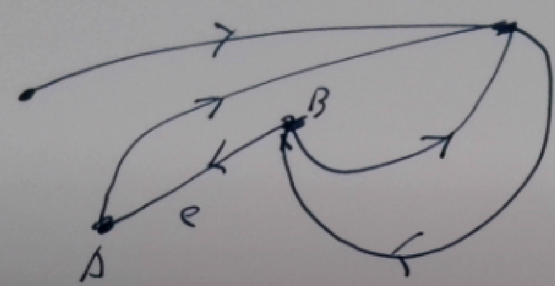
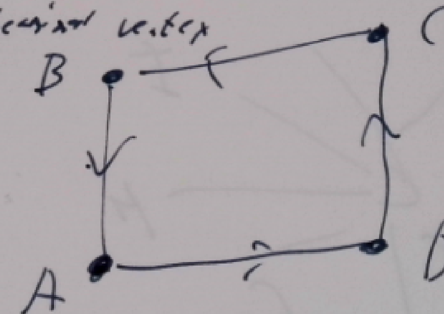


e is incident to B & A

e is an edge from B to A

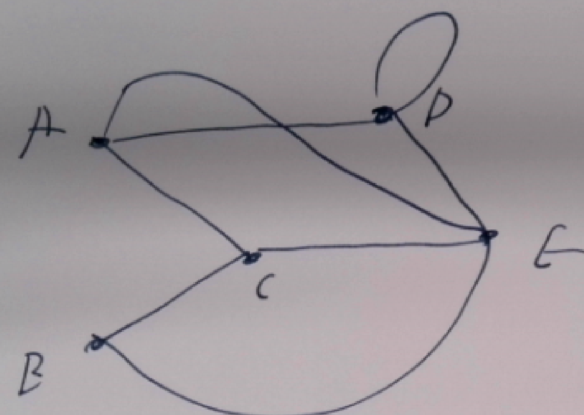
B is the initial vertex

A is the terminal vertex



The <u>degree</u> of a vertex V is the # of edges incident to V

(loops count twice)



