Signals & Systems - Chapter 6

1S. A real-valued signal x(t) is known to be uniquely determined by its samples when the sampling frequency is w_s = 10,000π. For what values of w is X(jw) guaranteed to be zero?

Solution:

From the Nyquist sampling theorem, it is know that X(jw) = 0 for |w| > $w_s/2$. In other word signal frequencies above $w_s/2$ are not recoverable. Therefore:

answer is any frequency w such that |w| > 5,000π

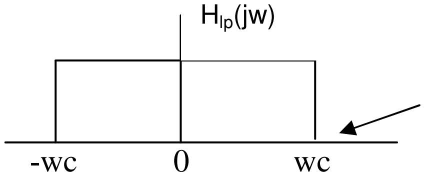
1U. A real-valued signal x(t) is known to be uniquely determined by its samples when the sampling frequency is f_s = 25,000. For what values of w is X(jw) guaranteed to be zero?

Solution:

2S. A continuous-time signal x(t) is obtained at the output of an ideal lowpass filter with cut off frequency wc = 1,000π. If impulse-train sampling is performed on x(t), which of the following sampling periods would guarantee that x(t) can be recovered from its sampled version using a appropriate lowpass filter?

a) T = 0.5 x 10^{-3} Sec.
b) T = 2 x 10^{-3} Sec.
c) T = 10^{-4} Sec.

Solution:



Signal with maximum frequency w_m = 1,000π pass through → Sampling rate w_s >2 w_m = 2,000π

So Sampling period, T_S, < ($2\pi/w_m$)= 2π/2,000π =1 x 10^{-3} Seconds

a and c meet this condition.

2U. A continuous-time signal x(t) is obtained at the output of an ideal lowpass filter with cut off frequency wc = 2,500 . If impulse-train sampling is performed on x(t), which of the following sampling periods would guarantee that x(t) can be recovered from its sampled version using a appropriate lowpass filter?

a) T = 1.0 x 10^{-3} Sec.
b) T =0.5 x 10^{-3} Sec.
c) T = 10^{-4} Sec.

Solution:

3S. The frequency which, under the sampling theorem, must be exceeded by the sampling frequency is called the Nyquist rate. Determine the Nyquist rate corresponding to each of the following signals:

a) x(t) = 1 + cos(2,000πt) + sin(4,000πt)

b) $x(t)=\dfrac{\sin(4{,}000\pi t)}{\pi t}$

c) $x(t)=\left(\dfrac{\sin(4{,}000\pi t)}{\pi t}\right)^2$

Solution:

Nyquist rate = 2 x maximum signal frequency
Sampling Rate must exceed Nyquist rate in order to be able to fully reconstruct the signal.

a) $x(t) = 1 + \cos(2{,}000\pi t) + \sin(4{,}000\pi t)$
The frequency for each term is a follows
Term 1 is DC → $w_1 = 0$
Term 2 → $w_2 = 2{,}000\pi$
Term 3 → $w_3 = 4{,}000\pi$

Maximum Signal Frequency → $w_m = 4{,}000\pi$
Another way of saying this is that $X(jw) = 0$ for $|w| > 4{,}000\pi$

Sampling theorem says that $w_s > 2w_m = 8{,}000\pi$

Therefore Nyquist rate is $8{,}000\pi$

b) $x(t) = \dfrac{\sin(4{,}000\pi t)}{\pi t}$

Using Fourier Transform table, we have $X(jw) = \begin{cases} 1 & \text{for } |w| < 4000\pi \\ 0 & \text{for } |w| > 4000\pi \end{cases}$

Therefore Maximum Signal Frequency → $w_m = 4{,}000\pi$

Sampling theorem says that $w_s > 2w_m = 8{,}000\pi$

Therefore Nyquist rate is $8{,}000\pi$

c) $x(t) = \left(\dfrac{\sin(4{,}000\pi t)}{\pi t}\right)^2$

We can rewrite the above function as $x(t) = x_{1(}t)x_1((t)$ where $x_1((t)\dfrac{\sin(4{,}000\pi t)}{\pi t}$

Using the Convolution property → $X(jw) = (1/2\pi)X_1(jw)^* X_1(jw)$

We know that convolving a signal with itself will double the maximum frequency therefore:

Therefore Maximum Signal Frequency → $w_m = 8{,}000\pi$

Sampling theorem says that $w_s > 2w_m = 16{,}000\pi$

Therefore Nyquist rate is $16{,}000\pi$

3U. The frequency which, under the sampling theorem, must be exceeded by the sampling frequency is called the Nyquist rate. Determine the Nyquist rate corresponding to each of the following signals:

a) $x(t) = 1 + \cos(3{,}000\pi t) + \sin(6{,}500\pi t)$

b) x(t)= $\dfrac{\sin(12{,}000\pi\, t)}{\pi\, t}$

c) x(t)= $\left(\dfrac{\sin(14{,}000\pi\, t)}{\pi\, t}\right)^2$

Solution:

4S. Let x(t) be a signal with Nyquist rate w_o. Determine the Nyquist rate for each of the following signals:

a) x(t) + x(t – 1)

b) $\dfrac{dx(t)}{dt}$

c) x^2(t)

d) x(t)cos(w_ot)

Solution:

Nyquist rate = 2 x maximum signal frequency

Sampling Rate must exceed Nyquist rate in order to be able to fully reconstruct the signal.

a) y(t) = x(t) + x(t-1)

Fourier transform → Y(jw) = X(jw) + e^{-jw}X(jw)

Since the Maximum Frequency for Y(jw) is the same as X(jw) then y(t) Nyquist rate is also w_0.

b) y(t) = $\dfrac{dx(t)}{dt}$

Fourier transform → Y(jw) =jwX(jw)

Since the Maximum Frequency for Y(jw) is the same as X(jw) then y(t) Nyquist rate is also w_0.

c) y(t) = x^2(t)

We can rewrite the above function as y(t) = x(t)x((t)

