Wed 13 Nov 2024, 12:04:03 Take a Photo

Take a Photo

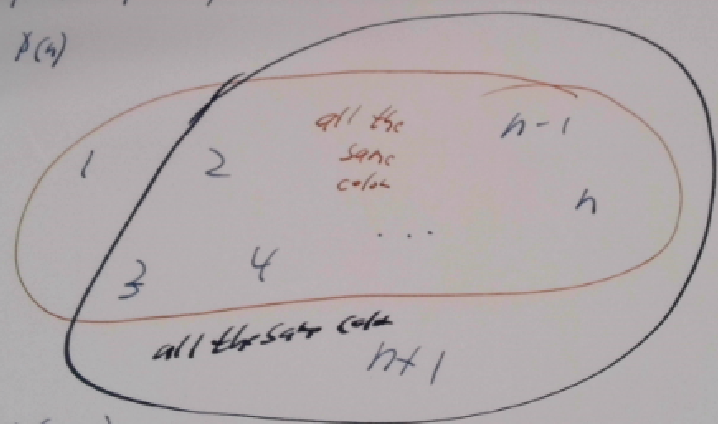
Prove: all horses are ~~brown~~ the same color

Let $P(n)$ be the prop: "in any group of n horses, all of them are ~~brown~~ the same color"

$P(1)$ True

Prove $P(n) \to P(n+1)$

given $P(n)$



$\therefore P(n+1)$

$\therefore P(n) \ \forall n \geq 1$

Photo Video Burst

Effects

BAD EXAMPLE

prove $n = n+1$ $(n \geq 1)$

Let $P(n)$ be the prop "$n = n+1$"

Prove $P(n) \to P(n+1)$

proof:

1)	$P(n)$	hyp
2)	$n = n+1$	def of P
3)	$n+1 = n+1+1$	add 1 to both sides of 2)
	$n+1 = n+2$	alg.
	$P(n+1)$	

$\therefore P(n) \to P(n+1)$

Counting

Product Rule

Suppose you wish to do some procedure P
and you can do P by first doing P_1
and then doing P_2.

If P_1 can be done n_1 ways
and P_2 can be done n_2 ways
then P can be done $n_1 \cdot n_2$ ways.

Ex: 2 people, 20 desks.
How many ways can you seat those 2 people?

P_1 : Choose where 1^{st} student sits. $n_1 = 20$
P_2 : ~ 2^{nd} ~. $n_2 = 19$

$20 \cdot 19 = 380$

Ex: A student ID# consists of one letter & 2 digits

How many student ID #s are there?

P_1: pick letter $\quad n_1 = 26$

P_2: pick 2 digit # $\quad n_2 = 100$

2600

P_1: pick letter $\quad n_1 = 26$

P_2: pick 1st digit $\quad n_2 = 10$

P_3: pick 2nd digit $\quad n_3 = 10$

$26 \cdot 10 \cdot 10 = 2600$

How many ways can you make a string of 4 bits?

P_1: pick 1^{st} bit $\qquad n_1 = 2$

P_2: — 2^{nd} — $\qquad n_2 = 2$

P_3: ~ 3^{rd} ~ $\qquad n_3 = 2$

P_4: ~ 4^{th} ~ $\qquad n_4 = 2$

$$2 \cdot 2 \cdot 2 \cdot 2 = 16$$

Shuffle a deck of 52 cards: How many outcomes?

~~pick 1^{st}~~

pick card to be 1^{st} $\qquad 52$

pick card to be 2^{nd} $\qquad 51$

$\qquad 50$

$\vdots \qquad \vdots$

pick card to be 52^{nd} $\qquad 1$

$$52 \cdot 51 \cdot 50 \cdots 2 \cdot 1 = 52!$$

Given a finite set S with $|S| = n$,

How many subsets can you make?

$$\left.\begin{array}{ll} \underline{\text{Decide if } 1^{st} \text{ element is in subset}} & 2 \\ \underline{\quad\quad}\ 2^{o}\ \underline{\quad\quad} & 2 \\ \vdots & \vdots \\ \text{decide if } n^{th} \text{ element is in subset.} & 2 \end{array}\right\} n$$

$$\underbrace{2 \times 2 \times \cdots \times 2}_{n} = 2^n$$

The Sum Rule

If a procedure P can be done by

either doing P_1

or doing P_2,

and P_1 can be done n_1 ways

and P_2 ——— n_2 ———

then P can be done $n_1 + n_2$ ways.

Ex: 5 teachers, 100 students

pick either a teacher or a student to give a speech

$$\begin{array}{r} 5 \\ 100 \\ \hline 105 \end{array}$$

5 Soups 5
4 Salads 4

9

Combos

A variable is 1 or 2 chars, 1st char: letter
2nd = letter or digit

How many vars?

