

Signals & Systems - Chapter 4

1S. Use the Fourier transform analysis equation to calculate the Fourier transform of:

a) $e^{-2(t-1)}u(t-1)$

b) $e^{-2|t-1|}$

Sketch and label the magnitude of each Fourier transform.

Solution: For Fourier Transform we have

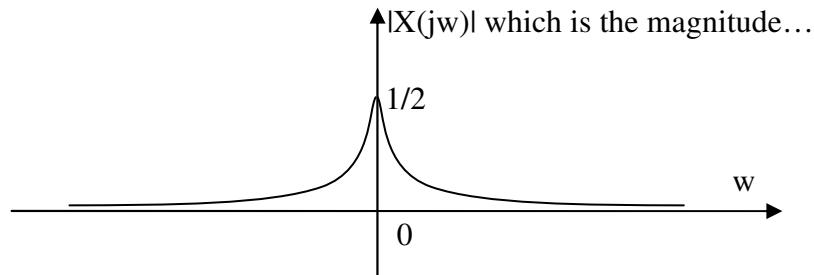
$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw)e^{jwt} dw \quad \text{Fourier Transform Synthesis equation}$$

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt} dt \quad \text{Fourier Transform Analysis equation}$$

a) Using the Fourier Transform Analysis equation

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt} dt = \int_{-\infty}^{\infty} e^{-2(t-1)}u(t-1)e^{-jwt} dt = \int_1^{\infty} e^{-2(t-1)}e^{-jwt} dt$$

$$X(jw) = \int_1^{\infty} e^{-(2+jw)t+2} dt = \frac{-1}{2+jw} e^{-(2+jw)t+2} \Big|_1^{\infty} = \frac{-1}{2+jw} (0 - e^{-(2+jw)+2}) = \frac{e^{-jw}}{2+jw}$$



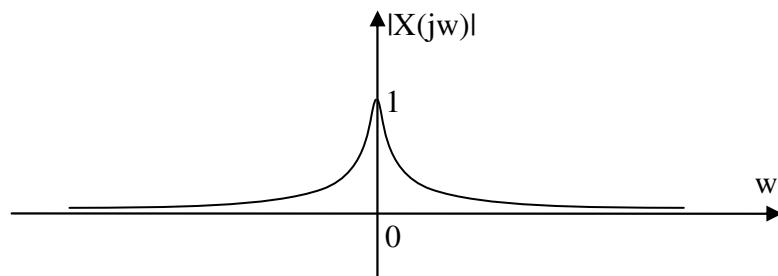
b) Using the Fourier Transform Analysis equation

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt} dt = \int_{-\infty}^{\infty} e^{-2|t-1|}e^{-jwt} dt = \int_1^{\infty} e^{-2(t-1)}e^{-jwt} dt + \int_{-\infty}^1 e^{2(t-1)}e^{-jwt} dt$$

$$X(jw) = \int_1^{\infty} e^{-(2+jw)t+2} dt + \int_{-\infty}^1 e^{(2-jw)t-2} dt$$

$$\frac{-1}{2+jw} e^{-(2+jw)t+2} \Big|_1^{\infty} + \frac{1}{2-jw} e^{(2-jw)t-2} \Big|_{-\infty}^1 = \frac{e^{-jw}}{2+jw} + \frac{e^{-jw}}{2-jw} =$$

$$X(jw) = \frac{4e^{-jw}}{4+w^2}$$



1U. Use the Fourier transform analysis equation to calculate the Fourier transform of:

a) $e^{4(t-5)}u(-t+3)$

b) $e^{-4|t-3|}$

Sketch and label the magnitude of each Fourier transform.

Solution:

2S. Use the Fourier transform analysis equation to calculate the Fourier transform of:

a) $\delta(t+1) + \delta(t-1)$

b) $\frac{d}{dt}\{u(-2-t) + u(t-2)\}$

Sketch and label the magnitude of each Fourier transform.

