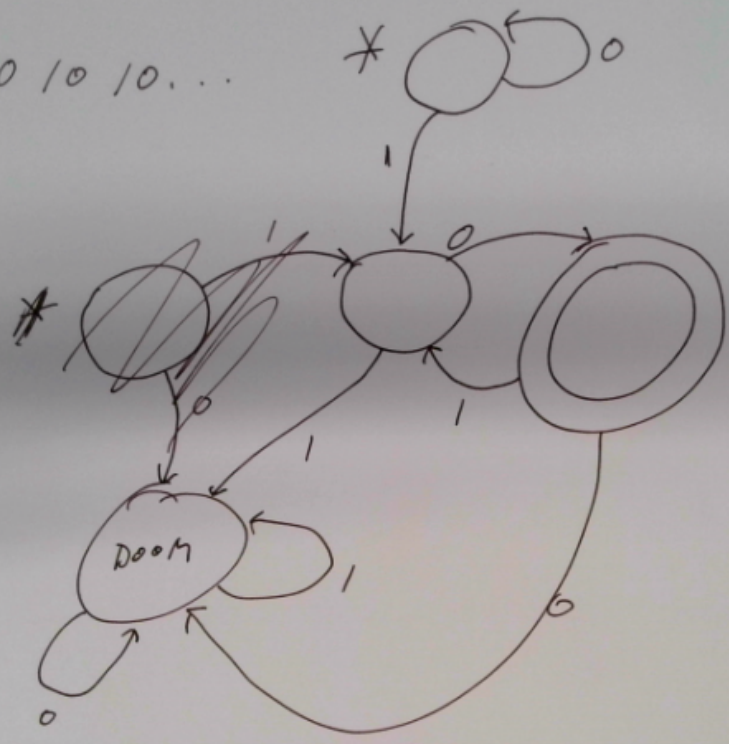
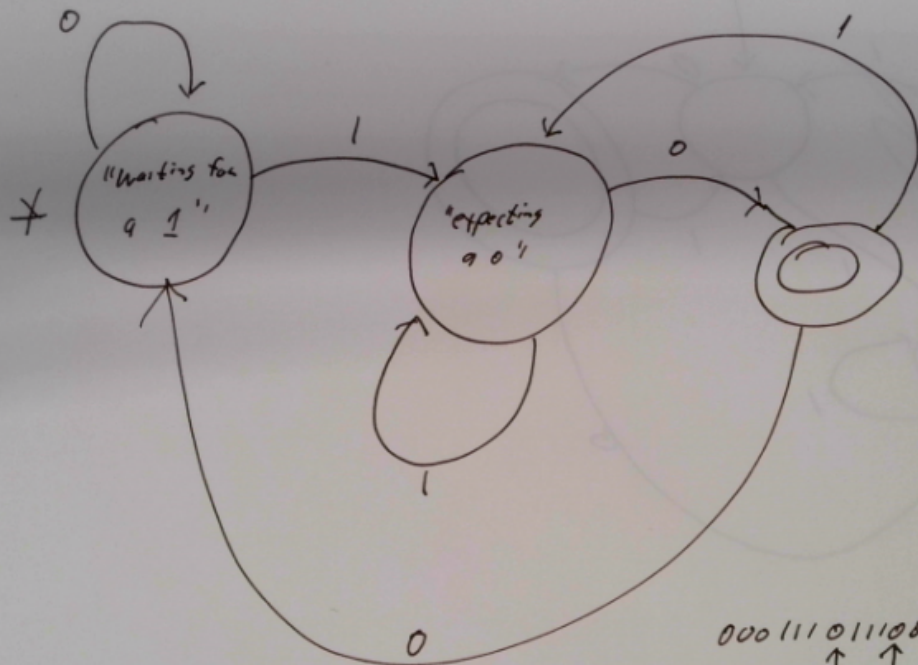


10 10 10...



101010...