Using the Convolution property → Y(jw) =(1/2π) X(jw)* X(jw)

We know that convolving a signal with itself will double the maximum frequency therefore:

Therefore Y(jw) =0 for |w| > w_0 in other word Maximum Signal Frequency → $w_m = w_0$

Therefore Nyquist rate is $2w_0$

d) $y(t) = x(t)\cos(w_0 t) \xrightarrow{FourierTransform} Y(jw) = \dfrac{X(j(w-w_0))}{2} + \dfrac{X(j(w+w_0))}{2}$

Note: Use cos Fourier transform and convolution property to find Y(jw)

We see that Y(jw) = 0 when |w| > w_0.+ w_0/2 since X(jw)=0 when |w|> w_0/2

Therefore Nyquist rate = $2w_m = 3w_0$

4U. Let x(t) be a signal with Nyquist rate w_o. Determine the Nyquist rate for each of the following signals:

a) x(-t) + x(t – 3)

b) $\dfrac{dx(t-3)}{dt}$

c) $x(t)e^{jw_0 t}$

d) x(t)sin(w_ot)

5S. Let x(t) be a signal with Nyquist rate w_o. Also, let y(t) -= x(t)p(t – 1) where

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \quad and \quad T < \frac{2\pi}{w_o}$$

Specify the constraints on the magnitude and phase of the frequency response of a filter that gives x(t) as its output when y(t) as the input.

Solution:

Nyquist rate = 2 x maximum signal frequency
Sampling Rate must exceed Nyquist rate in order to be able to fully reconstruct the signal.

$$p(t) \xrightarrow{FourierTransform} \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(w - k2\pi / T)$$

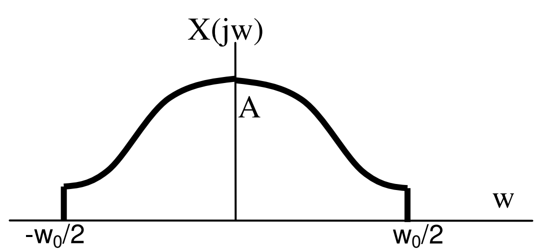
$$Shifting\ \Pr operty$$

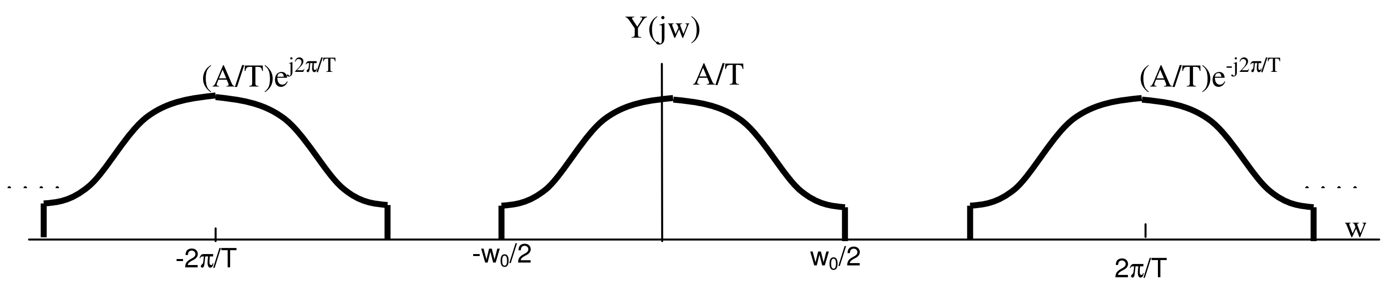
$$p(t-1) \xrightarrow{FourierTransform} \frac{2\pi}{T} e^{-jw} \sum_{k=-\infty}^{\infty} \delta(w - k2\pi / T) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(w - k2\pi / T) e^{-jk2\pi/T}$$

Since y(t) = x(t)p(t-1)

$$y(jw) = (\frac{1}{2\pi})[X(jw) * FT\{p(t-1)\} = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(w - k2\pi / T) e^{-jk2\pi/T}$$

Therefore Y(jw) consists of copies of X(jw) shifted by k2π/T and added together as shown below





In order to recover x(t) from y(t), we need to be able to isolate one copy of X(jw) from Y(jw). From the figure we see that if we multiply Y(jw) with filter H(jw):

H(jw) = T for $|w| \le w_c$
0 for $|w| > w_c$.

Where $(w_0/2) < w_c < (2\pi/T) - (w_0/2)$

5U. Let x(t) be a signal with Nyquist rate w_o. Also, let y(t) -= x(t)p(t – 3) where

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \quad and \quad T < \frac{2\pi}{w_o}$$

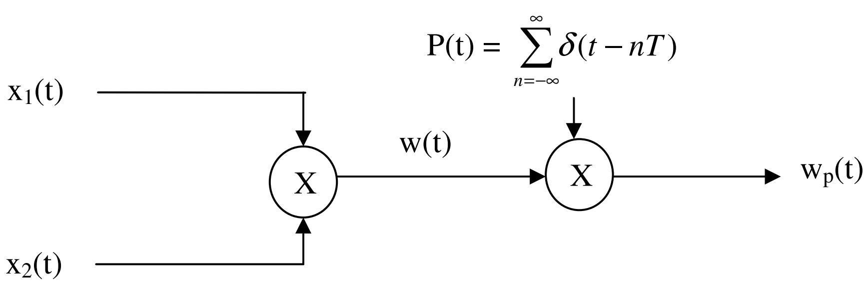
Specify the constraints on the magnitude and phase of the frequency response of a filter that gives x(t) as its output when y(t) us the input.

