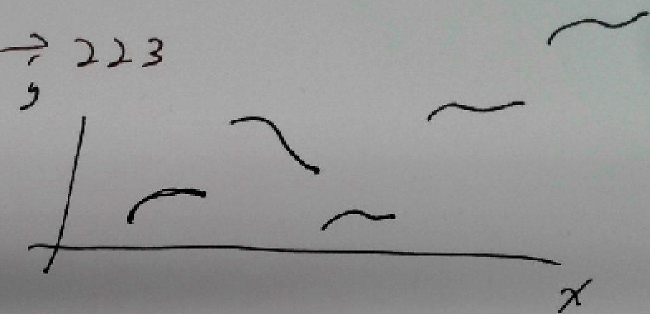
F W S

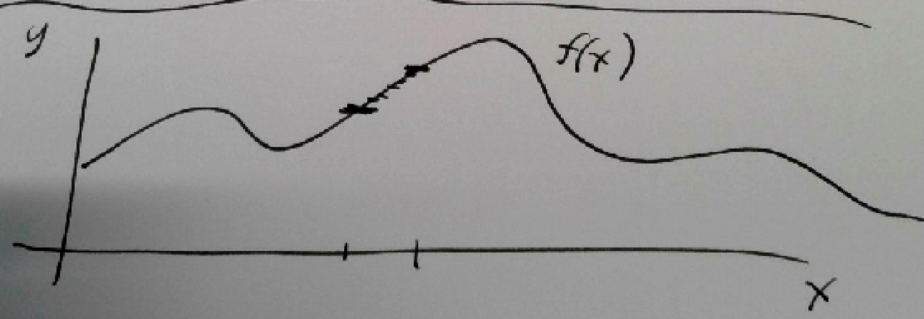
CSE 224 → 222 → 223

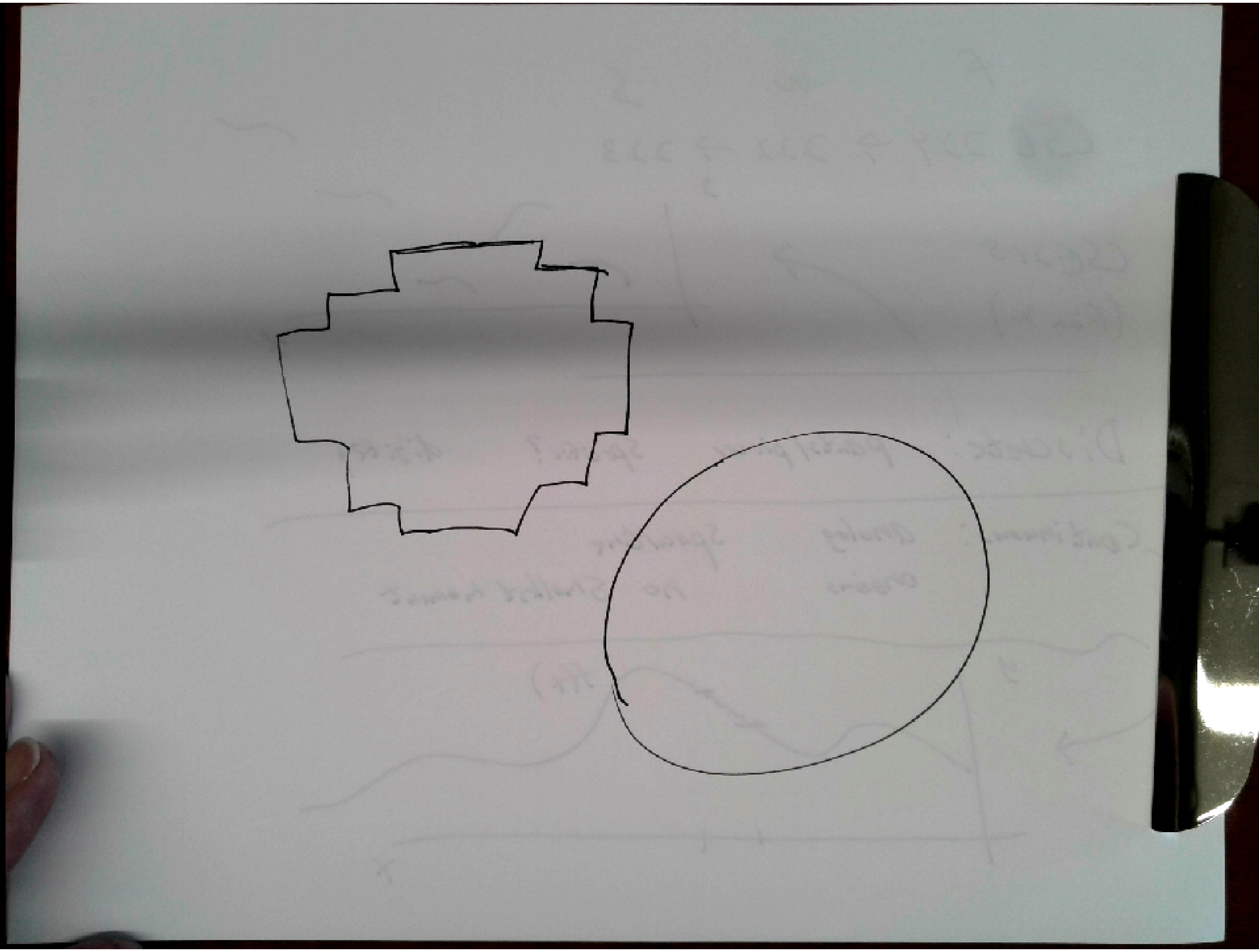
CSE215
(for W)



Discrete: parts/pieces specific? digital

Continuous: analog space/time
ongoing no smallest moment





1, 2, 3, 4, 5, . . .   integers

Real #s :   1, 1.1, 1.2, 1.3, 1.4 . . .