80011110  
↑



00011101110010  
↑ ↑ ↑

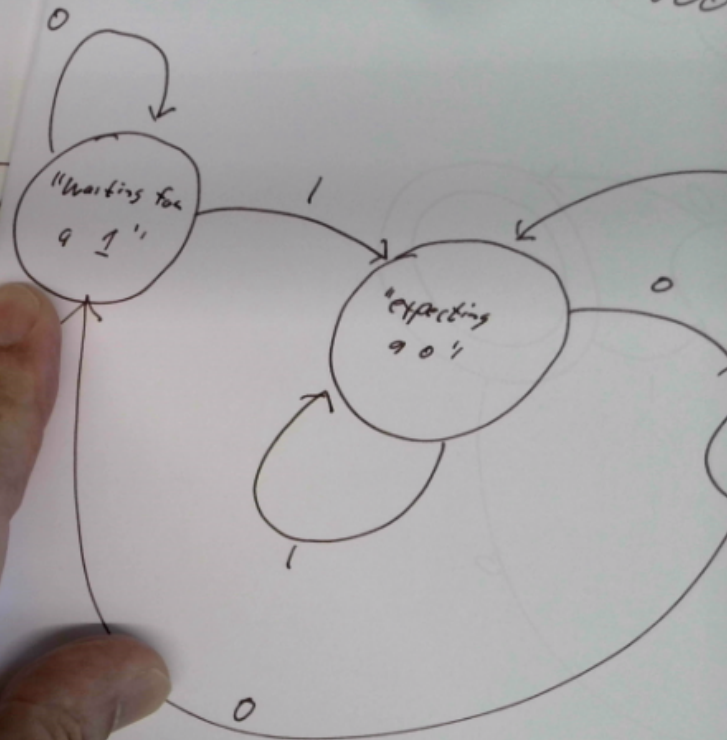


111(10)<sup>n</sup>



101010...

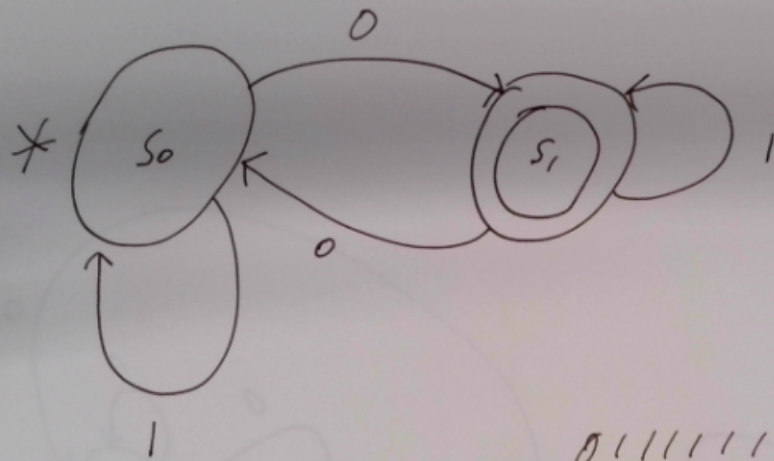
000



000111011  
↑



What strings does this recognize?



011111111

000111111

1110111

# BNF

## Backus-Naur Form

$AS ::= \langle VAR \rangle = \langle EXP \rangle$

$EXP ::= \langle EXP \rangle \langle OP \rangle \langle EXP \rangle \mid (\langle EXP \rangle) \mid \langle VAR \rangle \mid \langle NUM \rangle$

$VAR ::= \langle Letter \rangle \langle VAR_2 \rangle \mid \langle Letter \rangle$

$VAR_2 ::= \langle Symbol \rangle \langle VAR \rangle \mid \langle Symbol \rangle$

$NUM ::= - \langle UNUM \rangle \mid \langle UNUM \rangle$

$UNUM ::= \langle digit \rangle \langle UNUM \rangle \mid \langle digit \rangle$

$digit ::= 0 \mid 1 \mid 2 \mid \dots \mid 9$

$letter ::= A \mid D \mid \dots \mid z$

$Symbol ::= \langle letter \rangle \mid \langle digit \rangle \mid \_$

# BNF

## Backus-Naur Form

$AS ::= \langle VAR \rangle = \langle EXP \rangle$

$EXP ::= \langle EXP \rangle \langle OP \rangle \langle EXP \rangle \mid ( \langle EXP \rangle ) \mid \langle VAR \rangle \mid \langle NUM \rangle$

$VAR ::= \langle Letter \rangle \langle VAR_2 \rangle \mid \langle Letter \rangle$