P_1: make a 1-char var $n_1 = 26$

OR

P_2: make a 2-char var $n_2 = 26 \cdot 36$

pick 1st char 26
pick 2nd char 36

$26 \cdot 36$

$26 + 26 \cdot 36$

$26 \cdot 37$

SUBTRACTION

$$|A \cup B| = |A| + |B| - |A \cap B|$$

"principle of inclusion/exclusion"

How many 8-bit strings start with 1 or end with 00?

$A = \{ \underline{1}\ \_\ \_\ \_\ \_\ \_\ \_\ \_ \}$ $|A| = 128$ (2^7)

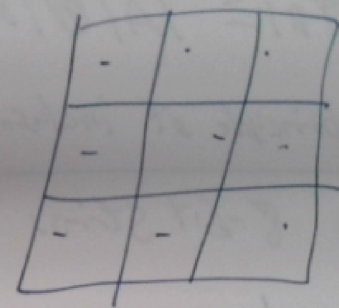
$B = \{ \_\ \_\ \_\ \_\ \_\ \_\ \underline{0}\ \underline{0} \}$ $|B| = 64$ (2^6)

$|A \cup B|$ $128 + 64 - 32 = 160$

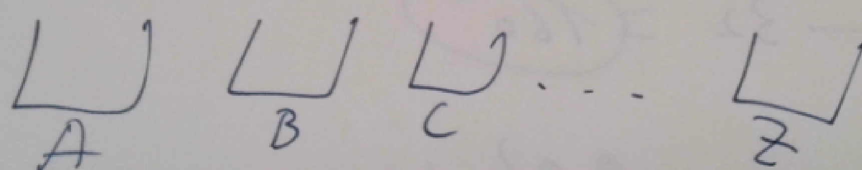
$A \cap B = \{ \underline{1}\ \_\ \_\ \_\ \_\ \_\ \underline{0}\ \underline{0} \}$ $|A \cap B| = 32$ (2^5)

The Pigeonhole Principle

If k is a positive integer and $k+1$ objects are placed in k boxes, then at least one box has 2 or more objects in it.



Ex: ~~2?~~ given 27 people, at least 2 have the same 1st initial



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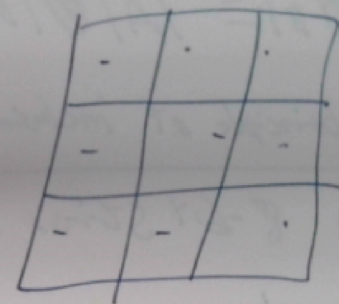

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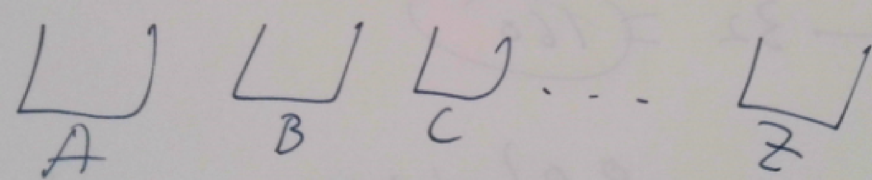


fact: Every positive integer has some multiple consisting of only 0's & 1's.

examples: $1 \cdot 1 = 1$

$2 \cdot 5 = 10$

$3 \cdot 3370 = 11010$

$4 \cdot 25 = 100$

$5 \cdot 20 = 100$

$6 \cdot$

$17 \cdot 653 = 11101$

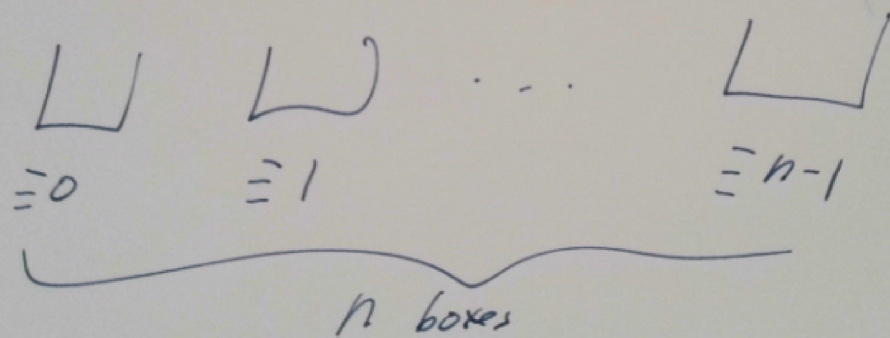
Given $n \geq 1$

Consider the $n+1$ numbers

$$n+1 \left\{ \begin{array}{l} 1 \\ 11 \\ 111 \\ 1111 \\ \vdots \\ \underbrace{1111\ldots111}_{n+1} \end{array} \right.$$