$deg(A) = \cancel{2}\ 1$

$deg(B) = 2$

$deg(C) = 3$

$deg(D) = \cancel{2}\ 4$

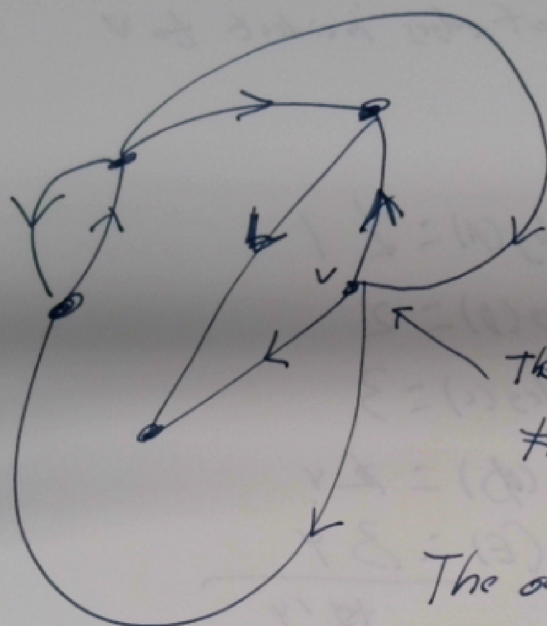
$deg(E) = \cancel{3}\ 4$

$\cancel{12}\ 14$

~~#egs~~ #edges = ~~6~~ 7

In a graph $G = (V, E)$, $V = \{v_1, v_2, \ldots, v_n\}$

$$\boxed{\sum_{i=1}^{n} deg(v_i) = 2|E|}$$



The in-degree $\deg^-(v) =$ # of edges that terminate at v

$= 1$

The out-degree $\deg^+(v) =$ # edges starting at v $= 3$

$$\sum_{v\in V} \deg^-(v) = \sum_{v\in V} \deg^+(v) = |E|$$

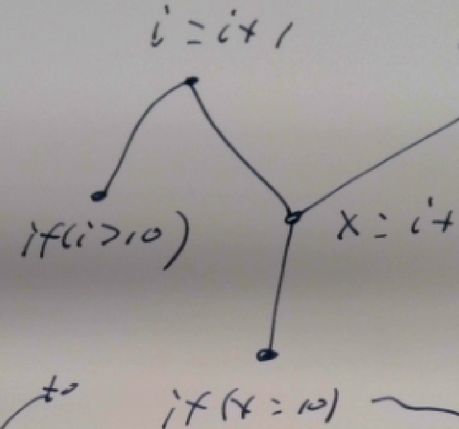
i = i+1

j = j+1

x = i+j

if (i > 10) ~

if (x = 10) ~



from to

Adjacency Matrix

	A	B	C	D	E
A	0	0	1	0	1
B	1	0	1	1	0
C	1	1	0	0	1
D	0	1	0	0	1
E	1	0	1	1	0

Adjacency List

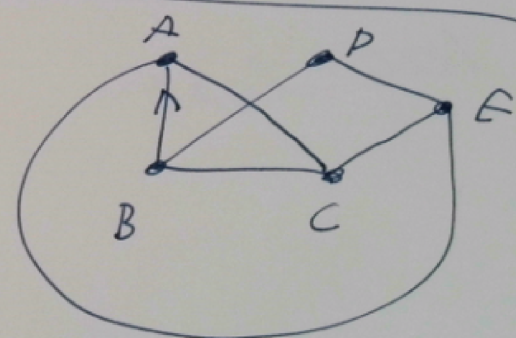
A: B, C, E

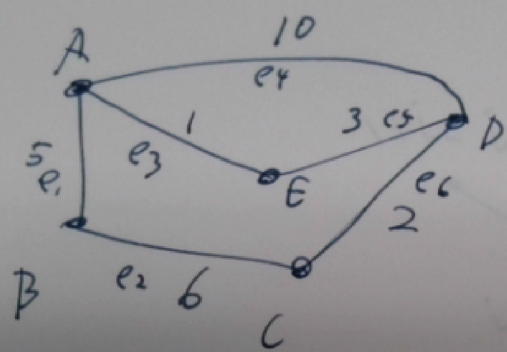
B: A, C, D

C: A, B, E

D: B, E

E: A, C, D



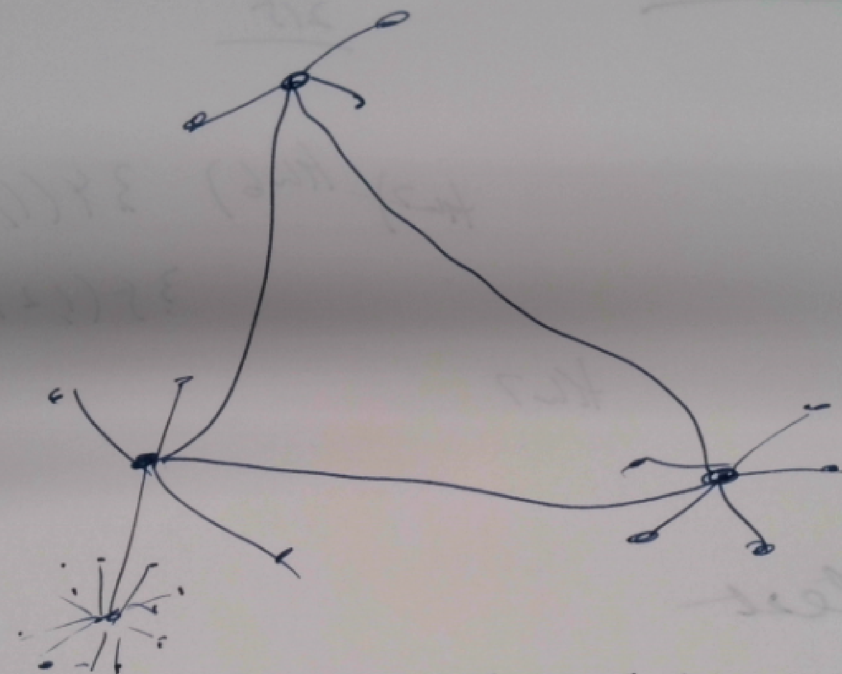


Weighted Graph

	A	B	C	D	E
A	0	5	0	10	1
B					
C					
D					
E					

~~Incide~~ Incidence Matrix

	e_1	e_2	e_3	e_4	e_5	e_6
A	1	0	1	1	0	0
B	1	1	0	0	0	0
C	0	1	0	0	0	1
D	0	0	0	1	1	1
E	0	0	1	0	1	0



Sparse Matrix

mostly 0's

10^6 vertices

10^6

10^6

10^{12} = 1 trillion entries

Isomorphisms of Graphs

2 Simple graphs $G_1 = (V_1, E_1)$ & $G_2 = (V_2, E_2)$

are <u>isomorphic</u> if $\exists$ a bijective map

$f: V_1 \to V_2$ such that vertices $a, b \in V_1$

are adjacent in G_1 iff $f(a), f(b)$ are

adjacent in G_2.

