FUNCTIONS

"Domain"

$f: A \to B$ "CO-DOMAIN"

A <u>function</u> f from A to B is an <u>assignment</u> (or mapping) that takes each element of A and gives you an element of B.

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ex: $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$

If $a \in A$, $f(a)$ is called the "image of a"

The <u>range</u> of f is the set of all $f(a)$ for any $a \in A$

Ex! Let $f: \mathbb{Z} \to \mathbb{Z}$, $f(x) = x^2$

Domain: $\mathbb{Z}$

Co-domain: $\mathbb{Z}$

Range $= \{0, 1, 4, 9, 16, \ldots\}$

$f(x) = 2x$ $\qquad$ $g(x) = x + x$

~~Ex~~

$f: \mathbb{Z} \to \mathbb{Z}$

$f(x) =$ last digit of x

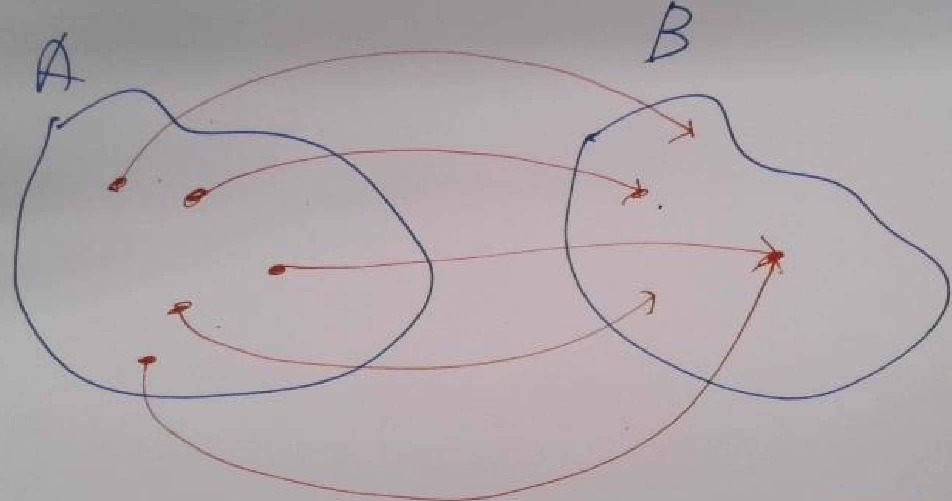
$f(17) = 7$

$f(2344) = 4$

$f(-138) = 8$

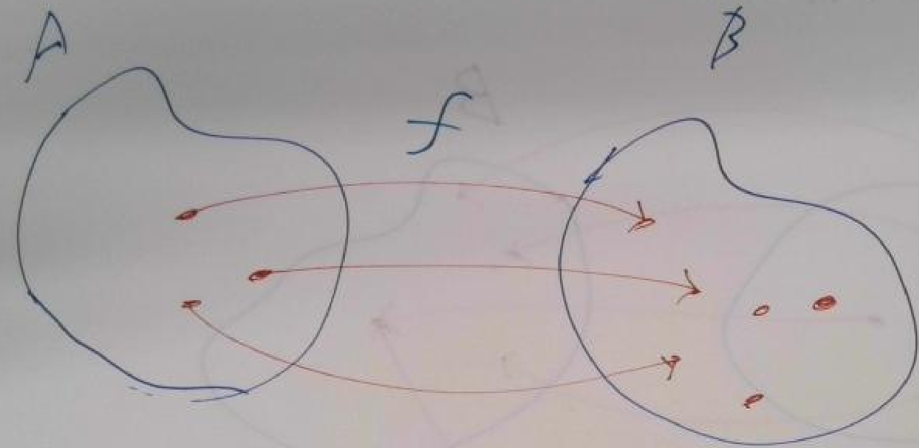
Properties of functions

Suppose $f: A \to B$



in this case, f is not ~~"onto"~~ ~~or "surjective"~~

"injective" or
"one-to-one"



If there are pts in B not mapped to,
then f is not "onto"
or "surjective"

Def: ~~A funct~~ Given a function $f: A \to B$

If $f(a_1) \neq f(a_2)$ whenever $a_1 \neq a_2$,
then f is "one-to-one" (injective)

If for any $b \in B$ there exists some $a \in A$
such that $f(a) = b$
then f is "onto" (surjective)