$\underline{\text{mod } n}$

$$\begin{array}{r} 1111111 \\ -\ 111 \\ \hline 1111000 \end{array} \equiv 0 \pmod{n}$$



Generalized Pigeonhole Principle

N objects, k boxes, then at least one box has $\left\lceil \frac{N}{k} \right\rceil$ objects

$\lceil x \rceil$ = smallest int $\geq x$

$\lceil 2.5 \rceil = 3$

$\lceil 2.99 \rceil = 3$

$\lceil 3 \rceil = 3$

$\lceil 3.001 \rceil = 4$

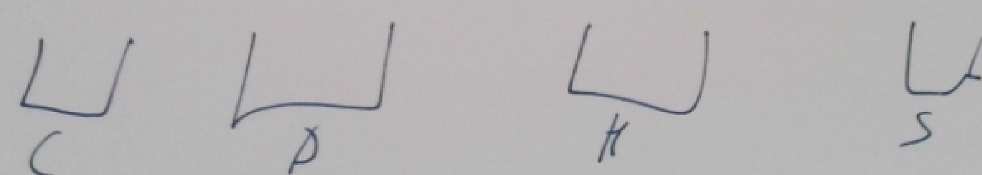
Ex: 52 cards, 13 of each suit

How many cards do you need to pick to guarantee you have at least 3 of the same suit?

find smallest N such that

$$\left\lceil \frac{N}{4} \right\rceil = 3$$

$N = 9$



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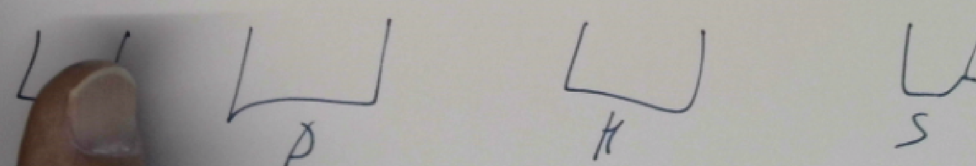

Permutations

A permutation is an <u>ordered</u> arrangement of objects

An r-permutation is an ordered arrangement of r objects.

ex: $\{a, b, c\}$

cab is a permutation

so is abc

acb

etc.

cb is a 2-permutation.

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ex: 5 Students

pick 3 to present to the class

P_1: pick 1^{st} Student — $n_1 = 5$

P_2: — 2^{nd} — $n_2 = 4$

P_3 — 3^{rd} — $n_3 = 3$

$5 \cdot 4 \cdot 3 = 60$

$P(n,r) = $ # of r-permutations of n objects

ex: $P(5,3) = 60$

$P(n,r)$ $\quad P_1 \quad n_1 = n$

$P_2 \quad n_2 = n-1$

$P_3 \quad n_3 = n-2$

$\vdots \quad \vdots$

P_n

$$P(n,r) = \underbrace{n(n-1)(n-2)(n-3)\cdots}_{r \text{ times}}$$

$$\frac{n(n-1)(n-2)\cdots(3)(2)(1)}{\underbrace{n(n-1)(n-2)\cdots}_{r \text{ terms}}}$$

$$P(12,7) = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6$$

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 (5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)}{(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)}$$

$$P(12,7) = \frac{12!}{5!} = \frac{12!}{(12-7)!}$$

$$P(n,r) = \frac{n!}{(n-r)!}$$

10 runners, 3 medals

$$P(10,3) = \frac{10!}{7!} = 10 \cdot 9 \cdot 8 = \underline{720}$$

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5 people in a race: how many finishing orders?

$$P(5,5) = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5!}{1} = 5! = 120$$

# of perms of A-H, but must have ABC in order

DGFABC

HABCE

ABC, D, E, F, G, H

$P(6,6)$

DEF: A combination is a unordered ~~arrangement~~ collection

an r-comb is an unordered collection of r objects

ex: 4 people, pick 3.

$$\frac{4!}{3!\,1!} = 4$$

$C(n,r) =$ # of r-comb. of n objects

$$= \frac{P(n,r)}{r!} = \frac{n!}{r!(n-r)!}$$

A B C D

A B C
A B D
B C D
A C D

POKER! 52 cards

5 in your hand

$\cancel{C(52,5)} = \frac{52!}{5!\,47!} = 2,598,960$

$C(52,5)$

$C(n,r)$ can be written as $\binom{n}{r}$

"n choose r"

Q: How many n-bit strings have r 1's?

$C(n,r)$

$$C(n,0) + C(n,1) + C(n,2) \cdots + C(n,n) = 2^n$$

$$\frac{n!}{0!\,n!} + \frac{n!}{1!\,(n-1)!} + \frac{n!}{2!\,(n-2)!} + \cdots + \frac{n!}{n!\,0!} = 2^n$$