$VAR_2 ::= \langle Symbol \rangle \langle VAR \rangle \mid \langle Symbol \rangle$

$NUM ::= - \langle UNUM \rangle \mid \langle UNUM \rangle$

$UNUM ::= \langle digit \rangle \langle UNUM \rangle \mid \langle digit \rangle$

$digit ::= 0 \mid 1 \mid 2 \mid \dots \mid 9$

$letter ::= A \mid D \mid \dots \mid z$

$Symbol ::= \langle letter \rangle \mid \langle digit \rangle \mid \_$

$OP ::= + \mid - \mid * \mid /$



# BNF

## Backus-Naur Form

$AS ::= \langle VAR \rangle = \langle EXP \rangle$

$EXP ::= \langle EXP \rangle \langle OP \rangle \langle EXP \rangle \mid ( \langle EXP \rangle ) \mid \langle VAR \rangle \mid \langle NUM \rangle$

$VAR ::= \langle Letter \rangle \langle VAR_2 \rangle \mid \langle Letter \rangle$

$VAR_2 ::= \langle Symbol \rangle \langle VAR \rangle \mid \langle Symbol \rangle$

$NUM ::= - \langle UNUM \rangle \mid \langle UNUM \rangle$

$UNUM ::= \langle digit \rangle \langle UNUM \rangle \mid \langle digit \rangle$

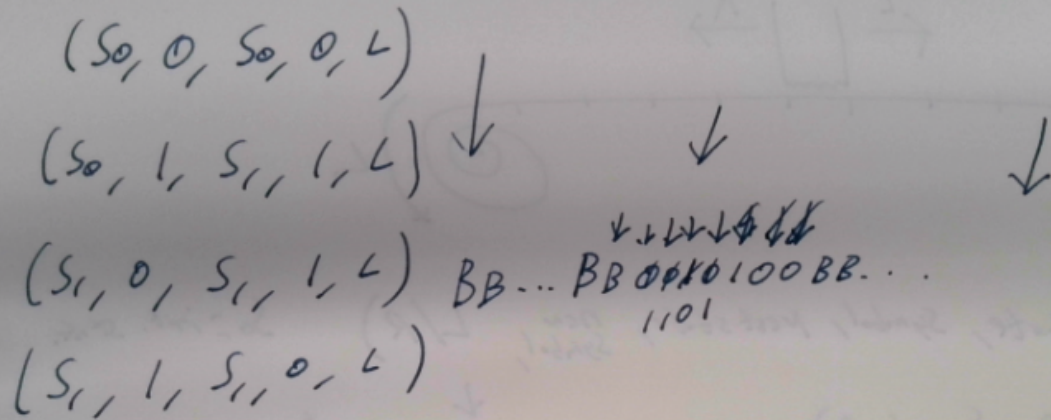
$digit ::= 0 \mid 1 \mid 2 \mid \dots \mid 9$

$letter ::= A \mid D \mid C \mid \dots \mid Y \mid Z$

$Symbol ::= \langle letter \rangle \mid \langle digit \rangle \mid \_$

$OP ::= + \mid - \mid * \mid /$





State =  $S_0 S_1$

in: 0010100  
 out: 1101100

- $(S_{10}, 0, S_{10}, 0, R)$
- $(S_{10}, 1, S_{10}, 1, R)$
- $(S_{10}, B, S_0, B, L)$



Tape  
a11111b01010cyxy01d101  
Head

Current state  
50

Running...