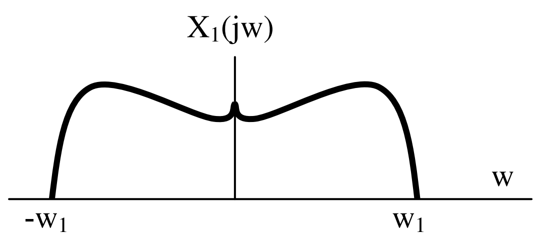
Solution:

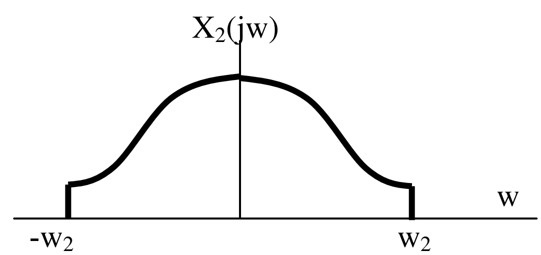
6S. In the system shown below, two functions of time, $x_1(t)$ and $x_2(t)$, are multiplied together, and the product w(t) is sampled by a periodic impulse train. $x_1(t)$ is band limited to w_1, and $x_2(t)$ is band limited to w_2; that is

$$X_1(jw) = 0 \quad \text{for} \quad |w| \geq w_1$$
$$X_2(jw) = 0 \quad \text{for} \quad |w| \geq w_2$$

Determine the maximum sampling interval T such that w(t) is recoverable from $w_p(t)$ through the use of an ideal lowpass filter.







Solution:

$w(t) = x_1(t)x_2(t) \rightarrow W(jw) = (1/2\pi)\{X_1(jw) * X_2(jw)\}$

We have the following facts:

1) $X_1(jw)=0$ for $|w| > w_1$
2) $X_2(jw)=0$ for $|w| > w_2$

Convolution two signal will result a signal that is non-zero with at least on of the signals is non-zero

Therefore:
$W(jw)=0$ for $|w| > (w_1 + w_2)$

Nyquist rate = $2\, w_M = 2(w_1 + w_2)$ which is also the minimum sampling frequency for the signal to be recoverable.

Maximum sampling period = 2π / (minimum sampling frequency) = $2\pi / 2(w_1 + w_2) = \pi / (w_1 + w_2)$

6U. In the system two functions of time, $x_1(t)$ and $x_2(t)$, are multiplied together, and the product w(t) is sampled by a periodic impulse train where:

$$x_1(t) = \frac{d(e^{j2000t} + 10e^{-j2500\pi t})}{dt}$$

$$x_2(t) = e^{j2000t} Cos(15000\pi t) Sin(12000\pi t)$$

Determine the maximum sampling interval T such that w(t) is recoverable from the samples.

Solution:

7S. Determine whether each of the following statement is true or false:

a) The signal $x(t) = u(t + T_o) – u(t – T_o)$ can undergo impulse-train sampling without aliasing, provided that the sampling period $T < 2T_o$.

b) The signal x(t) with Fourier transform $X(jw) = u(w + w_o) – u(w – w_o)$ can undergo impulse-train sampling without aliasing, provided that the sampling period $T < \pi/w_o$.

c) The signal x(t) with Fourier transform $X(jw) = u(w) – u(w - w_o)$ can undergo impulse-train sampling without aliasing, provided that the sampling period $T< 2\pi/w_o$.

Solution:

a) $x(t) = u(t + T_o) – u(t – T_o)$ → $X(jw) = e^{+jwT_0}\left\{\frac{1}{jw} + \pi\delta(w)\right\} - e^{-jwT_0}\left\{\frac{1}{jw} + \pi\delta(w)\right\}$

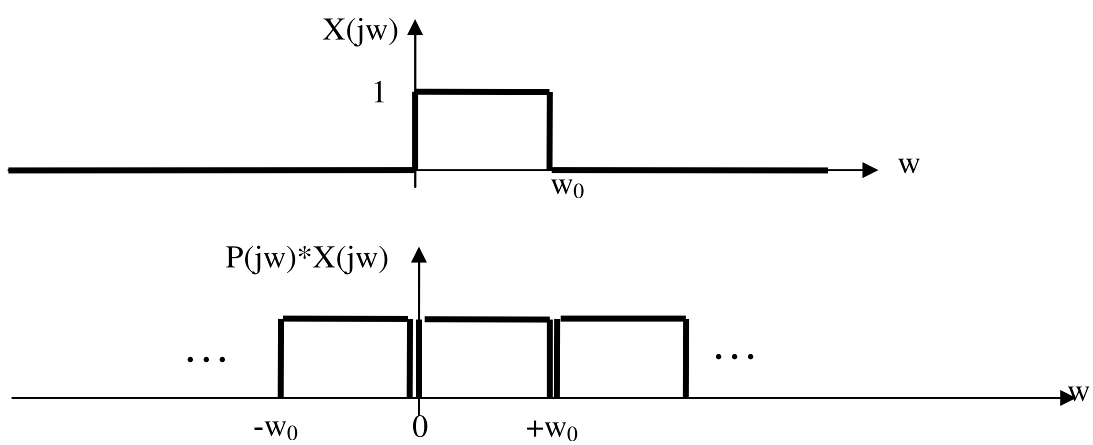
Meaning that x(t) is not a band-limited signal (w_M is not finite) therefore we can not sample it at a high enough rate so that it can be reconstructed . {Answer: False}

b) $X(jw) = u(w+w_0) –u(w-w_0)$ → $X(jw)=0$ for $|w| > w_0$ → x(t) is band limited

Nyquist rate = $2w_M = 2w_0$ → $w_s > 2w_0$ for no aliasing → $(2\pi/T_s) > 2w_0$

Therefore sampling period without aliasing is $T_s < (\pi/w_0)$
{Answer: True}

c) First draw X(jw) and its convolution with Impulse train with Sampling frequency = $2\pi/T > w_0$



So if we Filter the x(t)p(t) through a low pass filter with the cut off frequency of $wc = w_0$ we can recover the signal.