Solution: For Fourier Transform we have

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw)e^{jwt} dw \quad \text{Fourier Transform Synthesis equation}$$

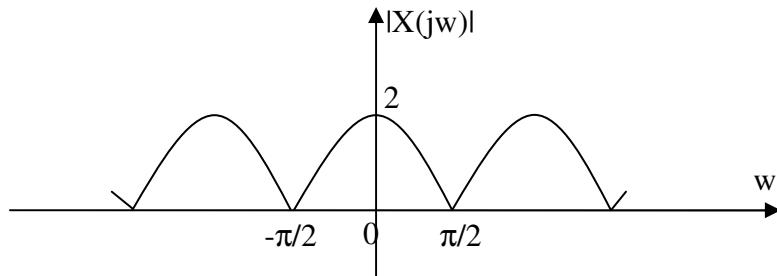
$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt} dt \quad \text{Fourier Transform Analysis equation}$$

a) Use the Fourier Transform Analysis Equation

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt} dt = \int_{-\infty}^{\infty} \{\delta(t+1) + \delta(t-1)\}e^{-jwt} dt$$

we know that when $t=0$ then $\delta(t)=1$ otherwise $\delta(t)=0$ therefore

$$X(jw) = e^{jw} + e^{-jw} = 2\cos w$$

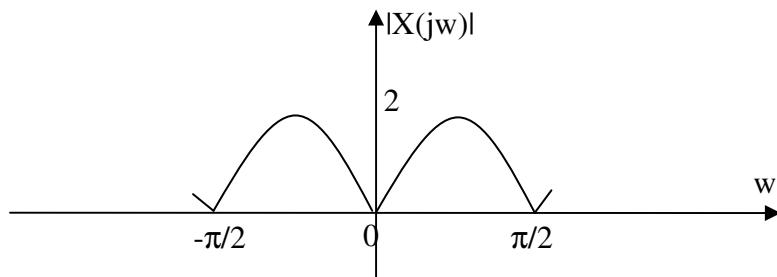


b) Use the Fourier Transform Analysis Equation

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt} dt = \int_{-\infty}^{\infty} \frac{d}{dt}\{u(-2-t) + u(t-2)\}e^{-jwt} dt$$

we know that $\frac{d\{u(t)\}}{dt} = \delta(t)$ therefore

$$X(jw) = \int_{-\infty}^{\infty} \{\delta(t-2) - \delta(t+2)\}e^{-jwt} dt = e^{-2jw} - e^{2jw} = -2j\sin(2w)$$



2U. Use the Fourier transform analysis equation to calculate the Fourier transform of:

a) $\delta(t+2) + \delta(-t+2)$

b) $\frac{d}{dt}\{u(-3-t) + u(t-3)\}$

Sketch and label the magnitude of each Fourier transform.

Solution:

3S. Determine the Fourier transform of each of the following periodic signals:

a) $\sin(2\pi t + \frac{\pi}{4})$

b) $1 + \cos(6\pi t + \frac{\pi}{8})$

Solution:

a) signal $x(t) = \sin(2\pi t + \frac{\pi}{4})$ is periodic with period (Fundamental Period, $T=1 \rightarrow w_0 = 2\pi$)

The easiest way would be use Euler's Identity to find the Fourier Series coefficients.

$$x(t) = \sin(2\pi t + \frac{\pi}{4}) = \frac{1}{2j}(e^{j(2\pi t + \pi/4)} - e^{-j(2\pi t + \pi/4)}) = \frac{1}{2j}e^{j\pi/4}e^{j2\pi t} - \frac{1}{2j}e^{-j\pi/4}e^{-j2\pi t}$$

$$\text{Therefore } a_1 = \frac{1}{2j}e^{j\pi/4} \quad a_{-1} = -\frac{1}{2j}e^{-j\pi/4}$$