Steps  
11975

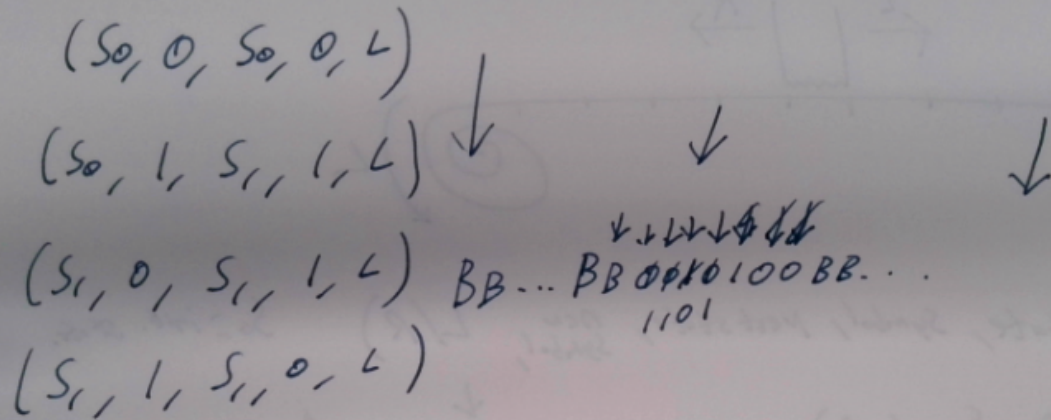
Turing machine program

```
1 ; Tests if a given number is prime.  
2 ; Input: a single natural number in binary.  
3  
4 ; This is very inefficient and slow, so be prepared to wait!  
5  
6 ; set up environment  
7 0 * * l 1  
8 1 * a r 2  
9 2 _ b l 3  
10 2 * * r 2  
11 3 a a r 4  
12 3 x x r 4  
13 3 y y r 4  
14 3 * * l 3  
15 4 0 x r 5x  
16 4 1 y r 5y  
17 4 b b l 9  
18 9 x 0 l 9  
19 9 y 1 l 9  
20 9 a a r 10  
21 5x b b r 6x  
22 5x * * r 5x  
23 5y b b r 6y  
24 5v * * r 5v
```

Controls

Run  Run at full speed  
Pause  
Step Undo  
Reset

Initial input: 11111  
[Advanced options](#)  
[Load an example program](#)  
[Save to the cloud](#)



State =  $S_0 S_1$

in: 0010100  
 out: 1101100

- $(S_{10}, 0, S_{10}, 0, R)$
- $(S_{10}, 1, S_{10}, 1, R)$
- $(S_{10}, B, S_0, B, L)$

DETERMINISTIC TURING MACHINE (DTM)

NON-DETERMINISTIC TURING MACHINE

(NDTM)

