$\begin{array}{l} P \to Q \\ P \\ \hline Q \end{array}$$

Direct Reasoning

$$\begin{array}{l} P \\ \hline P \vee Q \end{array}$$

Disjunctive Addition

$$\begin{array}{l} P \wedge Q \\ \hline P \end{array}$$

Conjunctive Simplification

$$\begin{array}{l} P \to Q \\ \hline \neg P \vee Q \end{array}$$

Conditional Equivalence

(See pg 50)

Let p be the prop "today is sunday"

Let q be the prop "I slept late today"

Let r be the prop "I teach CSE 215 today"

$$\begin{array}{l} q \to \neg r \\ r \\ p \to q \\ \hline \neg p \end{array}$$

$$\big((q \to \neg r) \wedge (r) \wedge (p \to q)\big) \to (\neg p)$$

PROVE: $q \to \neg r$

r

$p \to q$

$\neg p$

PROOF:

1. $q \to \neg r$ Hypothesis

2. $\neg\neg r \to \neg q$ Contrapositive, 1.

2.1 $r \to \neg q$ Double negation // you could skip 2

3. $p \to q$ Hypothesis

4. $\neg q \to \neg p$ Contrapositive, 3.

5. r Hypothesis

6. $r \vee p$ Disj addition, 5.

7. $\neg q$ Direct reasoning, 5. 2.1

8. $\neg p$ Direct reasoning 7. 4.

QED

PROVE: $q \to \neg r$

r

$p \to q$

$\neg p$

PROOF:

1. $q \to \neg r$ Hypothesis
2. $\neg\neg r \to \neg q$ Contrapositive, 1.

2.1 $r \to \neg q$ Double negation // you could skip 2

3. $p \to q$ Hypothesis
4. $\neg q \to \neg p$ Contrapositive, 3.
5. r Hypothesis
6. $r \vee p$ Disj addition, 5.
7. $\neg q$ Direct reasoning, 5. 2.1
8. $\neg p$ Direct reasoning 7. 4.

QED

PROVE: $P \to Q$

$Q \to R$

$P \wedge I$

R

Proof:

1. $P \to Q$ Hypothesis
2. $Q \to R$ Hypothesis
3. $P \to R$ Chain rule, 1. 2.
4. $P \wedge I$ Hypothesis
5. P Conjunctive Simplification
6. R Direct reasoning, 5, 3

Q.E.D.

PROVE: $P \to Q$

$Q \to R$

$P \wedge I$

R

Proof:

1. $P \to Q$ Hypothesis
2. $Q \to R$ Hypothesis
3. $P \to R$ Chain rule, 1. 2.
4. $P \wedge I$ Hypothesis
5. P Conjunctive Simplification
6. R Direct reasoning, 5, 3

Q.E.D.

PROVE the following

If you send me an email, then I'll do my homework.

If you don't send me an email, then I'll go to sleep early.

If I go to sleep early, then I'll wake up happy.

Therefore, if I don't do my homework then I'll wake up happy.

Let E be the prop "you send me an email"

Let W be the prop "I'll do my Homework"

Let S be the prop "I'll go to sleep early"

Let H be the prop "I'll wake up happy"

$$
\begin{array}{l}
E \to W \\
\neg E \to S \\
S \to H \\
\hline
\neg W \to H
\end{array}
$$