{Answer: True}

7U. Determine whether each of the following statement is true or false:

a) The signal $x(t) = 7u(t + 2T_o) – 12u(t – 2T_o)$ can undergo impulse-train sampling without aliasing, provided that the sampling period $T < 4T_o$.

b) The signal x(t) with Fourier transform $X(jw) = u(w + w_o) – u(w – 2w_o)$ can undergo impulse-train

sampling without aliasing, provided that the sampling period $T < 2\pi/w_o$.
c) The signal x(t) with Fourier transform $X(jw) = 5u(w) - 21u(w - w_o/2)$ can undergo impulse-train sampling without aliasing, provided that the sampling period $T < \pi/w_o$.

Solution:

8S. A signal x(t) with Fourier transform X(jw) undergoes impulse-train sampling to generate

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

where $T=10^{-4}$. For each of the following sets of constraints on x(t) and/or X(jw). Does the sampling theorem guarantee that x(t) can be recovered exactly from $x_p(t)$?

a) X(jw) = 0 for $|w| > 5{,}000\pi$
b) X(jw) = 0 for $|w| > 15{,}000\pi$
c) {Real X(jw)} = 0 for $|w| > 5{,}000\pi$
d) x(t) is real and X(jw)=0 for $w > 5{,}000\pi$
e) x(t) is real and X(jw)=0 for $w < -15{,}000\pi$
f) X(jw)*X(jw)=0 for $|w| > 15{,}000\pi$
g) |X(jw)|=0 for $w > 5{,}000\pi$

Solution:

For all the section sampling frequency is $Ws = 2\pi/T = 20{,}000\pi$.
for signal to be recoverable → 2x(Max. Signal Frequency, W_M) < Ws

a) Maximum signal Frequency = $w_M = 5{,}000\pi$ →
$2 W_M = 10{,}000\pi < Ws = 20{,}000\pi$ Therefore X(jw) is fully recoverable.

b) Maximum signal Frequency = $w_M = 15{,}000\pi$ →
$2 W_M = 30{,}000\pi > Ws = 20{,}000\pi$ Therefore X(jw) is not fully recoverable.

c) Since we do not have the imaginary portion of X(jw), we can determine Nyquist rate is indeterminate which means we cannot guarantee recovery.

d) Maximum signal Frequency = $w_M = 5{,}000\pi$ →
$2 W_M = 10{,}000\pi < Ws = 20{,}000\pi$ Therefore X(jw) is fully recoverable.

e) Maximum signal Frequency = $w_M = 15{,}000\pi$ →
$2 W_M = 30{,}000\pi > Ws = 20{,}000\pi$ Therefore X(jw) is not fully recoverable.

f) Convolution property says that:
X(jw) = 0 for $|w|>w_1$ → X(jw)*X(jw) = 0 for $|w|>w_1$
Therefore in this problem:
X(jw) = 0 for $|w| > 15{,}000\pi/2$
Maximum signal Frequency = $W_M = 15{,}000\pi/2$ →
$2 W_M = 15{,}000\pi < Ws = 20{,}000\pi$ Therefore X(jw) is fully recoverable.

g) $f_s=10{,}000$ → $w_s = 20{,}000\pi$.
Maximum signal Frequency = $w_M = 5{,}000\pi$ →
$2 W_M = 10{,}000\pi < Ws = 20{,}000\pi$ Therefore X(jw) is fully recoverable.

8U. A signal x(t) with Fourier transform X(jw) undergoes impulse-train sampling to generate

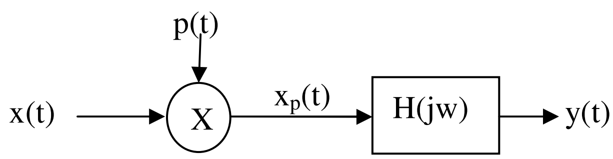
$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

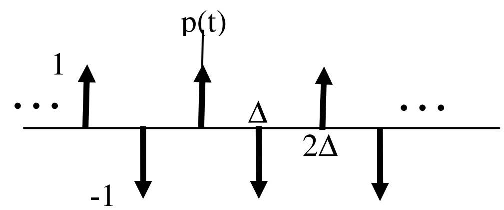
where T=2x10^{-5}. For each of the following sets of constraints on x(t) and/or X(jw), does the sampling theorem guarantee that x(t) can be recovered exactly from $x_p(t)$?

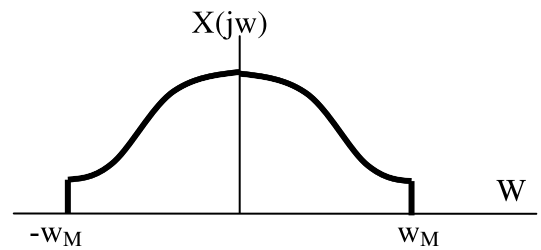
a) X(jw) = 0 for |w| > 51,000π
b) X(jw) = 0 for |w| > 15,000π
c) X(jw)*X(jw)=0 for |w| > 30,000π
d) |X(jw)|=0 for w > 49,000π

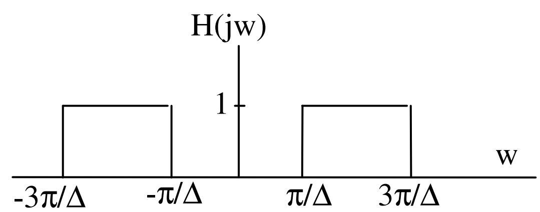
Solution:

9S. Using the following system in which sampling signal is an impulse train with alternating sign.









a) For Δ < π/(2w_M), sketch the Fourier transform of $x_p(t)$ and y(t)
b) For Δ < π/(2w_M), determine a system that will recover x(t) from $x_p(t)$.
c) For Δ < π/(2w_M), determine a system that will recover x(t) from y(t).
d) what is the maximum value of Δ in relations to w_M for which x(t) can be recovered from either $x_p(t)$ or y(t)?