One-to-one/onto (injective AND surjective)
"bijective"

"one-to-one correspondence"

SEQUENCES

things like $1, 3, 5$

or $2, 3, 5, 7, 11, 13, \ldots$

or $1, 2, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 2, \ldots$

We can think of a sequence of elements of a set S as a function

$$f : \mathbb{Z}^+ \to S$$

positive ints

$f(1), f(2), f(3), f(4), \ldots$

a_1, a_2, a_3, a_4 $\{a_n\}, n = 1, 2, 3, 4, \ldots$

Example: $a_n = \frac{1}{n}$, $n = 1, 2, 3, 4, 5 \ldots$

$a_1 = \frac{1}{1} = 1$

$a_2 = \frac{1}{2} = .5$

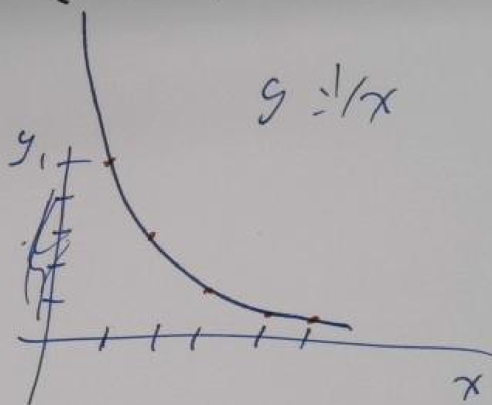
$a_3 = \frac{1}{3} = .33\overline{3}$

$a_4 = \frac{1}{4} = .25$

$a_5 = \frac{1}{5} = .2$

$\vdots$

$\{a_n\} = 1, .5, .3\overline{3}, .25, .2, \ldots$



$f: A \to B$

$(a_1, f(a_1))$

$(a_2, f(a_2))$

$(a_3, f(a_3))$

$\vdots$

$A \times B = \{(a,b) : a \in A, b \in B\}$

$A \times B$ contains every function $f: A \to B$

$f: \{1,2,3,4\} \to \mathbb{Z}$

$f(x) = x^2$

$(1,1)$

$(2,4)$

$(3,9)$

$(4,16)$

$\{(1,1), (2,4), (3,9), (4,16)\}$

$\mathbb{R} \times \mathbb{R}$

A Recurrence Relation for a sequence

$a_1, a_2, a_3, a_4, \dots$

is a rule defining a_n as some function

of $a_1, a_2, \dots, a_{n-1}$

Ex: $\boxed{a_n = a_{n-1} * 2}$

if $a_1 = 1$

$a_2 = a_1 * 2 = 2$

$a_3 = a_2 * 2 = 2*2 = 4$

$a_4 = a_3 * 2 = 4*2 = 8$

$1, 2, 4, 8, 16, \dots$

$\boxed{a_n = n + a_{n-1}}$

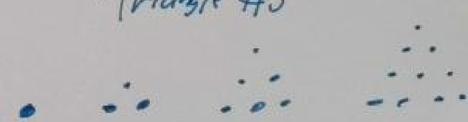
if $a_1 = 1$

$a_2 = 2+1 = 3$

$a_3 = 3+3 = 6$

$1, 3, 6, 10, 15, 21, \dots$

"triangle #s"



A Recurrance Relation for a sequence

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A Recurrence Relation for a sequence

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Ex: $\boxed{a_n = a_{n-1} * 2}$

if $a_1 = 1$ "initial value"

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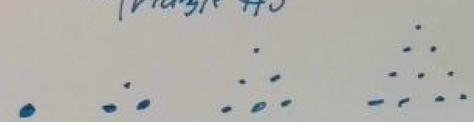
$1, 3, 6, 10, 15, 21, \ldots$

"triangle #s"