1.15   1.16

1.155
1.156
1.157
⋮

3.96

3.959999

Set Theory

A <u>set</u> is a collection of <u>things</u>.

A set is an <u>unordered</u> collection of <u>objects</u>.

elements
members

ex: Let S be a set containing the numbers 1, 3 & 5.

$S = \{1, 3, 5\}$ $\qquad$ $S = \{5, 1, 3\}$

$\in$ "membership" $\qquad$ $\underbrace{3 \in S}_{3 \text{ is an element of } S}$

$17 \notin S$

Examples

$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

= Set of all integers

$\mathbb{R}$ = Set of all real #s

"rational numbers"

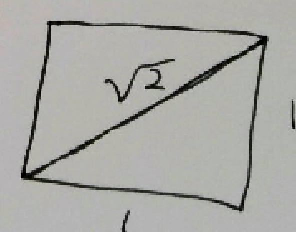
$\mathbb{Q}$ = Set of all fractions ($\frac{m}{n}$, $m, n \in \mathbb{Z}$, $n \neq 0$)

$\sqrt{2} \in \mathbb{R}$ $\quad$ $\sqrt{2} \notin \mathbb{Q}$

$\mathbb{N}$ = Set of "natural #s"

$= \{1, 2, 3, 4, \dots\}$

(Sometimes! $\{0, 1, 2, 3, 4, \dots\}$



$\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, ?

$i^2 = -1$

$x + 2 = 5$

$x^2 = 2$

$2x = 4$

$2x = 1$

$x^2 = -1$

Notation: Let S be the set of integers bigger than 10.

$$S = \{ \underbrace{x \in \mathbb{Z}}_{\text{elements in } \mathbb{Z} \text{ (integers)}} : \underbrace{x > 10}_{x \text{ is bigger than } 10} \}$$

the set of

"such that"

$$E = \{ x \in \mathbb{Z} : x \text{ is divisible by } 2 \}$$

$$= \{ x : x \in \mathbb{Z} \text{ and } x \text{ is div. by } 2 \}$$

$$D = \{ x+1 : x \in \mathbb{Z} \text{ and } x \text{ is div. by } 2 \} = \text{set of all odd \#s}$$

$\{a\}$ is an example of a "singleton set"

$\{\ \}$ "the empty set" $\emptyset$

$\{x \in \mathbb{Z} : x > x+1\} = \emptyset$

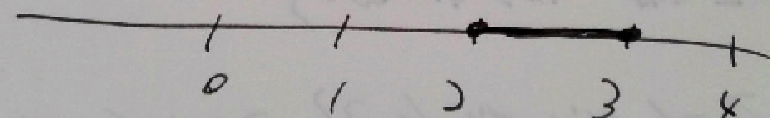
Interval Notation

$[a,b] = \{x \in \mathbb{R} : x \geq a \text{ and } x \leq b\}$

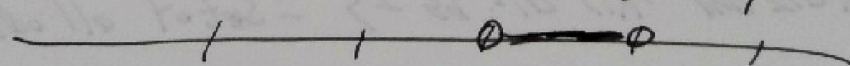
$[2,3]$

$(a,b) = \{x \in \mathbb{R} : x > a \text{ and } x < b\}$

"closed" $[2,3]$



"open" $(2,3)$



$B = \{\mathbb{Z}, \mathbb{R}\}$

$\{\{\ldots, -3, -2, -1, 0, 1, \ldots\}, \mathbb{R}\}$

$\{☺, \triangle\}$

$\mathbb{Z} \in B$

$5 \notin B$

$B = \{\mathbb{Z}, \mathbb{R}\}$

$\{\{\ldots, -3, -2, -1, 0, 1, \ldots\}, \mathbb{R}\}$

{☺, △}

$\mathbb{Z} \in B$

$5 \notin B$

There exists an $x \in B : 5 \in x$

$5 \in \mathbb{Z} \in B$

If A, B are sets,

"$A = B$" means $x \in A$ if and only if $x \in B$

Ex: $A = \{1, 2\}$

$B = \cancel{\{1, 2\}}\ \{2, 1\}$

$A = B$

SUBSETS

A is a subset of B if every element in A is also in B.

$$A \subseteq B$$

ex: $\{1, 3, 5\} \subseteq \{1, 2, 3, 4, 5, 6\} \subseteq \mathbb{Z} \subseteq \mathbb{R}$

$\{2, 4, 6\}$ is not a subset of $\{2, 3, 6, 8, 10\}$

$$\{2, 4, 6\} \not\subseteq \{2, 3, 6, 8, 10\}$$

If S is a set, is $S \subseteq S$?

yes!

$S \subseteq S$ $\qquad$ $\emptyset \subseteq S$ $\qquad$ (TRUE)

If $T \subseteq S$ and $T \neq S$, we can say

"T is a proper subset of S"

$$T \subsetneq S$$

$\{\}$

Def: A set S is called "self-consuming" if $S \in S$

ex: Let $S = \{1, 2, 3\}$ not S.C.

most sets are not S.C. (all?)

$B = \{$ all sets that are not self-consuming $\}$

Question: Is B self-consuming?

1) B is S.C. $B \in B$ $\therefore$ B is not S.C.

2) B is not S.C. so $B \in B$ so B is S.C.

Gödel

ZF

ZFC