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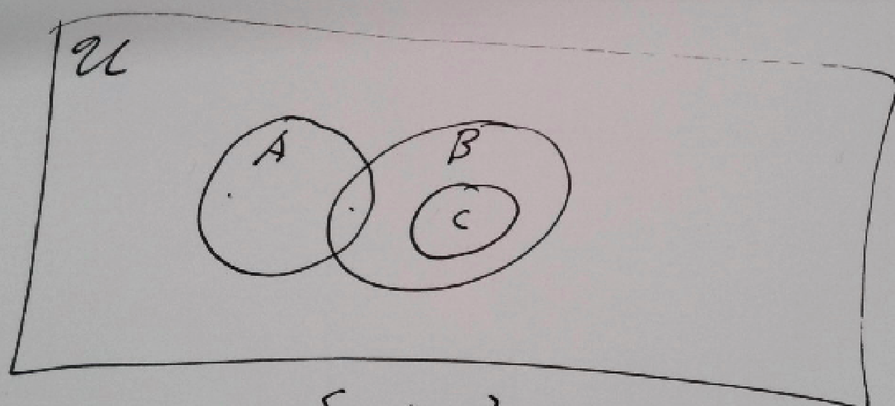
Guvcview (22.61 fps)

$A \subseteq B$ "A is a subset of B"

$\emptyset \subseteq B$ $B \subseteq B$ $\mathcal{U}$ = universal set (set of all things)

Venn Diagram



$\{2, 3, 4\}$

Note: $A = B$ $\underbrace{\text{iff}}_{\text{if and only if}}$ $A \subseteq B$ and $B \subseteq A$

Size of a set

Let $S = \{A, 3, ☺, \triangle, x^2\}$

S contains 5 things

$|S| = 5$ $\quad$ $|S|$ means "size of S"

If S is <u>finite</u> (finite # of elements),

then $|S| = \#$ of elements.

$|S|$ is also called "The cardinality of S"

BUT... what does $|\mathbb{Z}|$ or $|\mathbb{R}|$ mean?

The <u>Power Set</u> of a set S

$$\mathcal{P}(S) = \{\text{all subsets of } S\}$$

Example: Let $S = \{3, 6\}$ $\qquad |S| = 2$

$$\mathcal{P}(S) = \{\emptyset, \{3\}, \{6\}, \{3, 6\}\} \qquad |\mathcal{P}(S)| = 4$$

Let $T = \{1, 2, 3\}$ $\qquad |T| = 3$

$$\mathcal{P}(T) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$$

$|\mathcal{P}(T)| = 8$

$\emptyset \subseteq T$

$$2^{|T|} = |\mathcal{P}(T)|$$

$|\mathcal{P}(S)| = 2^{|S|}$

$\mathcal{P}(\phi) = \{\phi\}$

$\mathcal{P}(\mathcal{P}(\phi)) = \mathcal{P}(\{\phi\}) = \{\phi, \{\phi\}\}$

$\mathcal{P}(\mathcal{P}(\mathcal{P}(\phi))) = ?$

$\mathcal{P}(\{\phi, \{\phi\}\})$

Let $A = \{1, 2, 3\}$ and $B = \{x, y\}$

Def: an "ordered n-tuple" is an ordered collection

$$(a_1, a_2, a_3, \ldots, a_n)$$

Two tuples $(a_1, a_2, \ldots, a_n)$ and

$(b_1, b_2, \ldots, b_n)$ are <u>equal</u>

iff $a_1 = b_1$ & $a_2 = b_2$ & ... & $a_n = b_n$

$(1,2) = (1,2) \neq (2,1) \neq (2)$

DEF: Given two sets A, B, the

<u>Cartesian Product</u> $A \times B = \{(a,b) : a \in A, b \in B\}$

Ex: Let $A = \{1,2,3\}$ $B = \{x,y\}$

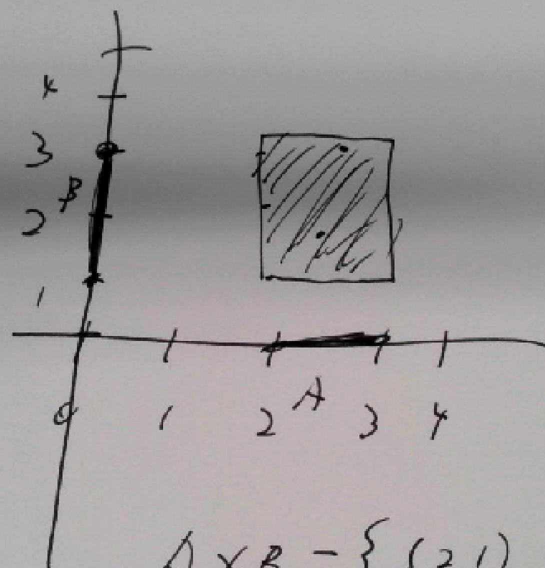
$$A \times B = \{ (1,x), (1,y), (2,x), (2,y), (3,x), (3,y) \}$$

