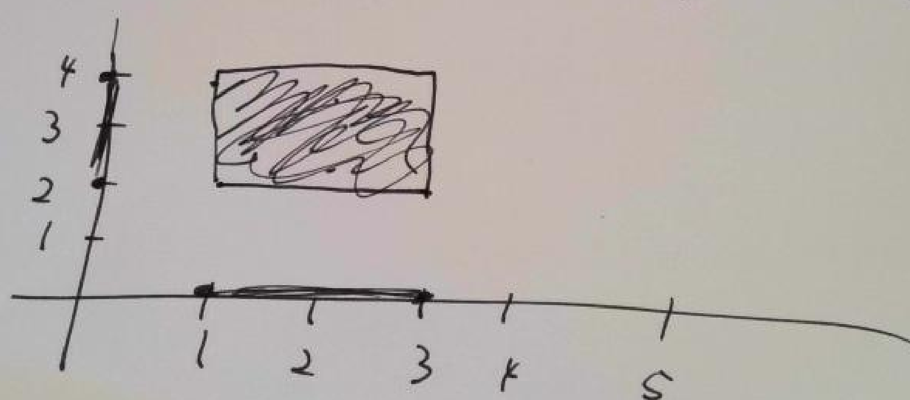
$\emptyset \subseteq A$ $\qquad$ $\emptyset \notin A$

$A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$

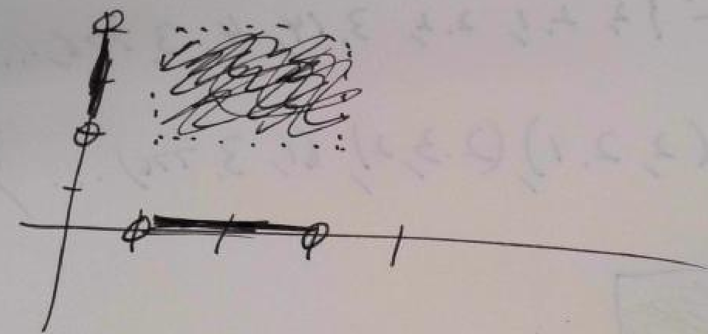
Let $A = [1, 3] = \{1, 2, 3, 2.3, 1.13, 2.57 \ldots)$

$B = [2, 4] = \{2, 2.1, 2.2, 3.14, 4, 3.776, \ldots\}$

$A \times B = \{(1,2), (2, 2.1), (2.3, 2), (1, 3.776) \ldots\}$



$(1,3) \times (2,4)$



$A = (a, b, c)$ $B = (1, 2)$ $C = (☺, ☹)$

$A \times B = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$

$A \times B \times C = \{(a,1,☺), (a,1,☹), (a,2,☺), (a,2,☹), (b,1,☺), \ldots\}$

~~SETS~~ SET OPERATIONS

Let A, B be sets

The <u>union</u> of A and B

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

$$\{1,2\} \cup \{1,3,5\} = \{1,2,3,5\}$$

The <u>intersection</u> $A \cap B = \{x : x \in A \text{ and } x \in B\}$

$$\{1,2\} \cap \{1,3,5\} = \{1\}$$

$$\{1,3,5\} \cap \{2,4,6\} = \emptyset$$

Def: A and B are called <u>disjoint</u> if $A \cap B = \emptyset$

$$|A \cup B| \leq |A| + |B|$$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

SUBTRACTING SETS

$A - B$ (or $A \setminus B$)