$a_1 = 1$ Fibonacci Sequence

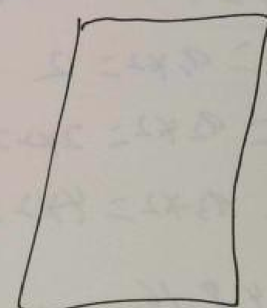
$a_2 = 1$

$a_n = a_{n-1} + a_{n-2}$ if $n \geq 3$

$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$

$$\frac{a_n}{a_{n-1}}$$

$1, 2, 1.5, 1.6\overline{6}, 1.6, \ldots$



SUMMATION

$$\sum_{i=1}^{n} f(i) \quad \text{means} \quad f(1)+f(2)+f(3)+\dots+f(n)$$

"The sum from $i=1$ to n of $f(i)$"

$$\sum_{j=1}^{n} f(j)$$

$$\sum_{i=0}^{n-1} f(i+1)$$

```
int sum=0;
for i=1 to n{
    sum=sum+f(i)
}
```

$$\left[\sum_{i=1}^{n} f(i)\right] + \left[\sum_{i=1}^{n} g(i)\right] =$$

$$\sum_{i=1}^{n} \left(f(i) + g(i)\right)$$

$$\cancel{\sum_{i=1}^{n} \sum_{j=1}^{k} (i \cdot j)}$$

$$\sum_{i=1}^{3} \left[\sum_{j=2}^{5} (i \cdot j) \right] =$$

$$\underbrace{\sum_{j=2}^{5} (1 \cdot j)}_{1\cdot 2 + 1\cdot 3 + 1\cdot 4 + 1\cdot 5} + \underbrace{\sum_{j=2}^{5} (2 \cdot j)}_{2\cdot 2 + 2\cdot 3 + 2\cdot 4 + 2\cdot 5} + \underbrace{\sum_{j=2}^{5} (3 \cdot j)}_{3\cdot 2 + 3\cdot 3 + 3\cdot 4 + 3\cdot 5}$$

$1\cdot 2 + 1\cdot 3 + 1\cdot 4 + 1\cdot 5 + 2\cdot 2 + 2\cdot 3 + 2\cdot 4 + 2\cdot 5 + 3\cdot 2 + 3\cdot 3 + 3\cdot 4 + 3\cdot 5$

Matrices

Rectangular array of numbers

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 2 & 4 & -1 \\ 6 & 0 & 18 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

$$5 \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 4 & -1 \\ 6 & 0 & 18 \end{bmatrix} =$$

$$\begin{bmatrix} (1\ 2)\cdot(2\ 6) & (1\ 2)\cdot(4\ 0) & (1\ 2)\cdot(-1\ 18) \\ 1\cdot 2 + 2\cdot 6 = 14 & 1\cdot 4 + 2\cdot 0 = 4 & 1\cdot -1 + 2\cdot 18 = 35 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix} =$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \end{bmatrix}$$

$$p_{23} = a_{21} \cdot b_{13} + a_{22} \cdot b_{23}$$

$5x = 15$

$x = \frac{15}{5} = 3$

$5x + 3y + 2z = 10$

$3x - y + z = 20$

$2x + 2y - 6z = 42$

$$\begin{pmatrix} 5 & 3 & 2 \\ 3 & -1 & 1 \\ 2 & 2 & -6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 20 \\ 42 \end{pmatrix}$$

$$C \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = B$$

$$\underbrace{C^{-1} \times C}_{I} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = C^{-1} \times B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = C^{-1} \times B$$

PROPOSITIONAL LOGIC

DEF: A proposition is a declarative statement that is true or false.

ex: My name is nick.

$1+1=2$

$1+1=5$

Cows can fly.

$x^2+2x+1=(x+1)^2$

NOT PROPOSITION

Hello.

What should I eat?

x^2+2x+1

Propositional Variable — a letter that represents a proposition.

TRUE T

FALSE F

Propositional Calculus

Let p be any proposition.

$\neg p$ "not p" or "the negation of p"

Jane

Let p be the proposition "My name is Nick."

Jane

$\neg p$ is the proposition "My name is not Nick"

Jane

$\neg\neg p$ is the proposition "My name is Nick"

$$p \equiv \neg\neg p$$

logically equivalent

TRUE T

FALSE F

Propositional Calculus

Let p be any proposition.

$\neg p$ "not p" or "the negation of p"

p	$\neg p$
F	T
T	F

CONJUNCTION (AND)

Let p and q be propositions

$p \wedge q$ is the proposition "p and q"

DISJUNCTION (OR)

$p \vee q$ is the proposition "p or q"

"exclusive or"

p	q	$p \wedge q$	$p \vee q$	$p \oplus q$
F	F	F	F	F
F	T	F	T	T
T	F	F	T	T
T	T	T	T	F