Set builder notation

$$\{ \quad \underbrace{x \in \mathbb{Z}} \quad : \quad x^2 < 10 \} = \{0, 1, 2, 3, -1, -2, -3\}$$

the set of

x contained in $\mathbb{Z}$

Such that

(or $|$)

$$\left\{ \frac{1}{x} : x \in \mathbb{Z},\ 1 \le x \le 5 \right\} = \left\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \right\}$$

(c) $A \oplus B = \{3, 4, 5\}$

6. Suppose that U is an infinite universal set, and A and B are infinite subsets of U. Answer the following questions with a brief explanation.

(a) Must A^c be finite?

(b) Must $A \cup B$ be infinite?

(c) Must $A \cap B$ be infinite?

7. Given that U = all students at a university, D = day students, M = mathematics majors, and G = graduate students. Draw Venn diagrams illustrating this situation and shade in the following sets:

(a) evening students

(b) undergraduate mathematics majors

(c) non-math graduate students

(d) non-math undergraduate students

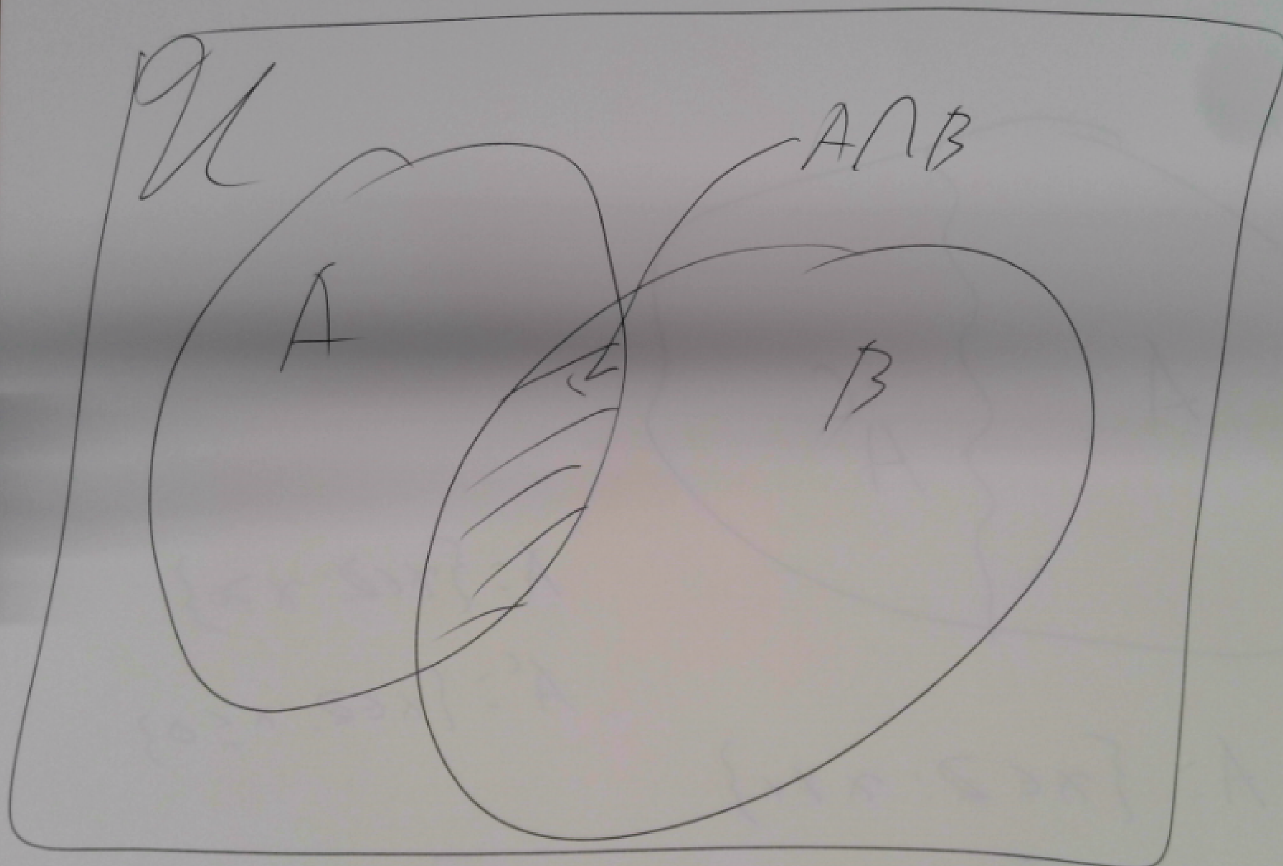
8. Let the sets D, M, G, and U be as in exercise 7. Let $|U| = 16,000$, $|D| = 9,000$, $|M| = 300$, and $|G| = 1,000$. Also assume that the number of day students who are mathematics majors is 250, 50 of whom are graduate students, that there are 95 graduate mathematics majors, and that the total number of day graduate students is 700. Determine the number of students who are:

(a) evening students

(b) nonmathematics majors

(e) evening graduate students

(f) evening graduate mathemat-



Let $E = \{\text{all evens}\}$

Let $O = \{\text{all odds}\}$

$E \cap O = \emptyset = \{\ \}$

Let $P = \{x \in \mathbb{Z} : x \geq 0\}$

Let $N = \{x \in \mathbb{Z} : x \leq 0\}$

$P \cap N = \{0\}$

Propositional Logic

A *proposition* is a declarative statement that is either true or false.