We know this signal is periodic with period 1 and we know that a periodic signal in the above form has transfer function as shown below:

$$X(jw) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(w - kw_0) = 2\pi a_{-1} \delta(w + w_0) + 2\pi a_1 \delta(w - w_0)$$

$$X(jw) = -\frac{\pi}{j}e^{-j\pi/4} \delta(w + 2\pi) + \frac{\pi}{j}e^{j\pi/4} \delta(w - 2\pi)$$

b) signal $x(t) = 1 + \cos(6\pi t + \frac{\pi}{8})$ is periodic with period (Fundamental Period, $T=1/3 \rightarrow w_0 = 6\pi$)

The easiest way would be use Euler's Identity to find the Fourier Series coefficients.

$$x(t) = 1 + \cos(6\pi t + \frac{\pi}{8}) = 1 + \frac{1}{2}(e^{j(6\pi t + \pi/8)} + e^{-j(6\pi t + \pi/8)}) = 1 + \frac{1}{2}e^{j\pi/8}e^{j6\pi t} + \frac{1}{2}e^{-j\pi/8}e^{-j6\pi t}$$

$$\text{Therefore } a_1 = \frac{1}{2}e^{j\pi/8} \quad a_{-1} = \frac{1}{2}e^{-j\pi/8} \quad a_0 = 1$$

We know this signal is periodic with period 1 and we know that a periodic signal in the above form has transfer function as shown below:

$$X(jw) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(w - kw_0) = 2\pi a_{-1} \delta(w + w_0) + 2\pi a_0 \delta(w) + 2\pi a_1 \delta(w - w_0)$$

$$X(jw) = \pi e^{-j\pi/8} \delta(w + 6\pi) + 2\pi \delta(w) + \pi e^{j\pi/8} \delta(w - 6\pi)$$

3U. Determine the Fourier transform of each of the following periodic signals:

a) $\cos(2\pi t - \frac{\pi}{3})$

b) $10 + \sin(5\pi t + \frac{3\pi}{8})$

Solution:

4S. Use the Fourier transform synthesis equation to determine the inverse Fourier transforms of:

$$X_2(jw) = 2 \quad \text{for} \quad 0 \leq w \leq 2$$

$$\text{a) } X_1(jw) = 2\pi\delta(w) + \pi\delta(w - 4\pi) + \pi\delta(w + 4\pi) \quad \text{b) } \begin{cases} -2 & \text{for } -2 \leq w < 0 \\ 0 & \text{for } |w| \geq 2 \end{cases}$$

Solution:

a)

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw)e^{jwt} dw \quad \text{Fourier Transform Synthesis equation}$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{2\pi\delta(w) + \pi\delta(w - 4\pi) + \pi\delta(w + 4\pi)\} e^{j\omega t} dw$$

$$x(t) = \frac{1}{2\pi} \{ 2\pi e^0 + \pi e^{j4\pi} + \pi e^{-j4\pi} \} = 1 + \frac{1}{2} e^{j4\pi} + \frac{1}{2} e^{-j4\pi} = 1 + \cos(4\pi t)$$

b)

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) e^{jw t} dw \quad \text{Fourier Transform Synthesis equation}$$

$$x(t) = \frac{1}{2\pi} \int_0^2 2e^{j\omega t} dw + \frac{1}{2\pi} \int_{-2}^0 -2e^{j\omega t} dw = \frac{1}{i\pi} (e^{j2t} - 1) - \frac{1}{i\pi} (1 - e^{-j2t})$$

$$x(t) = \frac{1}{i\pi} (e^{j2t} + e^{-j2t}) - \frac{2}{i\pi}$$

4U. Use the Fourier transform synthesis equation to determine the inverse Fourier transforms of:

a) $X_1(jw) = 3j\pi\delta(w - 2\pi) + \pi\delta(w - 4\pi) - 3j\pi\delta(w + 2\pi) + \pi\delta(w + 4\pi) - 10$

$$X_2(jw) = 1 \quad \text{for} \quad 0 \leq w \leq 5$$

b) $-1 \quad for \quad -5 \leq w < 0$

$$0 \quad for \quad |w| > 5$$

Solution:

5S. Consider the signal

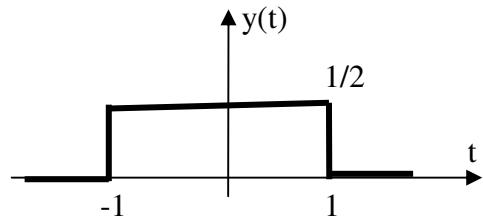
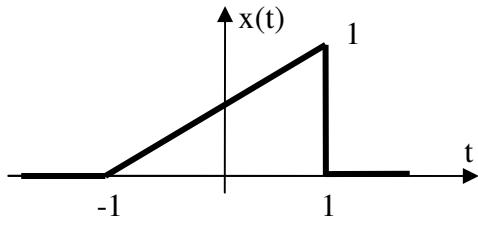
$$x(t) = \begin{cases} 0 & \text{for } |t| > 1 \\ (t+1)/2 & \text{for } -1 \leq t \leq 1 \end{cases}$$

- a) With the help of Fourier Transform pair table, determine the closed-form expression for $X(jw)$
 - b) Take the real part of your answer to part (a), and verify that it is the Fourier transform of the even part of $x(t)$.
 - c) What is the Fourier transform of the odd part of $x(t)$?

Solution:

a)

in order to be able to use the tables, we need to make the following conversions:



We can rewrite $x(t)$ as $x(t) = \frac{1}{2} \int_{-\infty}^t y(t) dt - u(t-1)$ and then use the following transformations

from the table:

$$\int_{-\infty}^t y(t) dt \xrightarrow{F} \frac{1}{jw} Y(jw) + \pi Y(0) \delta(w)$$

$$\text{Square wave : } y(t) = 1/2 \text{ for } |t| < 1 \text{ and } y(t) = 0 \text{ for } |t| > 1 \xrightarrow{F} \frac{\sin w}{w}$$

$$z(t-1) \xrightarrow{F} e^{-jw} Z(jw)$$

$$u(t) \xrightarrow{F} \frac{1}{jw} + \pi \delta(w)$$

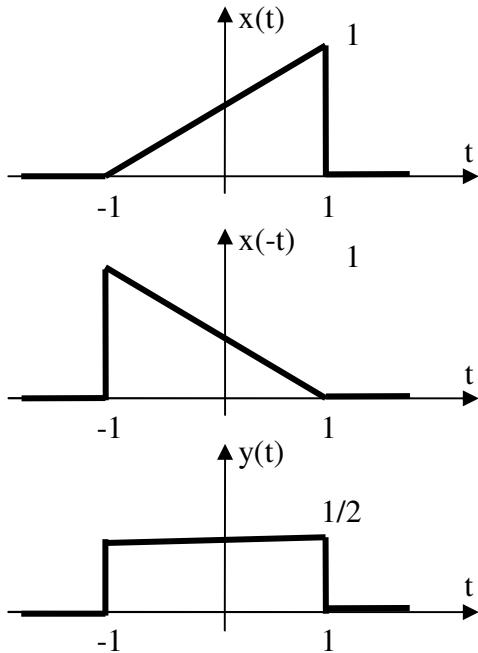
Which leads to:

$$X(jw) = \frac{1}{jw} \left\{ \frac{\sin w}{w} \right\} - e^{-jw} \left\{ \frac{1}{jw} + \pi \delta(w) \right\}$$

$$X(jw) = \frac{\sin w}{jw^2} - \frac{e^{-jw}}{jw} - \pi e^{-jw} \delta(w)$$

b)

Even part $\{x(t)\} = \{x(t) + x(-t)\} / 2$



$$\text{Square wave : } y(t) = 1/2 \text{ for } |t| < 1 \text{ and } y(t) = 0 \text{ for } |t| > 1 \xrightarrow{\text{F}} \frac{\sin w}{w}$$

