

```
* Welcome to the Engineering and Computer Science Department's Linux server! *
*
*           Use of this system is governed by Clark College's
*           Computer Policies and Computer Use Guidelines.
*           For more details, see
*
*   clark.edu/campus-life/student-support/computing_resources/policies.php *
*****

* Documentation:  https://help.ubuntu.com
* Management:    https://landscape.canonical.com
* Support:       https://ubuntu.com/pro
You do not have any new mail.
Last login: Wed Oct 16 11:57:30 2024 from 192.102.5.50
{1769} /usr/local/bin/CSE215PA1
Enter set A: abcdefg
Enter set B: cegijkl
A={a,b,c,d,e,f,g}
B={c,e,g,i,j,k,l}
|A|=7, |B|=7
A U B={a,b,c,d,e,f,g,i,j,k,l}
A ^ B={c,e,g}
A \ B={a,b,d,f}
Goodbye!
{1770}
```

```
|A|=7, |B|=7
A U B={a,b,c,d,e,f,g,i,j,k,l}
A ^ B={c,e,g}
A \ B={a,b,d,f}
Goodbye!
{1770} /usr/local/bin/CSE215PA1
Enter set A: aaaaaaaa
Enter set B: aaaaabbbbbbb
A={a}
B={a,b}
|A|=1, |B|=2
A U B={a,b}
A ^ B={a}
A \ B={}
Goodbye!
{1771} /usr/local/bin/CSE215PA1
Enter set A:
Enter set B: x
A={}
B={x}
|A|=0, |B|=1
A U B={x}
A ^ B={}
A \ B={}
Goodbye!
{1772} █
```

input: "abcabcd"

Set: empty

for each letter in input:

if letter  $\notin$  Set,

add to set

---

Union (A, B)

Set: empty

for each letter in A

if  $\notin$  A, add to A

---

for each letter in B

```
|A|=7, |B|=7
A U B={a,b,c,d,e,f,g,i,j,k,l}
A ^ B={c,e,g}
A \ B={a,b,d,f}
Goodbye!
{1770} /usr/local/bin/CSE215PA1
Enter set A: aaaaaaaaa
Enter set B: aaaaabbbbbbb
A={a}
B={a,b}
|A|=1, |B|=2
A U B={a,b}
A ^ B={a}
A \ B={}
Goodbye!
{1771} /usr/local/bin/CSE215PA1
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```

input: "abcabcd"

Set: empty

for each letter in input:

if letter  $\notin$  Set,

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---

Union (A, B)

Set: empty

for each letter in A

if  $\notin$  A, add to A

---

for each letter in B

# Number Theory

( $a, b, etc \in \mathbb{Z}$ )

## Divisibility

Definition: We say "a divides b" if

$$\exists q \in \mathbb{Z} : b = a \cdot q$$

(or, we might say:  $\frac{b}{a}$  is a whole number)

ex: 5 divides 20

3 divides 21

5 divides 5

1 divides 5

-3 divides 21

3 does not divide 20

a divides b  
b is divisible by a  
a is a factor of b  
b is a multiple of a

Def/new symbol

$a|b$  means "a divides b" ( $\frac{b}{a} \in \mathbb{Z}$ )

Note:  $a|b$  is a proposition.

$$5|20$$

$$5 \nmid 21$$

$$3|21$$

---

The Division Algorithm.

$$\frac{21}{5} = \sim R \sim$$

↑  
remainder

↙ quotient

$$\frac{20}{5} = \sim R 0$$



$$\frac{21}{5} = \underline{4} R \underline{1} R \geq 0$$

$$5 R \underline{-4} < 5$$

$$\frac{21}{5} = 3 R \underline{6}$$

$$2 R \underline{11}$$

$$1 R \underline{16}$$

$$0 R \underline{21}$$

$$-1 R \underline{26}$$

Theorem: If  $a|b$  and  $b|c$  then  $a|c$

→ If  $a|b$  and  $a|c$  then  $a|(b+c)$

prove this

Proof: 1.  $a|b$

Hyp.

2.  $a|c$

Hyp.

3.  $\exists q \in \mathbb{Z} : b = a \cdot q$

def of " $|$ ", 1.

4.  $\exists s \in \mathbb{Z} : c = a \cdot s$

def of " $|$ ", 2.

5.  $b+c = a \cdot q + a \cdot s$

Subs, 3, 4.

$= a(\underline{q+s})$

alg.

6.  $q+s \in \mathbb{Z}$

3, 4. closure of ints.

7.  $a|(b+c)$

5, 6. def of " $|$ "

Q.E.D.

Theorem: If  $a|b$  and  $b|c$  then  $a|c$

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prove this

Proof: 1.  $a|b$

Hyp.

2.  $a|c$

Hyp.

3.  $\exists q \in \mathbb{Z} : b = a \cdot q$

def of "1" 1.

4.  $\exists s \in \mathbb{Z} : c = a \cdot s$

def of "1" 2.

5.  $b+c = a \cdot q + a \cdot s$

Subs, 3, 4.

$= a(q+s)$

alg.

6.  $q+s \in \mathbb{Z}$

3, 4. closure of ints.

7.  $a|(b+c)$

5, 6. def of "1"

Q.E.D.

## Modular Arithmetic

Fix a ~~diver~~ divisor (such as 5)

& think about remainders when we divide by 5.

$$\frac{21}{5} R 1$$

$$21 + 7 = 28$$

$$\frac{7}{5} R 2$$

$$\frac{28}{5} R 3$$

$$\frac{56^{200}}{5} R 1^{200} = 1$$

$$\frac{8}{5} R 3$$

$$\frac{7+7}{5} R 4$$

$$2 \cdot 3 = 6$$
$$\frac{6}{5} R 1$$

$$\frac{56}{5} = 1$$

$$56 = 7 \cdot 8$$

### 3 Assumptions

You may assume input lines are no more than 120 characters each, and include only letters and digits.

### 4 Recommendations

Start this assignment by writing a program that reads a membership table from a file. With those two files, calculate the union of the two sets.