$O(n^2)$   
 $O(n^3)$  }  $\overline{P}$  problem  
poly time

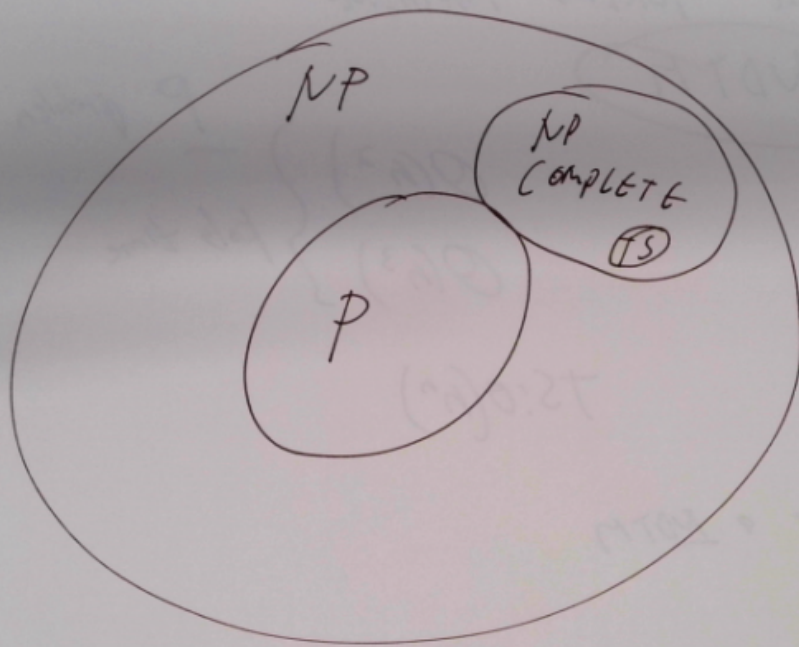
TS:  $O(n^n)$

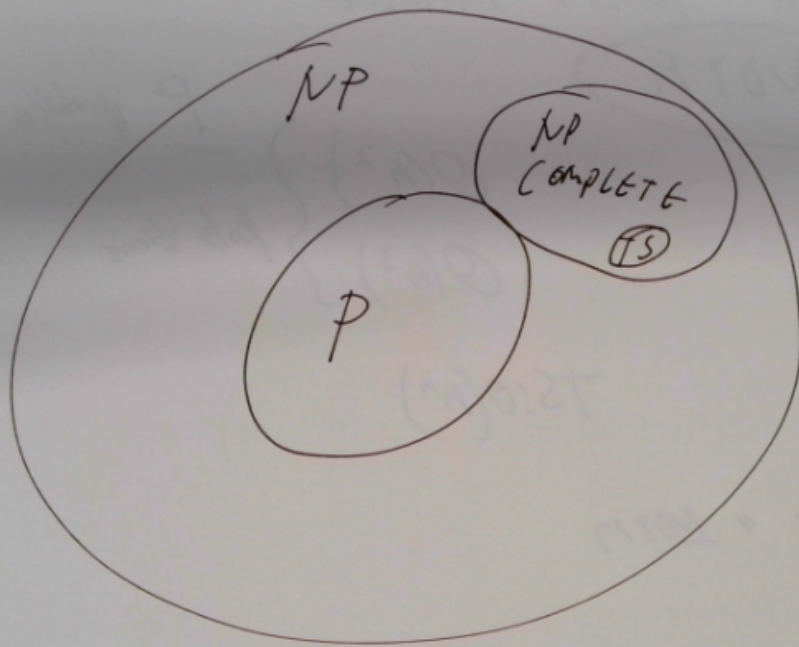
TS: poly time on a NDTM

NP problem

Q: Can TS be solved in poly time  
on a DTM







## Halting Problem

$(S_0, 0, S_0, 0, L)$   
 $(S_0, 1, S_0, 1, L)$   
 $(S_0, B, S_0, B, C)$



$$|z| = |2z| = |z > 1000|$$

↪ 1-1 contr. ↪

$$x \rightarrow 2x$$

---

$$|R| > |Z|$$

(1000, 1000)  
(1000, 1000)  
(1000, 1000)

---

Handwritten notes

Suppose  $f: \mathbb{Z}^+ \rightarrow \mathbb{R}$   ~~$[0, 1] \subseteq \mathbb{R}$~~

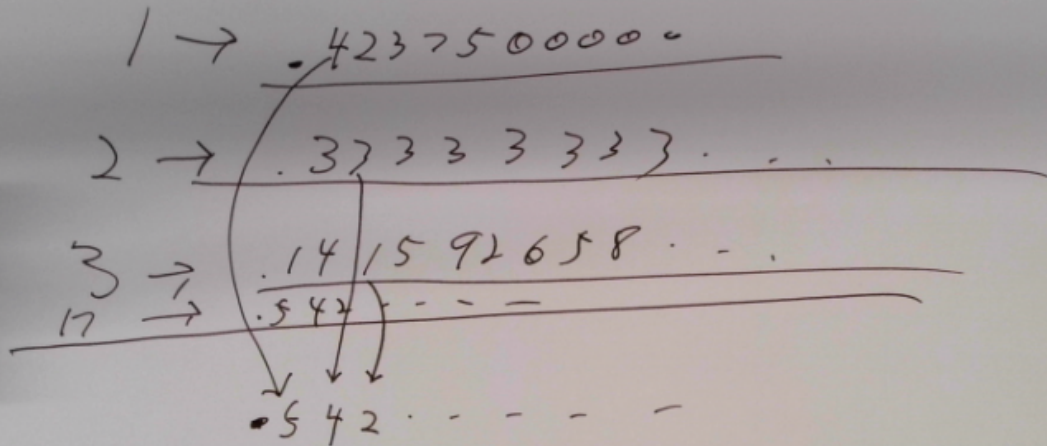
1  $\rightarrow$  .423750000.

2  $\rightarrow$  .33333333.

3  $\rightarrow$  .141592658.

17  $\rightarrow$  .542

.542



Suppose  $f: \mathbb{Z}^+ \rightarrow \mathbb{R}$   $[0, 1] \subseteq \mathbb{R}$

1  $\rightarrow$  .4237500000

2  $\rightarrow$  .33333333...

3  $\rightarrow$  .141592658...

17  $\rightarrow$  .542...

.542...