Solution:

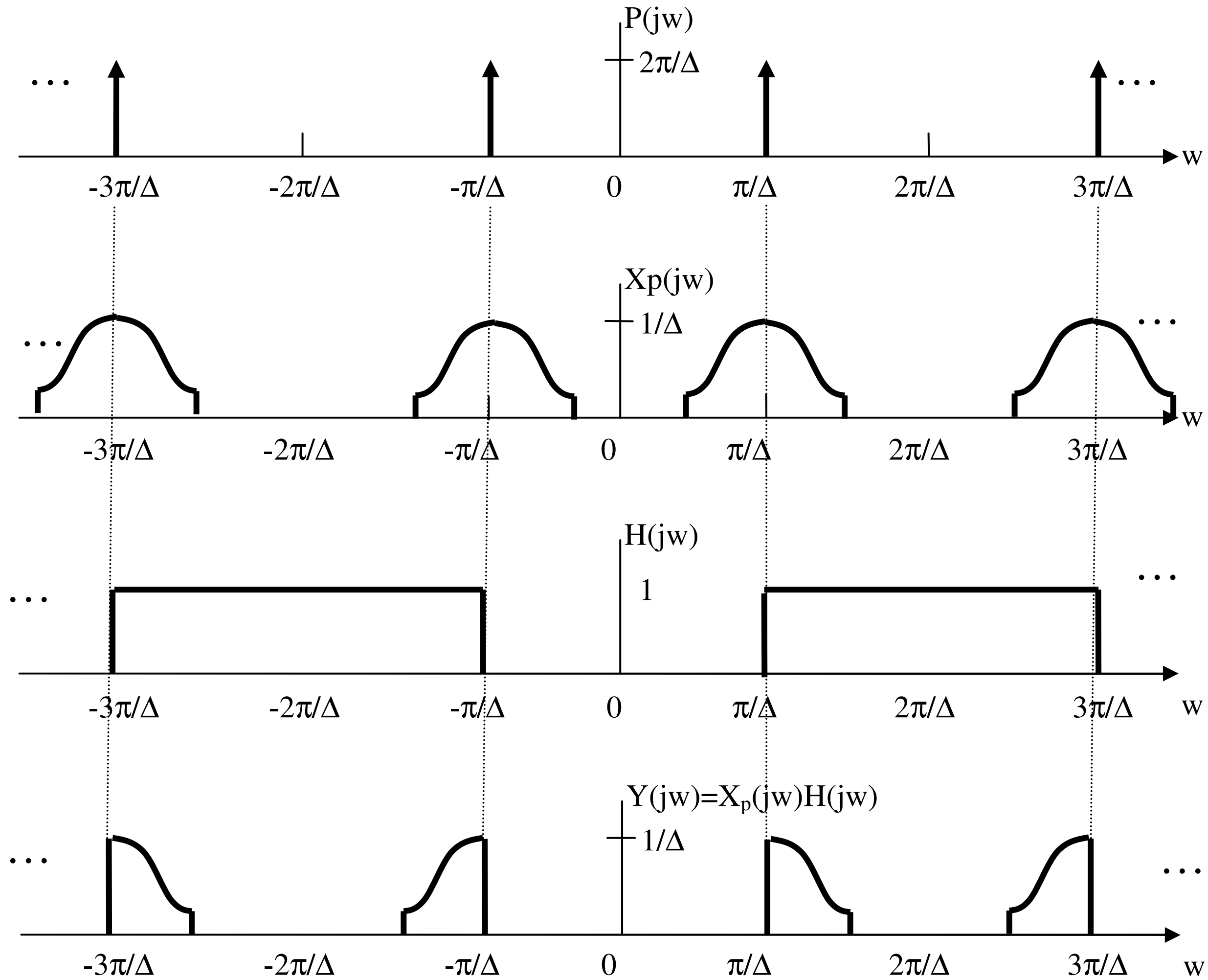
a) We can write p(t) = $p_1(t) - p_1(t-\Delta)$ where $p_1(t) = \sum_{k=-\infty}^{\infty} \delta(t - k2\Delta) \Rightarrow P_1(jw) = \frac{\pi}{\Delta} \sum_{k=-\infty}^{\infty} \delta(w - \pi / \Delta)$

using the above information and the time shifting property we can write P(jw) as:

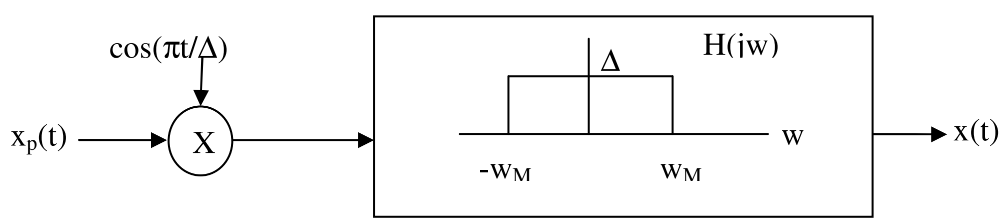
P(jw) = $P_1(jw) - e^{-jw\Delta} P_1(jw)$

$$P(jw) = \frac{\pi}{\Delta}\{\sum_{k=-\infty}^{\infty} \delta(w - \frac{\pi k}{\Delta}) - \sum_{k=-\infty}^{\infty} \delta(w - \frac{\pi k}{\Delta}) e^{-jw\Delta}\} \quad where \quad e^{-jw\Delta} = e^{-jwk} = (-1)^k \text{ where } w=\pi k/\Delta$$