$$= \{a \in A : a \notin B\}$$

$\{1,2,3,4,5\} - \{2,4\} = \{1,3,5\}$

$\{1,2,3,4,5\} - \{2,3,4,5,6,7\} = \{1\}$

If $A - B = \emptyset$, then $A \subseteq B$

$\mathcal{U}$ = universal set = set of all things

$\mathcal{U} - A$ is called the complement of A

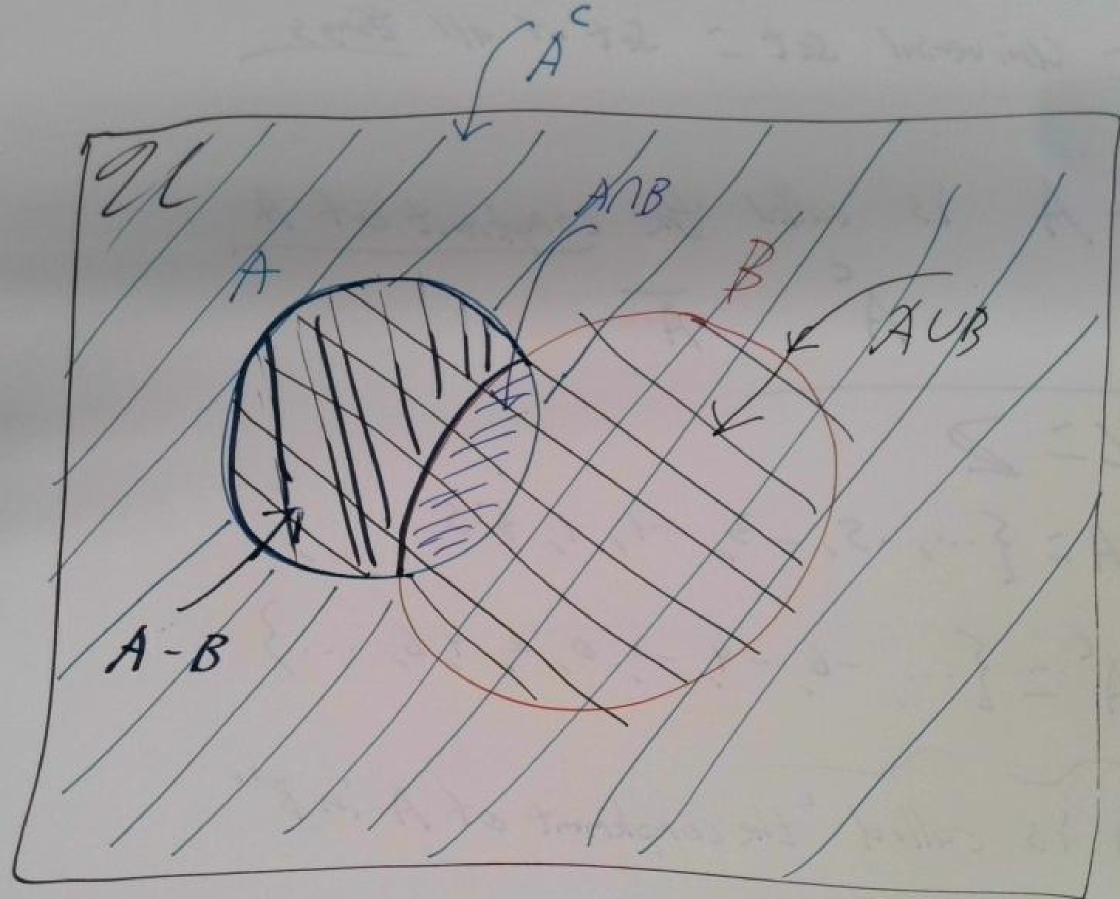
A^c $\quad \overline{A}$

If $\mathcal{U} = \mathbb{Z}$

and $A = \{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$

then $A^c = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$

$B - A$ is called "the complement of A in B"



$\mathcal{U}$
A^c
A
$A \cap B$
B
$A \cup B$
$A - B$

SET IDENTITIES

(Let A, B, C be sets)

$A \cup \emptyset = A$ $\qquad A \cup \mathcal{U} = \mathcal{U}$

$A \cap \emptyset = \emptyset$ $\qquad A \cap \mathcal{U} = A$

$A \cup A = A \cap A = A$ $\qquad \overline{\overline{A}} = A$

$A \cup B = B \cup A$ $\qquad A \cap B = B \cap A$ "commutative"

$A \cup (B \cup C) = (A \cup B) \cup C$ $\qquad A \cap (B \cap C) = (A \cap B) \cap C$

"associative"

$$\left.\begin{array}{l} A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \end{array}\right\} \text{"distributive"}$$

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\qquad 2 \cdot (3+4) = 2 \cdot 3 + 2 \cdot 4$

SET IDENTITIES

(Let A, B, C be sets)