Examples:	NOT PROPOSITIONS
The earth is flat.	Hello.
$x = y$	Who are you?
My name is Nick	$x^2 + 2x + 1$
My name is Barbara	π
$x^2 + 2x + 1 = 0$	
$(x+1)^2 = x^2 + 2x + 1$	
$(x+1)^3 = x^2 + 2x + 1$	
I want pie.	

Propositional Varicble: a letter that represents a proposition.

ex: $\boxed{\text{Let } p \text{ be the proposition "My name is Nick"}}$

2 Special varicbles: T, F truth values

true false

Propositional Calculus

Making new props from old:

$\neg p$ "negation of p" "not p"

Let p be the prop. "$x = y$"

$\neg p$ is the prop ("not $x = y$") "$x \neq y$"

Propositional Varible: a letter that represents a proposition.

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2 Special varibles: T, F   truth values

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Propositional Calculus

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A truth table

P	$\neg P$
F	T
T	F

P	Q	$P \wedge Q$	$P \vee Q$
F	F	F	F
F	T	F	T
T	F	F	T
T	T	T	T

Conjunction: $P \wedge Q$ is the proposition "P and Q"

Let P be the prop "The earth is flat"

Let Q be the prop "My name is Nick"

Disjunction: $P \vee Q$ is the prop "P or Q"

Exclusive-Or $P \oplus Q$ P or Q but not both.

Conditional Statements

$P \to Q$ is the proposition

"if P then Q"

Let P be the prop "Today is Wednesday"

Let Q be the prop "HW is due"

$P \to Q$ is the prop "If today is wed. Then HW is due"

Let R be the prop "$P \to Q$"

Hypothesis

Conclusion

$P \to Q$

If P then Q.

If P, Q.

P is sufficient for Q.

Q if P.

⋮

In order to pass a class, you must study.

Let P be the prop "you pass a class"

Let S be the prop "you study"

$\cancel{(S \to P)}$? $P \to S$

P	Q	$P \to Q$
F	F	T
F	T	T
T	F	F
T	T	T

Let P be the prop "~~Today is Wed~~" $x > 1000$

Let Q be the prop "~~HW is due~~" $x > 0$

$P \to Q$ is the prop 'if $x > 1000$ then $x > 0$'

Let P be the prop "$1+1=3$"

Let Q be the prop "I am the Queen of England"

$P \to Q$ is the prop "If $1+1=3$ then I am the Queen of England"

P	Q	$P \to Q$
F	F	T
F	T	T
T	F	F
T	T	T

Given the prop $P \to Q$

$Q \to P$ is called the converse of $P \to Q$

is called the inverse

$\neg P \to \neg Q$

$\neg Q \to \neg P$

is called the contrapositive

Let P be the prop "Today is Wed"

Let Q be the prop "I have class today"

P	Q	$\neg P \to \neg Q$	$Q \to P$	$P \to Q$
F	F	T	T	T
F	T	F	F	T
T	F	T	T	F
T	T	T	T	T

$$\sum_{k=1}^{3} k^{n}$$

for $n = 1, 2, 3, 4$

$$\sum_{k=1}^{3} k^{1} \quad \Big| \quad \sum_{k=1}^{3} k^{2} \quad \Big| \quad \sum_{k=1}^{3} k^{3} \quad \Big| \quad \sum_{k=1}^{\cancel{4}3} k^{4}$$

$$\sum_{k=1}^{3} k^n$$

for $n = 1, 2, 3, 4$

$$\sum_{k=1}^{3} k^1 \quad \Bigg| \quad \sum_{k=1}^{3} k^2 \quad \Bigg| \quad \sum_{k=1}^{3} k^3 \quad \Bigg| \quad \sum_{k=1}^{\cancel{4}\,3} k^4$$

(c) Want a prop. P that implies X

$\boxed{P \to X \text{ is true}}$

Ex: If P is the prop "$A \vee B$"

$A \to P$

(c) Want a prop. P that implies X

($P \to X$ is true)

Ex: ~~If P is the prop~~ $A \vee B$:

$A \to P$

(d) find a prop Q such that $X \to Q$

Biconditional

$P \leftrightarrow Q$

Q $\underbrace{\text{if and only if}}$ P

$(P \to Q) \wedge (Q \to P)$

Q iff P

$P \leftrightarrow Q$

P	Q	$P \to Q$	$Q \to P$	$(P \to Q) \wedge (Q \to P)$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

Propositional Equivalence

$P \equiv Q$ means "P is logically equivalent to Q."

a **Tautology** is a proposition $\equiv T$.

(example: $P \vee (\neg P)$

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a **Contradiction** is a prop $\equiv F$.

ex: $P \wedge (\neg P)$

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anything else is a **contingency**

ex: $P \to Q \equiv \neg P \vee Q$

P	Q	$P \to Q$	$\neg P$	$\neg P \vee Q$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	T	F	T