$G_1 \cong G_2$

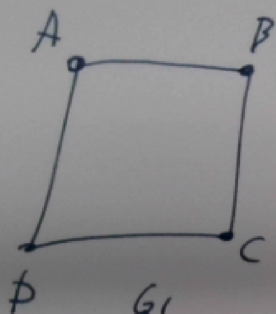
Graph ~~Invariants~~ Invariants

\# of vertices

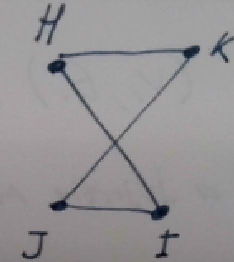
\# of edges

Set of degrees of all vertices

G_1



G_2



G_1

	A	B	C	D
A	0	1	0	1
B	1	0	1	0
C	0	1	0	1
D	1	0	1	0

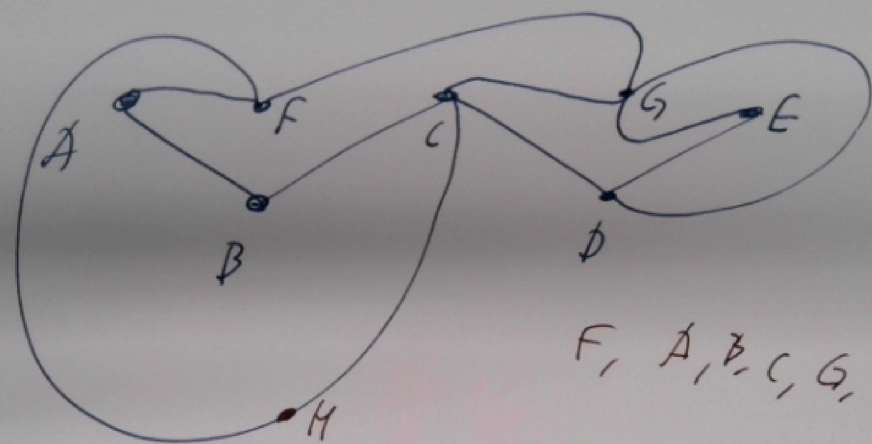
G_2

	H	I	J	K
H	0	1	0	1
I	1	0	1	0
J	0	1	0	1
K	1	0	1	0

G_2

	H	K	I	J
H	0	1	1	0
K	1	0	0	1
I	1	0	0	1
J	0	1	1	0

A path through a graph

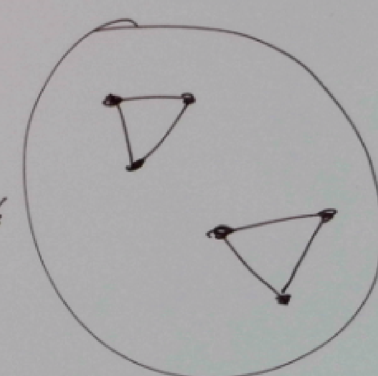


F, A, B, C, G, F, H

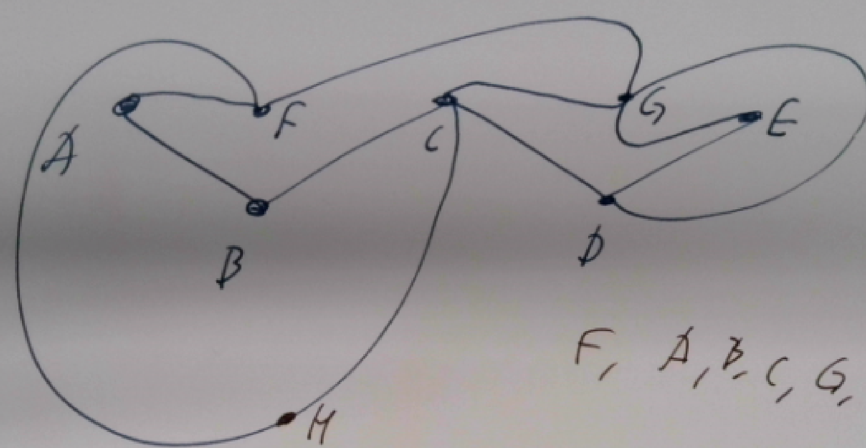
A to E: A, B, C, D, E
A, B, C, G, D, E
A, F, G, E
Simple