$A \cup \emptyset = A$ $\quad$ $A \cup \mathcal{U} = \mathcal{U}$

$A \cap \emptyset = \emptyset$ $\quad$ $A \cap \mathcal{U} = A$

$A \cup A = A \cap A = A$ $\quad$ $\overline{\overline{A}} = A$

$A \cup B = B \cup A$ $\quad$ $A \cap B = B \cap A$ "commutative"

$A \cup (B \cup C) = (A \cup B) \cup C$ $\quad$ $A \cap (B \cap C) = (A \cap B) \cap C$

"associative"

$$\left.\begin{aligned} A \cup (B \cap C) &= (A \cup B) \cap (A \cup C) \\ A \cap (B \cup C) &= (A \cap B) \cup (A \cap C) \end{aligned}\right\} \text{"distributive"}$$

$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\quad$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\quad$ $2 \cdot (3+4) = 2 \cdot 3 + 2 \cdot 4$

De Morgan's Theorem

De Morgan's theorem

$$\overline{A \cup B} = \bar{A} \cap \bar{B} \qquad \overline{A \cap B} = \bar{A} \cup \bar{B}$$

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Prove $\overline{A \cup B} = \bar{A} \cap \bar{B}$ MEMBERSHIP TABLE

A	B	$A \cup B$	$\overline{A \cup B}$	$\bar{A}$	$\bar{B}$	$\bar{A} \cap \bar{B}$
F	F	F	T	T	T	T
F	T	T	F	T	F	F
T	F	T	F	F	T	F
T	T	T	F	F	F	F

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

De Morgan's theorem

$\overline{A \cup B} = \overline{A} \cap \overline{B} \qquad \overline{A \cap B} = \overline{A} \cup \overline{B}$

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Prove $\overline{A \cup B} = \overline{A} \cap \overline{B}$ MEMBERSHIP TABLE

A	B	$A \cup B$	$\overline{A \cup B}$	$\overline{A}$	$\overline{B}$	$\overline{A} \cap \overline{B}$
F	F	F	T	T	T	T
F	T	T	F	T	F	F
T	F	T	F	F	T	F
T	T	T	F	F	F	F