$$B \times A = \{ (x,1), (x,2), (x,3) \ \ldots \} \neq A \times B$$

Let $f = \{$burger, sandwich$\}$ Let $d = \{$cola, water, orange$\}$

$f \times d =$

Let $A = [2, 3]$ Let $B = [1, 3]$



$$A \times B = \{ (2,1), (2,3)\ (2,2)\ (3,3), (2.5, 1.5), \dots \}$$

SET OPERATIONS

Union: $A \cup B = \{x : x \in A \text{ or } x \in B\}$

Intersection: $A \cap B = \{x : x \in A \text{ and } x \in B\}$

If $A \cap B = \emptyset$, we say A & B are <u>disjoint</u>.

Cool fact: $|A \cup B| \cancel{=} = |A| + |B| - |A \cap B|$

Complement of B in A (or "A minus B") $\begin{matrix} A \setminus B \\ A - B \end{matrix} = \{a \in A : a \notin B\}$

Ex: $A = \{1, 3, 5, 7\}$, $B = \{2, 3, 4, 5\}$ $\quad A \cup B = \{1, 2, 3, 4, 5, 7\}$

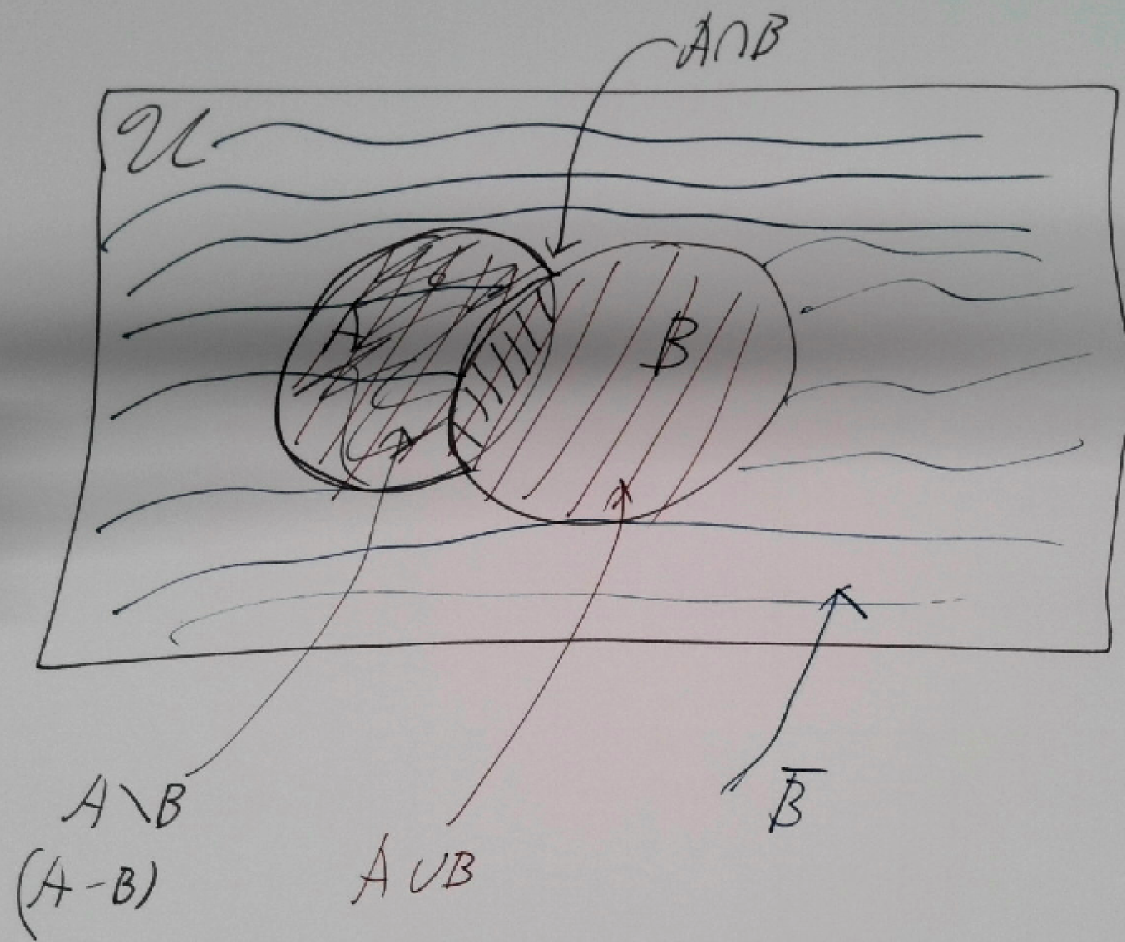
$A \cap B = \{3, 5\}$

$A \setminus B = A - B = \{1, 7\}$

$B - A = \{2, 4\}$

$\mathcal{U} \setminus B = \overline{B} = B^c$

$\overline{B} =$



A∩B
𝒰
A
B
A\B
(A-B)
A∪B
$\overline{B}$

Set Identities

$A \cup B = B \cup A$ $\qquad$ $A \cap B = B \cap A$

$A \cup (B \cup C) = (A \cup B) \cup C$ $\qquad$ $A \cap (B \cap C) = (A \cap B) \cap C$

$A \cup A = A$ $\qquad$ $A \cap A = A$

$A \cup \emptyset = A$ $\qquad$ $A \cap \emptyset = \emptyset$

$A \cup \mathcal{U} = \mathcal{U}$ $\qquad$ $A \cap \mathcal{U} = A$

$\overline{\overline{A}} = A$

$$\left.\begin{aligned} \overline{A \cup B} &= \bar{A} \cap \bar{B} \\ \overline{A \cap B} &= \bar{A} \cup \bar{B} \end{aligned}\right\} \text{de Morgans Theorem}$$