```
File Edit View Bookmarks Settings Help
67645225770308549126622920763081407530838118912409124760438945326377\
5494022488452059
60^200
42682522381202740079697489151877373234298874535448942949547907893511\
29295496197390190721393407570972968128154666761298309544652405175952\
423840155919198453760000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000\
59^202
51535483121569986891868869251430720438550662282618107481923992602438\
87674632750308604657405860190053541490926871339438210142405777868217\
94401819505720870367679484141592916380097584632938918507564453595111\
47132021468579766063831930110369014302352643965100886672774344944768\
12910683204482043984707523250218030443194490158321383608658977742562\
754147326818671481
59^205
10584305988024922337865136497989589932949096468941824296530067676696\
29406728399625630915933358159973006297865069908822480160837156252796\
73312451288265446635243644773516211573228061834328362144165079914910\
39886827437195443774423737974137477788402883664908455003967722190411\
53358783205853317711535246417606528874392841194225887444162772189789\
795884023834691930096299
[2002]
```

DEF: If  $\frac{a}{b}$  has a remainder of  $r$ ,

we say

$$a \equiv r \pmod{b}$$

↑  
"a is congruent to r, modulo b"

---

ex:  $21 \equiv 1 \pmod{5}$

$$57 \equiv 2 \pmod{5}$$

$$21 + 57 \equiv 1 + 2 \equiv 3 \pmod{5}$$

$$21 \cdot 57 \equiv 1 \cdot 2 \equiv 2 \pmod{5}$$

Theorem: If  $a \equiv b \pmod{m}$

and  $c \equiv d \pmod{m}$

Then  $a+c \equiv b+d \pmod{m}$

and  $a \cdot c \equiv b \cdot d \pmod{m}$

(and  $a-c \equiv b-d \pmod{m}$ )

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1. Define propositions P and Q and use them to give a real-world example of two fallacies we discussed (denying the hypothesis and affirming the conclusion).

Do the following problems using the formal proof techniques we've been discussing in class. Make sure:

- you begin by stating clearly what you are trying to prove
- you write the word "Proof"
- everything after that is a proposition
- each proposition is true (because it follows from prior steps and laws of logic; because it is a hypothesis; or because it is an assumption);
- each proposition has a number to its left
- each proposition has a justification (reason why it is true) to its right
- your final proposition is the proposition you are trying to prove
- you end with the words "QED"

Also, if you're trying to prove  $P \rightarrow Q$ , don't make the mistake of showing  $Q \rightarrow P$

2. Prove that if n is even, then  $10n-2$  is even.
3. Prove that if n is even, then  $n*n+n$  is even.
4. Prove that if  $n*n$  is even then n is even.
5. Prove that if  $n*n$  is odd then n is odd.
6. Prove (for all integers a, x and y) that if  $a|x$  and  $a|y$  then  $a|(2*x-3*y)$

All Bookmarks



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Empty content area

Think Mod 5

X	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

no 0  
1 in every row & column

mod 4

X	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	<del>3</del>

0 appears  
not nec. 1 in each  
row & column

Think Mod 5

X	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

no 0  
1 in every row & column

mod 4

X	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	<del>3</del>

0 appears  
not nec. 1 in each  
row & column

## Modular Exponentiation

Want to compute  $a^e \pmod{m}$

ex: Calculate  $7^{200} \pmod{5}$

STEP 1	$7^1 \equiv 2$
	$7^2 \equiv (7^1)^2 \equiv (2)^2 \equiv 4$
	$7^4 \equiv (7^2)^2 \equiv (4)^2 \equiv 16 \equiv 1$
	$7^8 \equiv (7^4)^2 \equiv (1)^2 \equiv 1$
	$7^{16} \equiv 1$
	$7^{32} \equiv 1$
	$7^{64} \equiv 1$
	$7^{128} \equiv 1$

STEP 2

$$200 = 128 + 64 + 8$$

STEP 3 (Law of Exponents)

$$\begin{aligned} 7^{200} &= 7^{(128+64+8)} \\ &= 7^{128} \cdot 7^{64} \cdot 7^8 \\ &\equiv 1 \cdot 1 \cdot 1 \\ &\equiv 1 \end{aligned}$$