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

family of Sets

$A_1, A_2, A_3, A_4 \ldots$

$A_i = \{i, i+1, i+2, i+3, i+4, \ldots\}$

ex: $A_1 = \{1, 2, 3, 4, 5, 6, 7, 8, \ldots\}$

$A_2 = \{2, 3, 4, 5, 6, 7, 8, 9, 10, \ldots\}$

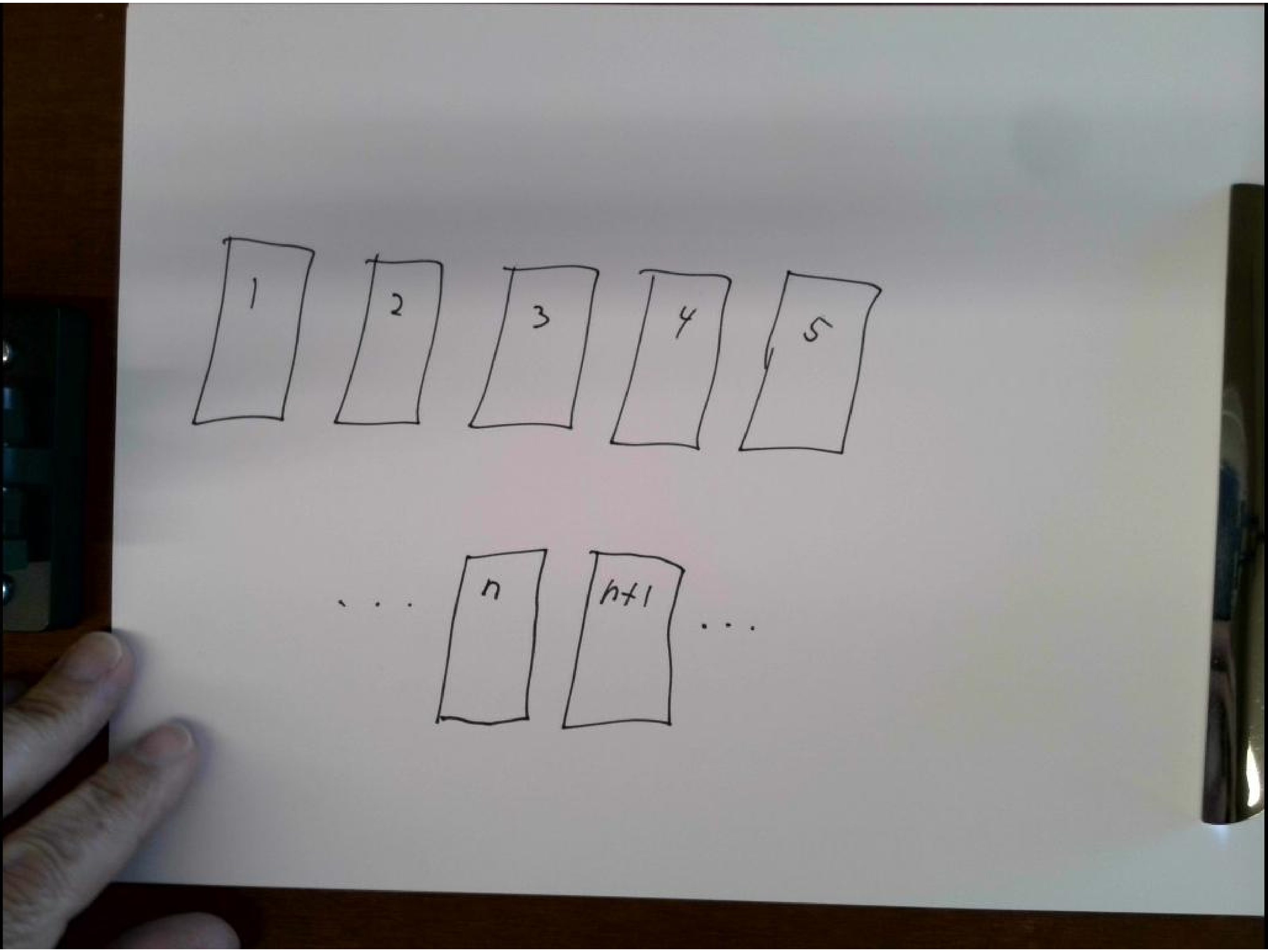
$A_3 = \{3, 4, 5, 6, 7, \ldots\}$

$A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 = \bigcup_{i=1}^{5} A_i$

$A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$

$$\bigcup_{i=1}^{n} A_i = A_1 \qquad \bigcap_{i=1}^{n} A_i = A_n$$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \qquad \bigcap_{i=1}^{\infty} A_i = \emptyset$$


1
2
3
4
5
...
n
n+1
...

Let R = set of rooms = $\{r_1, r_2, r_3, r_4, r_5, \ldots\}$

Let G = set of guests = $\{g_1, g_2, g_3, g_4, g_5 \ldots\}$

$$|R| = |G| = \aleph_0$$

"Cardinality of R"

(size of R)

$$|R| = |G \cup \{g_0\}| = |G|$$

Let R = set of rooms = $\{r_1, r_2, r_3, r_4, r_5, \ldots\}$

Let G = set of guests = $\{g_1, g_2, g_3, g_4, g_5, \ldots\}$

$$|R| = |G| = \aleph_0$$

"Cardinality of R"
(size of R)

$$|R| = |G \cup \{g_0\}| = |G|$$

	2	4	6	8	10	12
G	1	2	3	4	5	6

R 1 2 3 4 5 6

$\boxed{1}$ $\boxed{1.5}$ $\boxed{1.01001}$ $\boxed{1.999}$ ...

Cantor Diagonalization

$|\mathbb{R}| > |\mathbb{Z}|$

$|\mathbb{Z}| < |\mathbb{R}|$ Gödel

$\aleph_0$

$$|\mathbb{Z}| < |M| < |\mathbb{R}|$$

YES there is such a set 11

No 6

Continuum Hypothesis:

$|\mathbb{Z}| < |\mathbb{R}|$ Gödel

$\aleph_0$

$|\mathbb{Z}| < |M| < |\mathbb{R}|$

YES there is such a set 11

~~No~~ 6

Continuum Hypothesis: