Prove $q, \neg q \Rightarrow p$

p	q	$\neg q$	$q \wedge \neg q$	$(q \wedge \neg q) \to p$
F	F	T	F	T
F	T	F	F	T
T	F	T	F	T
T	T	F	F	T

A	B	$A \to B$
F	F	T
F	T	T
T	F	F
T	T	T

Let A be the prop 'I am sick'.
Let B be the prop 'Class is cancelled'
$A \to B$
(this is true)

Def: n is even if $\exists k \in \mathbb{Z} : n = 2k$

n is odd if $\exists k \in \mathbb{Z} : n = 2k+1$

Prove: if n is even, so is $5n$

Proof:

1.	n is even	Hypothesis
2.	$\exists k \in \mathbb{Z} : n = 2k$	1., def. of even
3.	$5n = 5 \cdot 2k = 10k$ $= 2(5k)$	2., algebra
4.	$5k \in \mathbb{Z}$	Closure of integers, 2
5.	$5n$ is even	Def of even, 3, 4

QED

Prove: If $5n$ is odd then n is odd

(Indirect) proof:

1.	n is even	Negation of conclusion
2.	$5n$ is odd	Hypothesis
3.	$\exists k \in \mathbb{Z} : n = 2k$	1., def. of even
4.	$5n = 5 \cdot 2k = 10k$ $= 2(5k)$	2., algebra
5.	$5k \in \mathbb{Z}$	Closure of integers, 2
6.	$5n$ is even	Def of even, 3, 4
	~~QED~~ $\times\!\!\times$	2, 6

$\therefore$ n is odd

QED

Prove: If $5n$ is odd then n is odd

(Indirect) proof:

1.	n is even	Negation of conclusion
2.	$5n$ is odd	Hypothesis
3.	$5n$ is even	previously shown
		1, 3

$\therefore$ n is odd

Prove: if n is even then n^2 is even.

Proof:

1.	n is even	Hypothesis
2.	$\exists k \in \mathbb{Z} : n = 2k$	Def of even, 1
3.	$n^2 = (2k)^2 = 4k^2 = 2 \cdot 2 \cdot k^2 = 2(2k^2)$	Subs. 2. and algebra
4.	$2k^2 \in \mathbb{Z}$	2. closure of integers
5.	n^2 is even	3, 4, def. of even

QED

Prove: If $5n$ is odd then n is odd

(Indirect) proof:

1.	n is even	Negation of conclusion
2.	$5n$ is odd	Hypothesis
3.	$5n$ is even	previously shown
	⨳	1, 3

$\therefore$ n is odd

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1. $5n$ is odd
2. $\exists k \in \mathbb{Z}: 5n = 2k+1$
3. $n = \frac{2}{5}k + \frac{1}{5}$
4. ???

$\cancel{5(2k+1) = 5n}$

$5n = 2k+1$

$25n = 5(2k+1)$

Prove: If $5n$ is odd then n is odd

(Indirect) proof:

1.	n is even	Negation of conclusion
2.	$5n$ is odd	Hypothesis
3.	$5n$ is even	previously shown
	$\Rightarrow\Leftarrow$	1, 3

$\therefore$ n is odd

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1. $5n$ is odd
2. $\exists k \in \mathbb{Z}: 5n = 2k+1$
3. $n = \frac{2}{5}k + \frac{1}{5}$
4. ???

~~$5(2k+1) = 4n$~~

$5n = 2k+1$

$25n = 5(2k+1)$

Prove: if n is odd, then n^2 is odd

Proof: 1. n is odd — Hyp.

2. $\exists k \in \mathbb{Z} : n = 2k+1$ — Def "odd", 1

3. $n^2 = (2k+1)^2 =$ — Sub. 2.

$4k^2 + 4k + 1 =$

$2(2k^2 + 2k) + 1$ — algebra

4. $2k^2 + 2k \in \mathbb{Z}$ — 2. + closure of ints.

5. n^2 is odd — 3, 4, def of "odd".

QED

Prove: if n^2 is odd then n is odd.

Proof:

1.	n is even	Neg. of cond.
2.	n^2 is even	Mon. Theorem
3.	n^2 is odd	Hyp
	⨳	2, 3

$\therefore$ n is odd

QED

If n^2 is even, then n is even.

Prove: The sum of 2 even numbers (a, b) is even.

1.	a is even	Hyp.
2.	b is even	Hyp.
3.	$\exists k \in \mathbb{Z}: a = 2k$	Def of even, 1.
4.	$\exists j \in \mathbb{Z}: b = 2j$	Def of even, 2
5.	$a+b = 2k+2j$	Subs. 3, 4
	$= 2(k+j)$	algebra
6.	$k+j \in \mathbb{Z}$	Closure of ints, 3, 4
7.	$a+b$ is even	5, 6, def of even

QED

Prove: if ($n = ab$) then ($a \le \sqrt{n}$ or $b \le \sqrt{n}$)

Proof:

1. $\neg(a \le \sqrt{n}$ or $b \le \sqrt{n})$ neg. of concl.
2. $\neg(a \le \sqrt{n})$ and $\neg(b \le \sqrt{n})$ DMT
3. $a > \sqrt{n}$ and $b > \sqrt{n}$ ~~Stats ""~~ algebra
4. $ab > \sqrt{n} \cdot \sqrt{n} = n$ algebra
5. $ab = n$ Hyp

$\times$ 4, 5.

$\therefore$ $a \le \sqrt{n}$ or $b \le \sqrt{n}$

QED

Prove: $\sqrt{2}$ can't be written as $\frac{a}{b}$ where $a, b \in \mathbb{Z}$

Proof: 1. $\exists a, b \in \mathbb{Z} : \sqrt{2} = \frac{a}{b}$, and a&b have no common factor — Neg. of Concl.

2. $2 = \frac{a^2}{b^2}$

$2b^2 = a^2$, so a^2 is even, so a is even. — alg, Mon Thm.

3. $a = 2k$, $k \in \mathbb{Z}$

$2b^2 = a^2 = (2k)^2 = 4 \cdot k^2$ — alg.

$b^2 = 2k^2$, so b^2 is even, so b is even — alg.

$\times\!\!\!\times$ a and b are both divisible by 2.

$\therefore \sqrt{2} \neq \frac{a}{b}$

QED