$1^n$ $\quad$ $10^n$ $\quad$ $(11)^n$ $\quad$ $0\cancel{(1)}$ $0(1^{2n})$

$I \to \lambda,\ I \to 1I$ $\qquad$ $I \to \lambda,\ I \to 11I$

$I \to \lambda,\ I \to 0I$ $\qquad$ $I \to 0A$

$\cancel{A \to \lambda}$ $\quad A \to 11$

$I \to 1A,\ A \to \lambda,\ A \to 0A$ $\qquad$ $A \to 11A$

$1^n$ $\quad 10^n \quad (11)^n \quad \cancel{o(1)} \; o(1^{2n})$

$I \to \lambda, \; I \to 1I \qquad I \to \lambda, \; I \to 11I$

$I \to \lambda, \; I \to 0I \qquad I \to 0A$

$\cancel{A \to \lambda} \quad A \to 11$

$I \to 1A, \; A \to \lambda, \; A \to 0A \qquad A \to 11A$

$10^n \qquad \cancel{I \to 1} \qquad I \to I0$

$\cancel{I \to 0I}0 \qquad I \to 1$

$I$

$1$

$I0$

$1$

$I00$

$1$

$100$

$\underbrace{1000}$
istas on $I$?

$IJ \to$

$\phi 000$
ot?

$000$
oI?

DIGIT ::= '0' | '1' | '2' | . . . | '9'

EXPSYM ::= 'E'

B 100101 B

(S0, 0, S0, 0, R)

(S0, 1, S0, 1, R)

(S0, B, S1, 1, L)

B 1001001100110111101 B

00

(S0, 0, S0, 0, R)

(S0, 1, S1, 1, R)

(S0, B, S99, B, R)

// found 1st one

(S1, 0, S0, 0, R)

(S1, B, S99, B, R)

(S1, 1, S2, 0, L)

(S2, 1, S99, 0, R)

A recurrence relation is a way to describe a sequence of numbers, where the $n^{th}$ number is some function of the first $(n-1)$ numbers.

---

Ex: Bacteria growth

pop. doubles every hour

init pop = 10

$$P(n) = 2 * P(n-1)$$

$$P(0) = 10$$

$$P(3) = 2 \cdot P(2)$$

$$P(2) = 2 \cdot P(1)$$

$$P(1) = 2 \cdot P(0)$$

$$P(2) = 2 \cdot P(1) = 2 \cdot 2 \cdot P(0)$$

$$= 2^2 \cdot P(0)$$

$$P(3) = 2 \cdot P(2) = 2 \cdot 2^2 P(0)$$

$$= 2^3 P(0)$$

? $P(n) = 2^n \cdot P(0)$

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```
#include <stdio.h>

int mylen(char *str);  // my string length function

void main()
{
  printf("length of HELLO is %d\n",mylen("hello"));
}

int mylen(char *str)
{
  if (str[0]=='\0') return(0);
  return(1+mylen(&str[1]));
}
~
~
~
~
~
~
~
~
~
~
~
~
"strlen.c" 14L, 226B
```

$n! = (n-1)!\,(n)$

$1! = 1$

---

$2^n = 2(2^{n-1})$

$2^0 = 1$

---

strlen("~")

$$\text{strlen}(S) = \begin{cases} 0 \text{ if } S[0] == \text{'\textbackslash 0'} \\ 1 + \text{strlen}(S \text{ starting from } 2^{\text{nd}} \text{ char}) \end{cases}$$

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```
{2707} towers
1 --> 2  1 --> 3  2 --> 3  1 --> 2  3 --> 1
3 --> 2  1 --> 2  # of calls to tower(): 7
{2708} !vi
vi towers.c
{2709} !gc
gcc -o towers towers.c
{2710} towers
1 --> 3  1 --> 2  3 --> 2  1 --> 3  2 --> 1
2 --> 3  1 --> 3  1 --> 2  3 --> 2  3 --> 1
2 --> 1  3 --> 2  1 --> 3  1 --> 2  3 --> 2
# of calls to tower(): 15
{2711} !vi
vi towers.c
{2712} !gc
gcc -o towers towers.c
{2713} Towers
-bash: Towers: command not found
{2714} towers
1 --> 2  1 --> 3  2 --> 3  1 --> 2  3 --> 1
3 --> 2  1 --> 2  1 --> 3  2 --> 3  2 --> 1
3 --> 1  2 --> 3  1 --> 2  1 --> 3  2 --> 3
1 --> 2  3 --> 1  3 --> 2  1 --> 2  3 --> 1
2 --> 3  2 --> 1  3 --> 1  3 --> 2  1 --> 2
1 --> 3  2 --> 3  1 --> 2  3 --> 1  3 --> 2
1 --> 2  # of calls to tower(): 31
{2715}
```