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Set Identities

$A \cup B = B \cup A$ $\quad$ $A \cap B = B \cap A$

$A \cup (B \cup C) = (A \cup B) \cup C$ $\quad$ $A \cap (B \cap C) = (A \cap B) \cap C$

$A \cup A = A$ $\quad$ $A \cap A = A$

$A \cup \emptyset = A$ $\quad$ $A \cap \emptyset = \emptyset$

$A \cup \mathcal{U} = \mathcal{U}$ $\quad$ $A \cap \mathcal{U} = A$

$\overline{\overline{A}} = A$

$$\left.\begin{array}{l} \overline{A \cup B} = \bar{A} \cap \bar{B} \\ \overline{A \cap B} = \bar{A} \cup \bar{B} \end{array}\right\} \text{de Morgans Theorem}$$

$$\left.\begin{aligned} A \cup (A \cap B) &= A \\ A \cap (A \cup B) &= A \end{aligned}\right\} \text{"absorption"}$$

$A \cup \bar{A} = \mathcal{U}$ $\quad A \cap \bar{A} = \phi$

Example: PROVE $A \cup (A \cap B) = A$

Membership table

A	B	$A \cap B$	$A \cup (A \cap B)$
F	F	F	F
F	T	F	F
T	F	F	T
T	T	T	T

$A = A \cup (A \cap B)$

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Guvcview (20.44 fps)

$\cancel{A \cup B \cup C}$ $\quad A_i$

$i = 1, 2, 3, 4 \ldots$

Let $A_i := \{i, i+1, i+2, i+3, \ldots\}$

ex: $A_1 := \{1, 2, 3, 4, 5, \ldots\}$

$A_2 := \{2, 3, 4, 5, \ldots\}$

$A_3 := \{3, 4, 5, \ldots\}$

$$\bigcup_{i=1}^{n} A_i = A_1$$

$$\bigcap_{i=1}^{n} A_i = A_n$$

$$A_1 \cup A_2 \cup A_3 = \bigcup_{i=1}^{3} A_i$$

$$A_1 \cap A_2 \cap A_3 = \bigcap_{i=1}^{3} A_i$$

$$\bigcup_{i=1}^{\infty} A_i = A_1$$

$$\bigcap_{i=1}^{\infty} A_i = \emptyset$$

$$|E| = |O| = |\mathbb{Z}|$$