Therefore $\text{F}\{\text{even part of } x(t)\} = \sin(w) / w$

c)

$$x(t) = \{\text{odd part of } x(t)\} + \{\text{even part of } x(t)\}$$

$$\{\text{odd part of } x(t)\} = x(t) - \{\text{even part of } x(t)\}$$

$$\text{F}\{\text{odd part of } x(t)\} = \frac{\sin w}{jw^2} - \frac{e^{-jw}}{jw} - \frac{\sin w}{w} = \frac{\sin w}{jw^2} - \frac{\cos w}{jw}$$

5U. Consider the signal

$$x(t) = 0 \quad \text{for } |t| > 2$$

$$t/2 \quad \text{for } -2 \leq t \leq 2$$

- a) With the help of Fourier Transform pair table, determine the closed-form expression for $X(jw)$
- b) Take the real part of your answer to part (a), and verify that it is the Fourier transform of the even part of $x(t)$.
- c) What is the Fourier transform of the odd part of $x(t)$?

Solution:

6S. Determine whether each of the following statements is true or false. Justify your answers.

- a) An odd and imaginary signal always has an odd and imaginary Fourier transform.
- b) The convolution of an odd Fourier transform with an even Fourier Transform is always odd.

Solution:

- a) From Fourier transform properties table we have
odd signal $x(t) \rightarrow$ purely imaginary and odd Fourier transform $X(jw)$
 $\text{F}\{\text{purely imaginary signal } jx(t)\} \rightarrow jX(jw)$ using linearity

Based on the above two statements we can conclude that an odd and imaginary signal $jx(t)$ always has an odd and real Fourier transform therefore the problem statement is **false**.

- b) We have
 - 1) odd Fourier transform corresponds to an odd signal
 - 2) even Fourier transform corresponds to an even signal

Convolution of an even F.T. with an odd F.T. in time domain is multiplication of even and odd function which results in an odd function therefore the problem statement is **True**.

6U. Determine whether each of the following statements is true or false. Justify your answers.

- a) A $\sin()$ function will always have pure imaginary even Fourier Transfer function.
- b) A $\tan()$ function will always have pure imaginary odd Fourier Transfer function.
- c) A $\cos()$ function will always have real and even Fourier Transfer function.

Solution:

7S. Compute the Fourier transform for each of the following Signals:

$$\text{a) } [e^{-at} \cos w_o t]u(t) \quad \text{for } a > 0 \qquad \text{b) } e^{-3|t|} \sin 2t$$

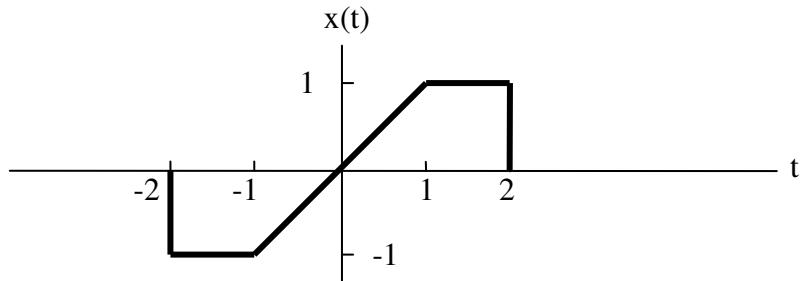
c) $x(t) = \begin{cases} 1 + \cos \pi t & \text{for } |t| \leq 1 \\ 0 & \text{for } |t| > 1 \end{cases}$

d) $\sum_{k=0}^{\infty} a^k \delta(t - kT) \quad \text{for } |a| < 1$

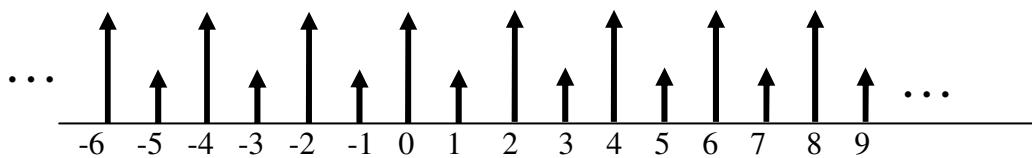
e) $[te^{-2t} \sin 4t]u(t)$

f) $\left[\frac{\sin \pi t}{\pi t} \right] \left[\frac{\sin 2\pi(t-1)}{\pi(t-1)} \right]$