$n$ disks $\qquad 2^n - 1$ moves

---

$(n+1)$ disks

$n$ disks

1

$n$ disks

Let $T(n)$ = # of moves for $n$ disks

$$T(n+1) = T(n) + 1 + T(n)$$
$$= 2T(n) + 1$$

$$T(1) = 1$$

AGE →

TYPE ↓

| | | | | | | | | TOTAL |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | 1 |
| | 1 | | | | | | | 1 |
| 1 | | 1 | | | | | | 2 |
| 1 | 1 | | 1 | | | | | 3 |
| 2 | 1 | 1 | | 1 | | | | 5 |
| 3 | 2 | 1 | 1 | | 1 | | | 8 |
| 5 | 3 | 2 | 1 | 1 | | 1 | | 13 |

| AGE→ TIME ↓ | | | | | | | TOTAL |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | $P(1) = 1$ |
| | 1 | | | | | | $P(2) = 1$ |
| 1 | | 1 | | | | | $P(3) = 2$ |
| 1 | 1 | | 1 | | | | $P(4) = 3$ |
| 2 | 1 | 1 | | 1 | | | $P(5) = 5$ |
| 3 | 2 | 1 | 1 | | 1 | | $P(6) = 8$ |
| 5 | 3 | 2 | 1 | 1 | | 1 | $P(7) = 13$ |

$P(8) = P(7) + P(6)$

Let $P(n)$ = pop at time $n$

Fibonacci Sequence

$f_n = f_{n-1} + f_{n-2}$ $\quad n \geq 3$

$f_1 = f_2 = 1$

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```
{2727} !vi
vi fib1.c
{2728} fib1
fib(1)=1. fib was called 1 times
fib(2)=1. fib was called 1 times
fib(3)=2. fib was called 2 times
fib(4)=3. fib was called 3 times
fib(5)=5. fib was called 5 times
fib(6)=8. fib was called 8 times
fib(7)=13. fib was called 13 times
fib(8)=21. fib was called 21 times
fib(9)=34. fib was called 34 times
fib(10)=55. fib was called 55 times
fib(11)=89. fib was called 89 times
fib(12)=144. fib was called 144 times
fib(13)=233. fib was called 233 times
fib(14)=377. fib was called 377 times
fib(15)=610. fib was called 610 times
fib(16)=987. fib was called 987 times
fib(17)=1597. fib was called 1597 times
fib(18)=2584. fib was called 2584 times
fib(19)=4181. fib was called 4181 times
```

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```
{2727} !vi
vi fib1.c
{2728} fib1
fib(1)=1. fib was called 1 times
fib(2)=1. fib was called 1 times
fib(3)=2. fib was called 2 times
fib(4)=3. fib was called 3 times
fib(5)=5. fib was called 5 times
fib(6)=8. fib was called 8 times
fib(7)=13. fib was called 13 times
fib(8)=21. fib was called 21 times
fib(9)=34. fib was called 34 times
fib(10)=55. fib was called 55 times
fib(11)=89. fib was called 89 times
fib(12)=144. fib was called 144 times
fib(13)=233. fib was called 233 times
fib(14)=377. fib was called 377 times
fib(15)=610. fib was called 610 times
fib(16)=987. fib was called 987 times
fib(17)=1597. fib was called 1597 times
fib(18)=2584. fib was called 2584 times
fib(19)=4181. fib was called 4181 times
```

| AGE → / TIME ↓ | | | | | | | TOTAL |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | $P(1) = 1$ |
| | 1 | | | | | | $P(2) = 1$ |
| 1 | | 1 | | | | | $P(3) = 2$ |
| 1 | 1 | | 1 | | | | $P(4) = 3$ |
| 2 | 1 | 1 | | 1 | | | $P(5) = 5$ |
| 3 | 2 | 1 | 1 | | 1 | | $P(6) = 8$ |
| 5 | 3 | 2 | 1 | 1 | | 1 | $P(7) = 13$ |

$P(8) = P(7) + P(6)$

Let $P(n)$ = pop at time $n$

Fibonacci Sequence

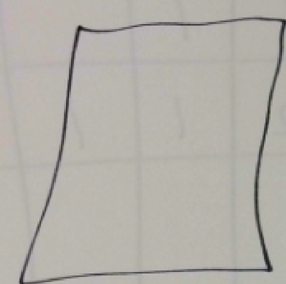
$f_n = f_{n-1} + f_{n-2} \quad n \geq 3$

$f_1 = f_2 = 1$

$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots$

$$\frac{34}{21} \qquad \frac{55}{34} \qquad \frac{89}{55}$$

Golden Ratio



How many $n$-bit strings do not have 2 consecutive 0's?

Let $p(n)$ = # of $n$-bit strings w/out 2 consec. 0's.

$p(5)$

_ _ _ $\underline{1}$ $\underline{0}$ $p(3)$

_ _ _ _ $\underline{1}$ $p(4)$

$p(5) = p(4) + p(3)$

$$\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$