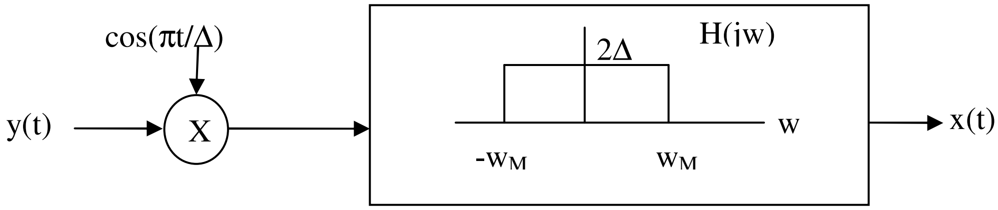
$$x_p(t) = x(t)p(t) \xrightarrow{FT} X_p(jw) = \frac{1}{2\pi}\left[X(jw) * P(jw)\right]$$



b) recovering x(t) from $x_p(t)$
Let's use:
1) FT{cos(w_0t)} = π[δ(w-w_0) + δ(w-w_0)
2) Convolutions FT{x_p(t)cos(πt/Δ)} = (1/2π)X_p(jw)*{ π[δ(w-π/Δ) + δ(w-π/Δ)}

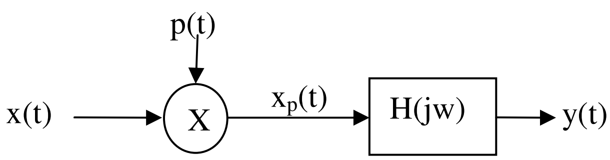


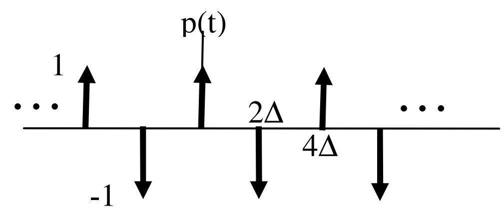
c) recovering x(t) from y(t) – use similar process as b.

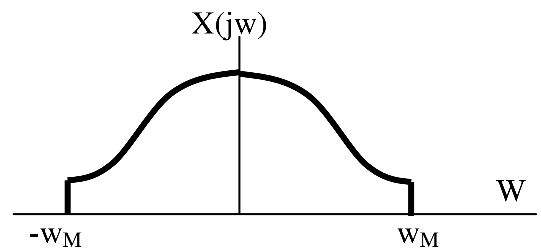


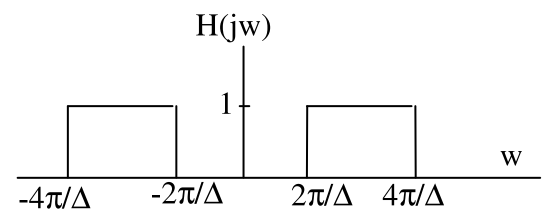
d) As can be seen from figure in section a, we can avoid aliasing by having $w_M < \pi/\Delta$ since sampling rate is $w_S = 2\pi/\Delta$ and it has to be larger the w_M for guaranteed recover.

9U. Using the following system in which sampling signal is an impulse train with alternating sign.









What is the maximum value of Δ in relations to w_M for which x(t) can be recovered from either $x_p(t)$ or y(t)?

Solution:

10S. The sampling theorem states that a signal x(t) must be sampled at a rate greater than its bandwidth (or equivalently, a rate greater than twice its highest frequency). This implies that if x(t) has a spectrum as indicated in figure (a) then x(t) must be samples at a rate greater than $2w_2$. However, since the signal has most of its energy concentrated in a narrow band, it would seem reasonable to expect that a sampling rate lower than twice the highest frequency could be used. A signal whose energy is concentrated in a frequency band is often referred to as a bandpass signal. There are a variety of techniques for sampling such signals, generally referred to as bandpass-sampling techniques.
To examine the possibility of sampling a bandpass signal at a rate less than the total bandwidth, consider the system shown in figure (b). Assuming the $w_1 > w_2 - w_1$, find the maximum value of T and the values of the constant A, w_a and w_b such that $x_r(t) = x(t)$.

Figure (a)

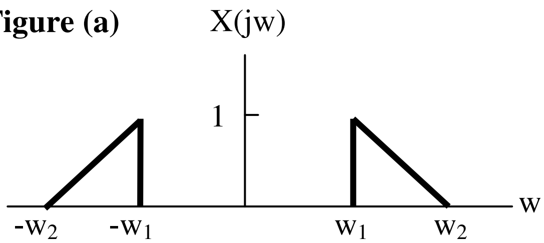
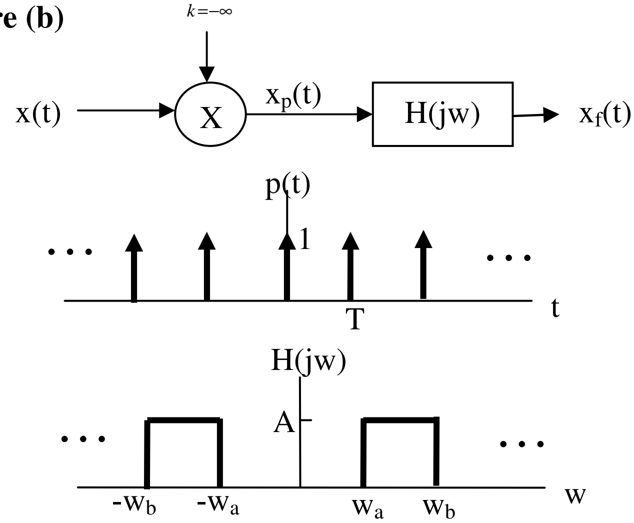


Figure (b)

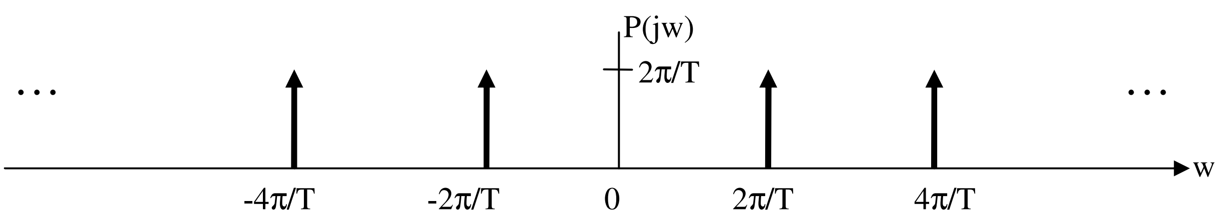
$$p(t)=\sum_{k=-\infty}^{+\infty}\delta(t-nT)$$