g)



h)



i) $x(t) = \begin{cases} 1 - t^2 & \text{for } 0 < t < 1 \\ 0 & \text{Otherwise} \end{cases}$

j) $\sum_{n=-\infty}^{+\infty} e^{-|t-2n|}$

Solution:

a)

$$x(t) = [e^{-at} \cos \omega_o t]u(t) \quad \text{for } a > 0$$

$$x(t) = \frac{1}{2} e^{-at} \{e^{j\omega_0 t} + j e^{-j\omega_0 t}\} u(t) = \left\{ \frac{1}{2} e^{-(a-j\omega_0)t} + \frac{1}{2} e^{-(a+j\omega_0)t} \right\} u(t)$$

From the Fourier Transform Table:

$$e^{-at} u(t) \xrightarrow{F} \frac{1}{a + j\omega}$$

Therefore \Rightarrow

$$X(j\omega) = \frac{1}{2(a - j\omega_0 + j\omega)} + \frac{1}{2(a + j\omega_0 + j\omega)}$$

b)

$$x(t) = e^{-3|t|} \sin(2t) = e^{-3t} \sin(2t)u(t) + e^{3t} \sin(2t)u(-t)$$

$$x(t) = \frac{1}{2j} e^{-3t} \{e^{j2t} - e^{-j2t}\} u(t) + \frac{1}{2j} e^{3t} \{e^{j2t} - e^{-j2t}\} u(-t)$$

$$x(t) = x_1(t) + x_2(t) \quad \text{where}$$

$$x_1(t) = \frac{1}{2j} e^{-(3-2j)t} u(t) - \frac{1}{2j} e^{-(3+2j)t} u(t)$$

$$x_2(t) = \frac{1}{2j} e^{-(3-2j)t} u(-t) - \frac{1}{2j} e^{-(3+2j)t} u(-t) = -x_1(-t) \Rightarrow X_2(jw) = -X_1(-jw)$$

$$X_1(jw) = \frac{1}{2j(3-j2+jw)} - \frac{1}{2j(3+j2+jw)}$$

$$X(jw) = X_1(jw) + X_2(jw) = \frac{1}{2j(3-j2+jw)} - \frac{1}{2j(3+j2+jw)} - \frac{1}{2j(3-j2-jw)} + \frac{1}{2j(3+j2-jw)}$$

$$X(jw) = \frac{1}{9+(w+2)^2} - \frac{1}{9+(w-2)^2}$$

c)

$$x(t) = 1 + \cos \pi t \quad \text{for } |t| \leq 1$$

$$0 \quad \text{for } |t| > 1$$

$$X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt = \int_{-1}^1 (1 + \cos \pi t) e^{-jwt} dt = \int_{-1}^1 \left(1 + \frac{1}{2} e^{+j\pi t} + \frac{1}{2} e^{-j\pi t}\right) e^{-jwt} dt$$

$$X(jw) = \left\{ \frac{e^{-jwt}}{-jw} + \frac{e^{j(\pi-w)t}}{2j(\pi-w)} + \frac{e^{j(\pi-w)t}}{2j(\pi-w)} \right\}_{-1}^1 = \frac{2 \sin w}{w} + \frac{\sin w}{\pi-w} - \frac{\sin w}{\pi+w}$$

d)

$$x(t) = \sum_{k=0}^{\infty} a^k \delta(t - kT) \quad \text{for } |a| < 1$$

$$X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt = \int_{-\infty}^{\infty} \sum_{k=0}^{\infty} a^k \delta(t - kT) e^{-jwt} dt$$

