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positive integer

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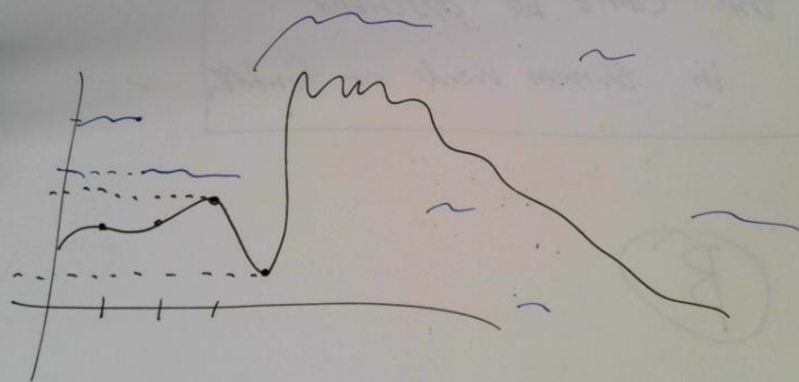
B

DISCRETE Structures



Real numbers: 2, 3, 5, 0.3, 3.141592, 2.718,

Integers: 2, 3, 4, 5, -6, 219, -1437

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all integers between 1 and 10

all real #s between 1 and 10

SET THEORY

A <u>set</u> is an unordered collection of <u>things</u>.

"elements"

We use letters to represent sets.

S is the set containing the numbers 1, 2 and 3.

$S = \{1, 2, 3\}$

$I = \{1, 2, 3, \ldots\}$ = all positive integers

$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ = <u>all</u> <u>integers</u>

$\mathbb{Z}$ = Set of all integers

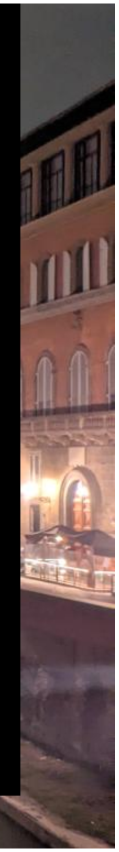
$\mathbb{N}$ = Set of natural #s (integers, ≥ 0)

$\in$ "membership"

Let $S = \{A, B, c\}$

$A \in S$ "A is a member of S)

$D \notin S$ "D is <u>not</u> a member of S"



Such that : or |

$$E = \{x \in \mathbb{Z} : x \text{ is divisible by } 2\}$$

E is the set of x contained in the integer

$$E = \{2, 4, 6, 8, 10, \dots, 0, -2, -4, -6, \dots\}$$

$$\frac{0}{2} = 0$$

$$0 = 0 \cdot 2$$

$$\frac{-4}{2} = -2$$

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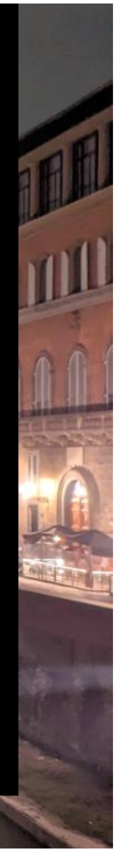
$\mathbb{R}$ = Set of Real #s

$\mathbb{Q}$ = Set of rational #s (fractions)

$\mathbb{C}$ = Set of complex #s



A Singleton set is a set that has only one element.

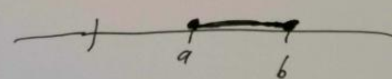
$\{5\}$

$\{x \in \mathbb{Z} : x < 0 \text{ and } x = |x|\} = \{\ \}$

empty set $= \emptyset$

Interval Notation (usually sets of real #s)

$[a,b] = \{x \in \mathbb{R} : x \geq a \text{ and } x \leq b\}$



$(a,b) = \{x \in \mathbb{R} : x > a \text{ and } x < b\}$

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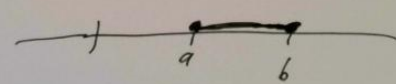

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$\{\mathbb{Z}, \mathbb{R}\}$