A, B, A, B, A, F, G, E – not simple

not connected



A *path* through a graph



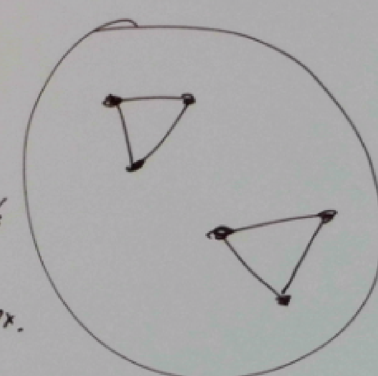
F, A, B, C, G, F, H

A to E: A, B, C, D, E
A, B, C, G, D, E
A, F, G, E

Simple

A, B, A, B, A, F, G, E – not simple

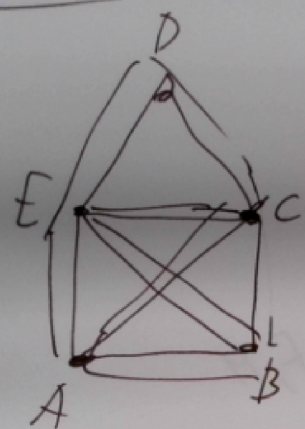
not connected



A *circuit* is a path that starts & ends @ same vertex.

An Euler path is a simple path that includes all edges

An Euler Circuit and is a circuit.



~~X~~ ~~D~~ C B E A C

~~A~~ E D C B E C A B

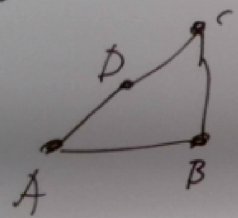
Theorem: A graph $G=(V,E)$ has an Euler Path

iff each vertex has an even degree (except possibly 2 vertices)

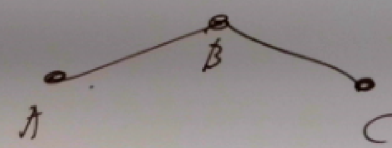
has an Euler circuit iff each vertex has an even degree.

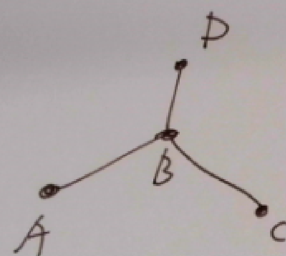
A **Hamilton path** visits each vertex exactly once.

yes:



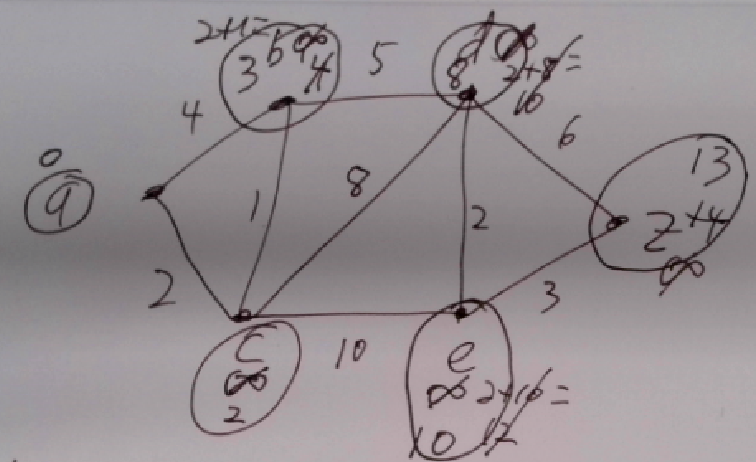
yes





NO

Dijkstra's Algorithm



from a to z

Write tentative distances

Circle starting vertex "visited"

Loop

- Relax distances to unvisited vertices
- Visit a vertex with smallest tentative distance from a

Dijkstra's Algorithm

A-H