Note: $\delta(t - kT) = 1$ for $t = KT$ otherwise $\delta(t - kT) = 0$ therefore:

$$X(jw) = \sum_{k=0}^{\infty} a^k e^{-jwkT} = \sum_{k=0}^{\infty} (a^1 e^{-jwT})^k$$

$$\text{Apply finite sum} \Rightarrow \sum_{k=0}^{\infty} a^k = \frac{1}{1-a} \text{ when } |a| < 1$$

$$X(jw) = \frac{1}{1 - ae^{-jwT}}$$

e)

$$x(t) = [te^{-2t} \sin 4t]u(t) = \frac{t}{2j} [e^{-2t} e^{j4t} - e^{-2t} e^{-j4t}] u(t) = \frac{t}{2j} [e^{-(2-j4)t} - e^{-(2+j4)t}] u(t)$$

$$\text{From table : } te^{-at} u(t) \xrightarrow{F} \frac{1}{(a+jw)^2}$$

$$X(jw) = \frac{1}{(2-j4+jw)^2} - \frac{1}{(2+j4+jw)^2}$$

f)

$$x(t) = \left[\frac{\sin \pi t}{\pi} \right] \left[\frac{\sin 2\pi(t-1)}{\pi(t-1)} \right]$$

From the Fourier Transform Table:

$$x_1(t) = \frac{\sin \pi t}{\pi} \xrightarrow{F} X_1(jw) = 1 \text{ for } |w| < \pi \text{ otherwise } X_1(jw) = 0$$

apply the time shifting and time scaling properties \Rightarrow

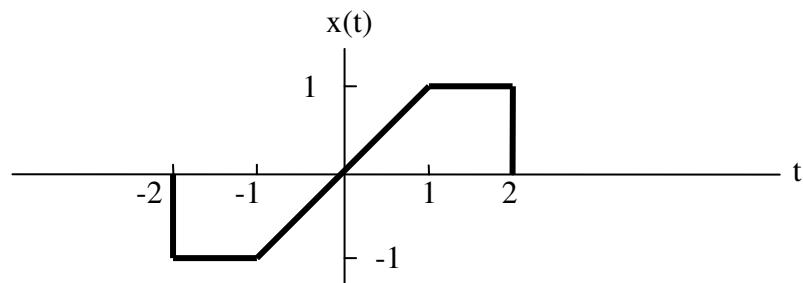
$$x_2(t) = \frac{\sin 2\pi(t-1)}{\pi(t-1)} \xrightarrow{F} X_2(jw) = e^{-j2w} \text{ for } |w| < 2\pi \text{ otherwise } X_2(jw) = 0$$

$$x(t) = x_1(t) x_2(t) \rightarrow X(jw) = \frac{1}{2\pi} \{X_1(jw) * X_2(jw)\}$$

$$X(jw) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_2(j\theta) X_1(j(w-\theta)) d\theta$$

$$X(jw) = \begin{cases} e^{-jw} & |w| < \pi \\ (1/2\pi)(3\pi+w)e^{-jw} & -3\pi < w < -\pi \\ (1/2\pi)(3\pi+w)e^{-jw} & \pi < w < 3\pi \\ 0 & \text{otherwise} \end{cases}$$

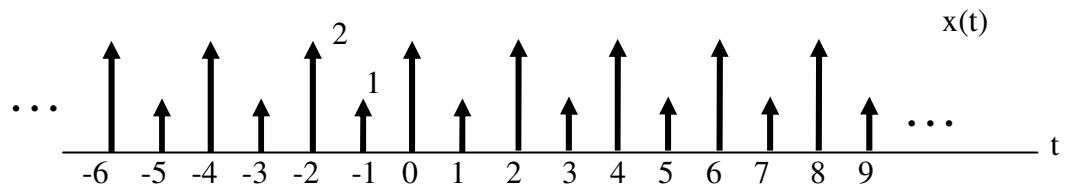
g)



$$X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt = \int_{-2}^{-1} -e^{-jwt} dt + \int_{-1}^1 te^{-jwt} dt + \int_1^2 e^{-jwt} dt$$

$$X(jw) = \frac{2j}{w} \left[\cos 2w - \frac{\sin w}{w} \right]$$

h)



