$x = (p \wedge \neg q) \vee (r \wedge p)$

(c) find a prop C such that $C \to X$

$\cancel{X \to}$ $(p \wedge \neg q) \vee (r \wedge p) \vee p$

$p = T$

$Q = T$

$r = F$

$p \wedge x \to x$

Let C be the prop $p \wedge \neg q \wedge r$

$x \vee \neg(q \wedge \neg r \wedge \neg p)$

$\neg q \vee r \vee p$

$(r \wedge p) \to x$

$(p \wedge \neg q) \to x$

$\cancel{(p \wedge q)}$

$? \to x$

$x := (p \wedge \neg q) \vee (r \wedge p)$

$(p \wedge r) \to x$

$x \to p$

$f \to x$

$A \to B$

if $A=1$ then $B=1$

p	q	r	$p \wedge \neg q$	$r \wedge p$	x
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	~~0~~	0	0
1	0	0	1	0	1
→ 1	0	1	1	1	1 ✓
1	1	0	0	0	0
→ 1	1	1	0	1	1 ✓

$$\begin{array}{c} A \\ B \\ C \\ \hline D \end{array}$$

If A, B & C are all true, then D is true.

$$(A \wedge B \wedge C) \rightarrow D$$

Different notation:

$$A, B, C \Rightarrow D$$

Prove: $q \to \neg r,\ p \to q,\ r \Rightarrow \neg p$

$$\begin{array}{l} q \to \neg r \\ p \to q \\ \underline{r} \\ \neg p \end{array}$$

Proof:

1.	$p \to q$	Hypothesis
2.	$q \to \neg r$	Hypothesis
3.	$p \to \neg r$	Chain rule, 1. & 2.
4.	r	Hypothesis
5.	$\neg p$	Indirect Reasoning, 3 & 4
	QED	

4.	r	Hypothesis
5.	$r \to \neg p$	Contrapositive 3.
6.	$\neg p$	Direct Reasoning, 4 & 5
	QED	

prove: $p \to q$

$q \to r$

$p \wedge t$

---

r

proof:

1. $p \to q$ Hypothesis
2. $q \to r$ Hypothesis
3. $p \to r$ Chain rule, 1. 2.
4. $p \wedge t$ Hypothesis
5. p Conjunctive Simplification, 4
6. r Detachment, 3. & 5.

QED

or Direct Reasoning

or Modus Ponens

prove: $p \to q$

$q \to r$

$p \wedge t$

---

r

proof:

1.	$p \to q$	Hypothesis
2.	$q \to r$	Hypothesis
3.	$p \to r$	Chain rule, 1. 2.
4.	$p \wedge t$	Hypothesis
5.	p	Conjunctive Simplification, 4.
6.	r	Detachment, 3. & 5.

QED

or Direct Reasoning

or Modus Ponens

If you send me an email, then I'll finish my HW.

If you don't send me an email, then I'll go to sleep early.

If I go to sleep early, then I'll wake up happy.

Therefore, if I don't finish my HW, I'll wake up happy.

Is this a valid theorem?

Let E be the prop "you send me an email"

Let W be the prop "I'll finish my homework"

Let S be the prop "I go to sleep early"

Let H be the prop "I wake up happy"

Prove:

$$\begin{array}{c} E \to W \\ \neg E \to S \\ S \to H \\ \hline \neg W \to H \end{array}$$

Prove: $E \to W$

$\neg E \to S$

$S \to H$

$\neg W \to H$

<u>Proof</u>:

1.	$\neg E \to S$	Hyp.
2.	$S \to H$	Hyp.
3.	$\neg E \to H$	Chain rule, 1 & 2
4.	$E \to W$	Hyp
5.	$\neg W \to \neg E$	Contrapositive, 4.
6.	$\neg W \to H$	Chain rule, 5. 3

QED

Indirect Proof

Want to prove $A \wedge B \wedge C \Rightarrow D$

Assume $\neg D$ and show that this leads to a contradiction.

Then we can say "therefore, D"

ex: prove $a \to b$

$$\frac{\begin{array}{c} a \to b \\ \neg(b \vee c) \end{array}}{\neg a}$$

proof:

1.	a	Negation of conclusion
2.	$a \to b$	Hypothesis
3.	b	Direct Reasoning, 1. 2.
4.	$\neg(b \vee c)$	Hyp.
5.	$(\neg b) \wedge (\neg c)$	DeMorgans, 4.
6.	$\neg b$	Conj. Simpl. 5.
$\#$		3. 6.

"contradiction"

Then we can say "therefore, b"

ex: prove $a \to b$

$$\frac{\neg(b \vee c)}{\neg a}$$

proof:

1.	a	Negation of conclusion
2.	$a \to b$	Hypothesis
3.	b	Direct Reasoning, 1. 2.
4.	$\neg(b \vee c)$	Hyp.
5.	$(\neg b) \wedge (\neg c)$	DeMorgans, 4.
6.	$\neg b$	Conj. Simpl. 5.
$\#$		3. 6.

"contradiction"

Therefore, $\neg a$

QED

$\therefore \neg a$

"therefore"

$$\begin{array}{c} A \\ B \\ C \\ \hline D \end{array}$$

$$\begin{array}{c} \neg D \\ A \\ B \\ C \\ \vdots \\ \hline F \end{array}$$

$\neg D \to F$

$T \to D$

Therefore, $\neg a$

$\therefore \neg a$

QED

"therefore"

$\cancel{P \to Q}$

$$\begin{array}{l} P \to Q \\ \underline{P} \\ Q \end{array}$$

Direct Reasoning

$$\begin{array}{l} P \to Q \\ \underline{\neg Q} \\ \neg P \end{array}$$

Indirect Reasoning

$$\begin{array}{l} P \to Q \\ \underline{Q} \\ P \end{array}$$

fallacy of affirming the conclusion.

$$\begin{array}{l} P \to Q \\ \underline{\neg P} \\ \neg Q \end{array}$$

fallacy of denying the hypothesis

NOTE: this is not an indirect proof:

prove $\begin{array}{c} A \\ B \\ \underline{C} \\ D \end{array}$

$\neg D$

proof: $\begin{array}{c} A \\ B \\ C \\ \vdots \\ D \\ \cancel{\neg D} \\ \# \end{array}$

Def: an int n is even if $n=2k$ for some $k \in \mathbb{Z}$

n is odd if $n=2k+1$ for some $k \in \mathbb{Z}$

Prove: If n is even, so is $3n+6$

Proof:

1.	n is even	Hypothesis
2.	$n=2k, k \in \mathbb{Z}$	Definition of "even"
3.	$3n+6=3(2k)+6$	Subs, 2.
4.	$3n+6= 6k+6$	
	$=2(3k+3)$	Algebra 3.
5.	$3k+3 \in \mathbb{Z}$	Closure of integers, 2.
6.	$3n+6$ is even	4, 5, Def. of "even"

QED