Conditional Statements

$p \to q$ "if p then q"

ex: Let p be the proposition "it is sunny"

Let q be the proposition "I go for a run".

$p \to q$ is the proposition "If it's sunny then I go for a run."

p implies q

q is implied by p

if p, q

q if p.

p is sufficient for q.

q unless $\neg p$

$p \to q$ if p then q

$q \to p$ "converse" of $p \to q$

$\neg p \to \neg q$ "inverse" of $p \to q$

$\neg q \to \neg p$ "contrapositive" of $p \to q$.

Let p be the prop "I win the lottery"

Let q be the prop "I eat breakfast tomorrow"

$\neg q \to \neg p$ says "If I don't eat breakfast tomorrow then I don't win the lottery."

$$P \to Q \equiv \neg Q \to \neg P$$

$$P \to Q \not\equiv Q \to P$$

$$P \to Q \not\equiv \neg P \to \neg Q$$

P	Q	$P \to Q$
F	F	T
F	T	T
T	F	F
T	T	T

Let P be the proposition "$x > 1000$"

Let Q be the proposition "$x > 0$"

$x = 500$

$P \to Q$ TRUE

$P \to Q$

"Hypothesis" "conclusion"

P iff Q

"P if and only if Q"

$P \leftrightarrow Q$ "Biconditional"

If P then Q; and
If Q then P

$(P \to Q) \wedge (Q \to P)$

P	Q	$P \to Q$	$Q \to P$	$P \leftrightarrow Q$ $(P \to Q) \wedge (Q \to P)$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

$$\overline{P \vee Q} = \bar{P} \wedge \bar{Q}$$

$$\overline{\boxed{\neg P} \vee Q} = \overline{\boxed{\neg P}} \wedge \bar{Q}$$

$$\overline{(P \vee \bar{Q} \vee R) \vee (P \wedge S)} = \overline{P \vee \bar{Q} \vee R} \wedge \overline{P \wedge S}$$

Ex: find the truth table for $(P \vee \neg Q) \rightarrow (P \wedge Q)$

P	Q	$\neg Q$	$P \vee \neg Q$	$P \wedge Q$	$(P \vee \neg Q) \rightarrow (P \wedge Q)$
F	F	T	T	F	F
F	T	F	F	F	T
T	F	T	T	F	F
T	T	F	T	T	T

$$(P \vee \neg Q) \rightarrow (P \wedge Q) \equiv Q$$

Propositional Equivalence

If $p \equiv T$ then p is called a "tautology"

If $p \equiv F$ then p is called a "contradiction"
or a "fallacy"

Otherwise, p is called a "contingency"

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Table 3.4.3 Basic Logical Laws - Equivalences

Commutative Laws	
$p \lor q \Leftrightarrow q \lor p$	$p \land q \Leftrightarrow q \land p$
Associative Laws	
$(p \lor q) \lor r \Leftrightarrow p \lor (q \lor r)$	$(p \land q) \land r \Leftrightarrow p \land (q \land r)$
Distributive Laws	
$p \land (q \lor r) \Leftrightarrow (p \land q) \lor (p \land r)$	$p \lor (q \land r) \Leftrightarrow (p \lor q) \land (p \lor r)$
Identity Laws	
$p \lor 0 \Leftrightarrow p$	$p \land 1 \Leftrightarrow p$
Negation Laws	
$p \land \neg p \Leftrightarrow 0$	$p \lor \neg p \Leftrightarrow 1$
Idempotent Laws	
$p \lor p \Leftrightarrow p$	$p \land p \Leftrightarrow p$
Null Laws	
$p \land 0 \Leftrightarrow 0$	$p \lor 1 \Leftrightarrow 1$
Absorption Laws	
$p \land (p \lor q) \Leftrightarrow p$	$p \lor (p \land q) \Leftrightarrow p$
DeMorgan's Laws	
$\neg(p \lor q) \Leftrightarrow (\neg p) \land (\neg q)$	$\neg(p \land q) \Leftrightarrow (\neg p) \lor (\neg q)$
Involution Law	
$\neg(\neg p) \Leftrightarrow p$	

Table 3.4.4 Basic Logical Laws - Common Implications and Equivalences

$$\prod_{i=1}^{n} a_i = a_1 \times a_2 \times a_3 \times \dots \times a_n$$

Let w be the prop "the world is flat"

Let Z be the prop "0 is even"

$$w \vee Z \equiv T$$

(c) find a prop C such that $C \to X$

(d) find a prop D such that $X \to D$

Prove $\neg(P \vee (\neg P \wedge Q)) \equiv (\neg P) \wedge (\neg Q)$

Proof:

$$\neg(P \vee (\neg P \wedge Q)) \equiv (\neg P) \wedge (\neg(\neg P \wedge Q))$$ DeMorgan's Theorem

$$(\neg P) \wedge (\neg(\neg P \wedge Q)) \equiv (\neg P) \wedge (\neg\neg P \vee \neg Q)$$ DeMorgan's Theorem

$$\equiv (\neg P) \wedge (P \vee \neg Q)$$ Involution

$$\equiv (\neg P \wedge P) \vee (\neg P \wedge \neg Q)$$ Distribution

$$\equiv F \vee (\neg P \wedge \neg Q)$$ Negation Law

$$\equiv \neg P \wedge \neg Q$$ Identity Law

Q.E.D.

Prove $\neg(P \vee (\neg P \wedge Q)) \equiv (\neg P) \wedge (\neg Q)$

Proof:

$$\neg(\boxed{P} \vee (\neg P \wedge Q)) \equiv (\neg P) \wedge (\neg(\neg P \wedge Q))$$ DeMorgan's Theorem

$$(\neg P) \wedge (\neg(\neg P \wedge Q)) \equiv (\neg P) \wedge (\neg\neg P \vee \neg Q)$$ DeMorgan's Theorem

$$\equiv (\neg P) \wedge (P \vee \neg Q)$$ Involution

$$\equiv (\neg P \wedge P) \vee (\neg P \wedge \neg Q)$$ Distribution

$$\equiv F \vee (\neg P \wedge \neg Q)$$ Negation Law

$$\equiv \neg P \wedge \neg Q$$ Identity Law

Q.E.D.

QUANTIFIERS

Suppose $p(x)$ is a "family of propositions"

Example: Let $P(x)$ be the proposition

"$x^2 > x$" $(x \in \mathbb{Z})$

$p(5)$ is the prop "$5^2 > 5$"

$P(-6)$ is the prop "$(-6)^2 > -6$"

"for all x" $\forall x$ "Universal Quantifier"

$\boxed{\forall x \in \mathbb{Z}\ P(x)}$ says "for all integers x, $x^2 > x$"

There exists an x such that

$\exists x : x^2 = x$

$\exists x \in \mathbb{Z} : x^2 = x$

"Existential Quantifier"