```
[2001] bc -l
bc 1.07.1
Copyright 1991-1994, 1997, 1998, 2000, 2004, 2006, 2008, 2012-2017 Free Software Foundation, Inc.
This is free software with ABSOLUTELY NO WARRANTY.
For details type `warranty'.
7*200
1400
7^200
10461838291314357175018899611816813659819188550170233659950140084035\
12576742426225177438261490936405029306524825254631417406318034368359\
1188150754267339816534637456120001
7^201
73232868039200500225132297282717695618734319851191635619650980588245\
88037196983576242067830436554835205145673776782419921844226240578513\
8317055279871378715742462192840007
3^17
129140163
(3^17)/7
18448594.71428571428571428571
((3^17)-5)/7
18448594.0000000000000000000000
```

```
[2001] bc -l
bc 1.07.1
Copyright 1991-1994, 1997, 1998, 2000, 2004, 2006, 2008, 2012-2017 Free Software Foundation, Inc.
This is free software with ABSOLUTELY NO WARRANTY.
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7*200
1400
7^200
10461838291314357175018899611816813659819188550170233659950140084035\
12576742426225177438261490936405029306524825254631417406318034368359\
1188150754267339816534637456120001
7^201
73232868039200500225132297282717695618734319851191635619650980588245\
88037196983576242067830436554835205145673776782419921844226240578513\
8317055279871378715742462192840007
3^17
129140163
(3^17)/7
18448594.71428571428571428571
((3^17)-5)/7
18448594.0000000000000000000000
```

Calculate  $3^{17} \pmod{7}$

Using modular exponentiation

$$\begin{aligned} 3^1 &\equiv 3 \\ 3^2 &\equiv 9 \equiv 2 \\ 3^4 &\equiv 2^2 \equiv 4 \\ 3^8 &\equiv 4^2 \equiv 16 \equiv 2 \\ 3^{16} &\equiv 2^2 \equiv 4 \end{aligned}$$

$$\begin{aligned} 17 &= 16 + 1 \\ 3^{17} &= 3^{(16+1)} \\ &= 3^{16} \cdot 3^1 \\ \hline 3^{17} &\equiv 4 \cdot 3 \equiv 12 \equiv 5 \end{aligned}$$

$$3 \equiv ? \pmod{7}$$

$$\frac{3}{7} = 0 R 3$$

$$3 \equiv ? \pmod{18000000}$$

$$\frac{3}{18000000} = 0 R 3$$