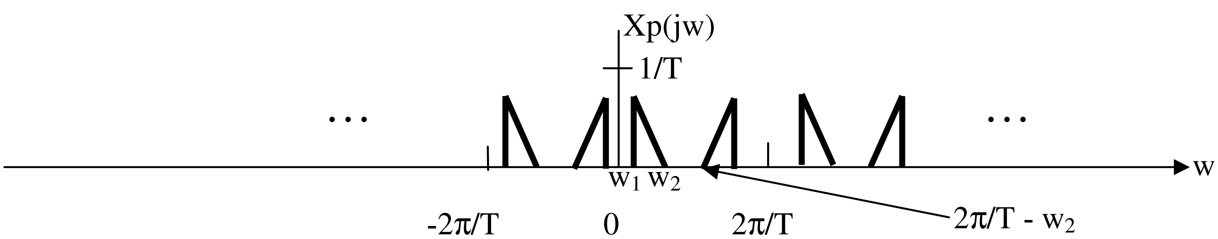


Solution:

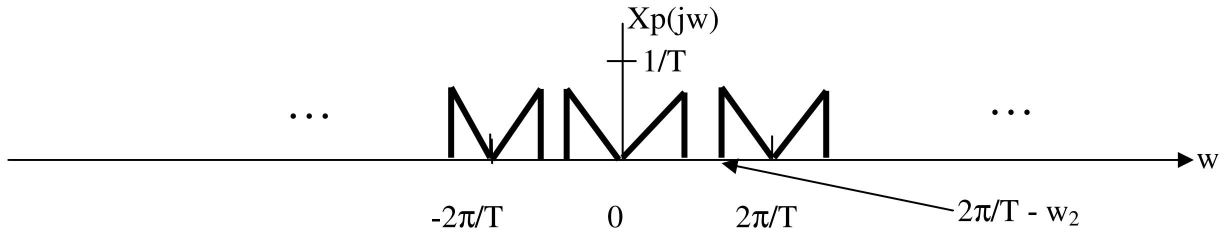
We have that $P(jw)=\dfrac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(w-k2\pi/T)$

Since $x_p(t) = x(t)p(t)$ → $X_p(jw)=\dfrac{1}{2\pi}[X(jw)*P(jw)]=\dfrac{1}{T}\sum_{k=-\infty}^{\infty}\delta(w-k2\pi/T)$





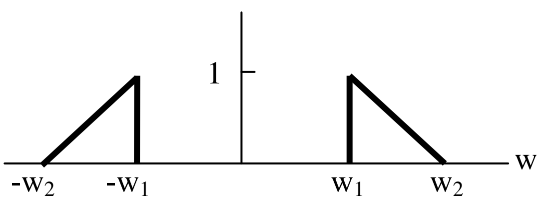
Consider that aliasing does not occurs when $(2\pi/T - w_2) > 0$ → Sampling period, $T < 2\pi/w_2$ which means $T_{Max} = 2\pi/w_2$ and result in following $X_P(jw)$:



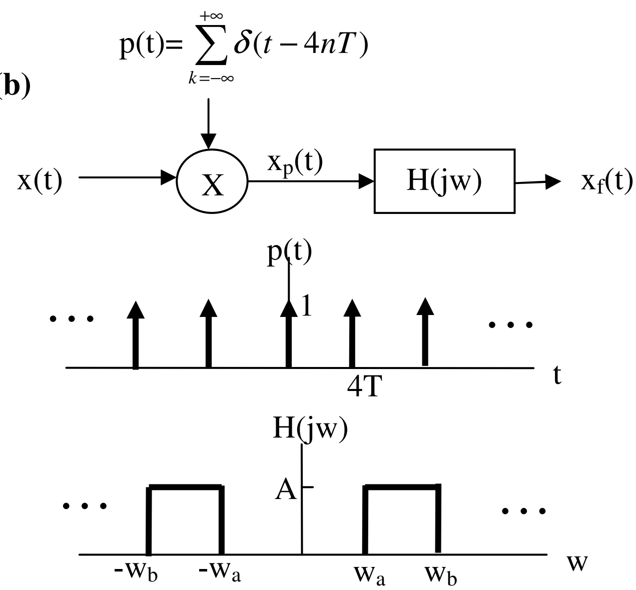
In order to have $x_r(t) = x(t) \rightarrow A = T$, $w_a = 2\pi/T$ & $w_a = w_b - w_1$

10U. Consider the system shown in figure (b). Assuming the $w_1 > w_2 - w_1$, find the maximum value of T and the values of the constant A, w_a and w_b such that $x_r(t) = x(t)$.

(a)



(b)



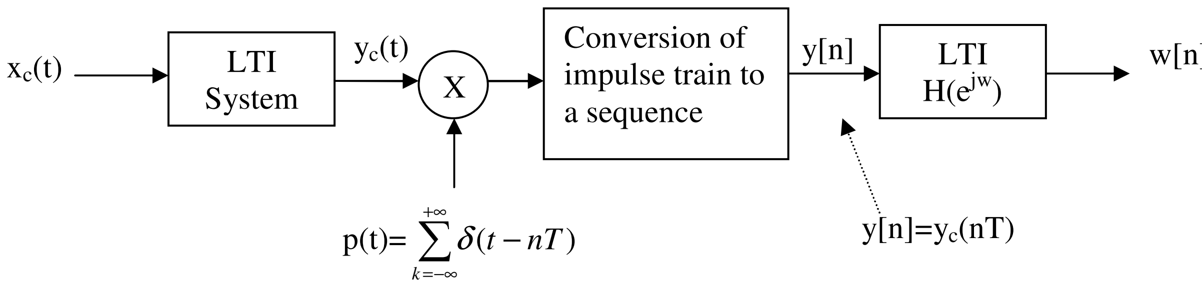
Solution:

11S. The system shown below consists of a continuous-time LTI system followed by a sampler, conversion to a sequence, and an LTI discrete-time system. The continuous-time LTI system is causal and satisfies the linear, constant-coefficient differential equation

$$\frac{dy_c(t)}{dt} + y_c(t) = x_c(t)$$

The input $x_c(t)$ is a unit impulse $\delta(t)$

a) Determine $y_c(t)$.
b) determine the frequency response $H(e^{jw})$ and the impulse response h[n] such that $w[n] = \delta[n]$.