Equality of Sets

$A = B$ means for any x, $x \in A$ if and only if $x \in B$

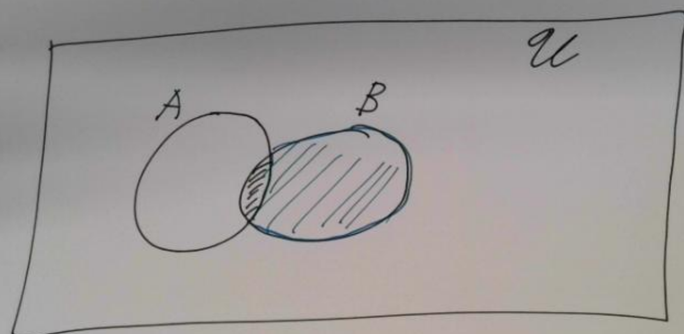
Example: $A = \{1, 2, 3\}$

$B = \{x \in \mathbb{Z} : x \ge 1 \text{ and } x \le 3\}$

$A = B$

$C = \{3, 2, 1\}$ $\quad A = C$

Venn Diagrams



$\mathcal{U}$ = Universal Set = Set of all things

SUBSETS

A is a <u>subset</u> of B ~~iff~~ if and only if every element of A is also an element of B.

Ex: $A = \{2, 3, 5, 7\}$

$B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A \subseteq B$ "A is a subset of B"

$C = \{-1, 0, 1\}$

$C \not\subseteq B$ "C is <u>not</u> a subset of B"

If S is a set, is $S \subseteq S$? <u>yes</u>

is $\emptyset \subseteq S$? <u>yes</u>

$A \subsetneq B$ means A is a subset of B
and $A \neq B$

"A is a proper subset of B"

Theorem $A = B$ if and only if
$A \subseteq B$ and $B \subseteq A$

Size of a set

If S is finite — S contains a finite # of elements

$|S|$ = # of elements in S

$|\{a, e, i, o, u\}| = 5$

$|\mathbb{Z}| = ?$

Power Set

$\mathcal{P}(S)$ = Set of all subsets of S.

Ex: Let $S = \{a, b, c\}$ $\qquad |S| = 3$

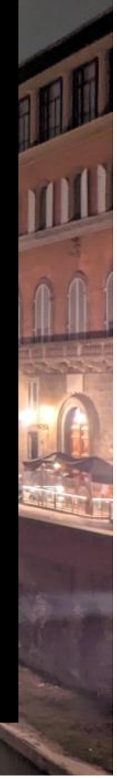
$$\mathcal{P}(S) = \{ \{a\}, \{b\}, \{c\}, \phi, \{a,b\}, \{b,c\}, \{a,b,c\}, \{a,c\} \}$$

$$|\mathcal{P}(S)| = 8$$

If S is finite,

$\cancel{|\mathcal{P}}$

$$\boxed{|\mathcal{P}(S)| = 2^{|S|}}$$



Power Set

$\mathcal{P}(S) =$ Set of all subsets of S.

Ex: Let $S = \{a, b, c\}$ $\quad |S| = 3$

$$\mathcal{P}(S) = \{ \{a\}, \{b\}, \{c\}, \emptyset, \{a,b\}, \{b,c\}, \{a,b,c\}, \{a,c\} \}$$

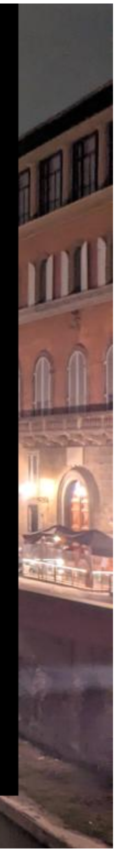
$|\mathcal{P}(S)| = 8$

If S is finite,

$$\boxed{|\mathcal{P}(S)| = 2^{|S|}}$$

$\mathcal{P}(\emptyset) = \{\emptyset\}$ $\quad 2^0 = 1$

$|\emptyset| = 0$ $\quad |\{\emptyset\}| = 1$



$$\mathcal{P}(\mathcal{P}(\emptyset)) = \mathcal{P}(\{\emptyset\})$$

$$= \{\emptyset, \{\emptyset\}\}$$

Cartesian Products

Def: an <u>ordered n-tuple</u> is an ordered collection $(a_1, a_2, \ldots, a_n)$ of n things.

Cartesian Product of 2 Sets A, B

$$A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$$

Ex: $A = \{x, y\}$ $\quad B = \{1, 2, 3\}$

$$A \times B = \{(x,1), (x,2), (x,3), (y,1), (y,2), (y,3)\}$$

$|A| = 2 \quad |B| = 3 \quad |A \times B| = 6$