Solution:

a) $x_c(t) = \delta(t)$ →

$$\frac{dy_c(t)}{dt} + y_c(t) = \delta(t)$$

Take F.T. of both side

$$jwY_c(jw) + Y_c(jw) = 1$$

$$Y_c(jw) = \frac{1}{jw+1} \xrightarrow{IFT} y_c(t) = e^{-t}u(t)$$

b)

$$y_c(t) = e^{-t}u(t)$$

$$y[n] = y_c(nT) = e^{-nT}u[n]$$

$$Y(e^{jw}) = \frac{1}{1-e^{-T}e^{-jw}}$$

$$given: H(e^{jw}) = \frac{W(e^{jw})}{Y((e^{jw})};$$

$$given: w[n] = \delta[n] \xrightarrow{FT} W(e^{jw}) = 1$$

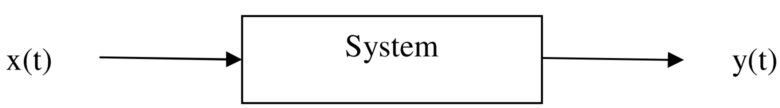
$$H(e^{jw}) = \frac{1}{\dfrac{1}{1-e^{-T}e^{-jw}}} = 1 - e^{-T}e^{-jw}$$

Therefore

$$h[n] = \delta[n] - e^{-T}\delta[n-1]$$

11U. A continuous-time LTI system is causal and satisfies the following linear, constant-coefficient differential equation:

$$3\frac{dy(t)}{dt} + 5y(t) = 2x(t)$$



Determine y(t) and h(t) if input x(t) is a unit impulse δ(t).

Solution:

12S. A signal, $x^2(t)$, undergoes sampling using the following pulse train:

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - \frac{T}{10{,}000})$$

Explain if $x^2(t)$ can be fully recovered from the sampled signal where |X(jw)|=0 for |w| > 8,000 rad/sec.

Solution:

Ts = 0.0001 → Ws = 2π/T = 20,000 π Sampling Frequecy
Mag. F{x(t)x(t)}=|X(jw)*X(jW)|=0 for |w|>16,000 → w_M = 16,000
Since Ws < 2 W_M → <u>signal is NOT recoverable from the sampled data</u>

13S. x(t) has a Nyquist rate of w_0. Determine the Nyquist rate of the following signal:

y(t) = x(t)cos(2w_ot)

Solution

Nyquist rate = 2 x maximum signal frequency
Sampling Rate must exceed Nyquist rate in order to be able to fully reconstruct the signal.

$$y(t) = x(t)\cos(w_0 t) \xrightarrow{FourierTransform} Y(jw) = \frac{X(j(w-w_0))}{2} + \frac{X(j(w+w_0))}{2}$$

Note: Use cos Fourier transform and convolution property to find Y(jw)

We see that Y(jw) = 0 when |w| > $2w_0$.+ $w_0/2$ since Y(jw)=0 when |w|> $5w_0/2$

Therefore Nyquist rate = $2w_m$ = $5w_0$

14S. What is the maximum allowable sampling period such that the following signal can be recovered from the sampled signal?

x(t) = Cos (2258πt) * sin(7742πt)
Note: "*" indicates convolution

Solutions:

X(jw) = π[δ(w-2258) + δ(w+2258)]+ π/j[δ(w-7742) + δ(w+7742)] = 0 for all w

T → ∞

<u>Wrong Approach</u>
This approach would be correct if the convolution was in time domain.
w_1 = 2258πt
w_2 = 7742πt

w_M = (2258π + 7742π)=10000π

$w_S > 2w_M$ must be true for the signal to be recoverable
$2\pi/T_s >$ 20000π → T_s < 1/10000 sec or 100 uSec.

15S. $x_p(t)$ is a sampled signal from x(t) with Fourier transform X(jw) as shown below:

$$x_p[t] = \sum_{m=-\infty}^{\infty} x(nT)\delta(t - nT)$$

Determine the limits of sampling period that guarantees x(t) is recoverable completely from the signal $x_p(t)$ when X(jw)*X(jw)=0 for |w| > 1500π.

Solutions:

Based on Convolution property we know that

If X(jw)=0 for |w| > w_1 then X(jw)*X(jw)=0 for |w|>2w_1

Therefore, we can conclude that x(jw)=0 for |w|>1500π/2 = 750π

In order to avoid aliasing and be able to recover x(t) from the $x_p(t)$

Sampling Frequency = $w_s > 2w_m = 1500\pi$
$2\pi/T_s > 1500\pi$
$T_s < 1/750$ Seconds

16S. Let x(t) be a signal with Nyquist rate 2000 rad/sec. Determine the Nyquist rate for the following signal:

x(t)cos(3000t)

Solution:

$$y(t) = x(t)\cos(3000t) \xrightarrow{FourierTransform} Y(jw) = \frac{X(j(w-3000))}{2} + \frac{X(j(w+3000))}{2}$$

Nyquist rate is 2 w_M therefore x(jw) = 0 when |w| > 1000 rad/sec.

Therefore Y(jw) = 0 when |w| > 1000+ 3000 = 4000 rad/sec.

Therefore Nyquist rate = $2w_m$ = 2 * 4000 = 8,000 rad/sec.