if $x_1(t) = \sum_{k=-\infty}^{\infty} \delta(t-2k)$ then $x(t) = 2x_1(t) + x_1(t-1)$ therefore

$$X(jw) = X_1(jw)[2 + e^{jw}] = \pi \sum_{k=-\infty}^{\infty} \delta(w - \pi k)[2 + e^{jw}]$$

i)

$$x(t) = 1 - t^2 \quad \text{for } 0 < t < 1$$

0 Otherwise

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt} dt = \int_0^1 (1 - t^2)e^{-jwt} dt$$

$$X(jw) = \frac{1}{jw} + \frac{2e^{-jw}}{-w^2} - \frac{2e^{-jw} - 2}{jw^2}$$

j) $x(t) = \sum_{n=-\infty}^{+\infty} e^{-|t-2n|}$ is periodic with period 2 therefore

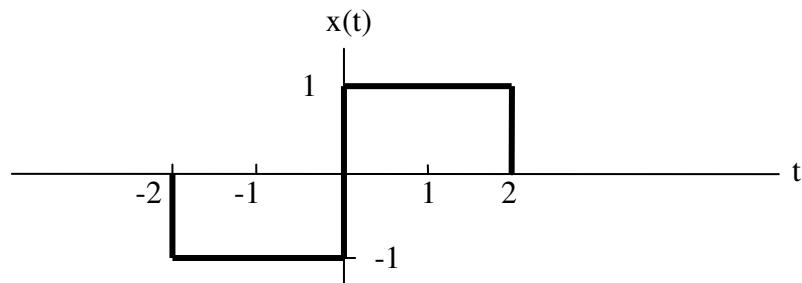
$$x(t) = \sum_{n=-\infty}^{+\infty} e^{-(t-2n)} u(t-2n) + \sum_{n=-\infty}^{+\infty} e^{(t-2n)} u(-t+2n)$$

$$X(jw) = \pi \sum_{k=-\infty}^{+\infty} \left[\frac{1}{1-e^{-2}} \left\{ \frac{1-e^{-2(1+jw)}}{1+jw} - e^{-2} \frac{1-e^{2(1+jw)}}{1-jw} \right\} \right] \delta(w - k\pi)$$

7U. Compute the Fourier transform for each of the following Signals:

a) $e^{-4|t|} \cos 2t$

b)



c) $x(t) = 4 + 3t^2 \quad \text{for } 0 < t < 1$
0 Otherwise

d)
$$\sum_{n=-\infty}^{+\infty} e^{-|2t-2n|}$$

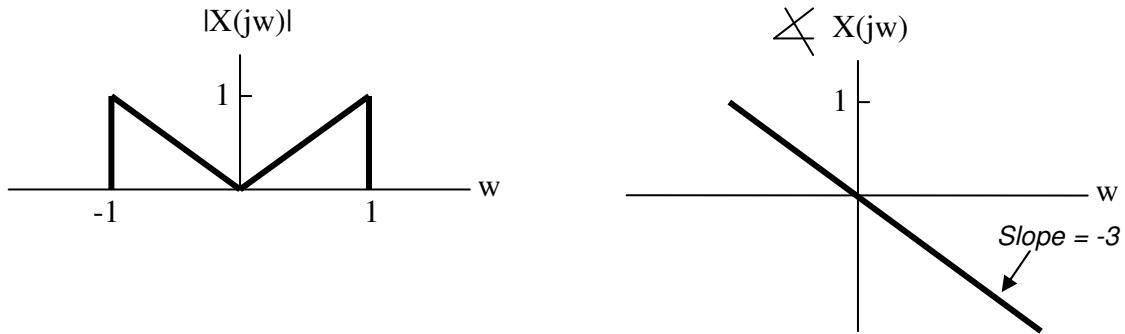
Solution:

8S. Determine the continuous-time signal corresponding to each of the following transforms.

a)
$$X(jw) = \frac{2 \sin[3(w - 2\pi)]}{(w - 2\pi)}$$

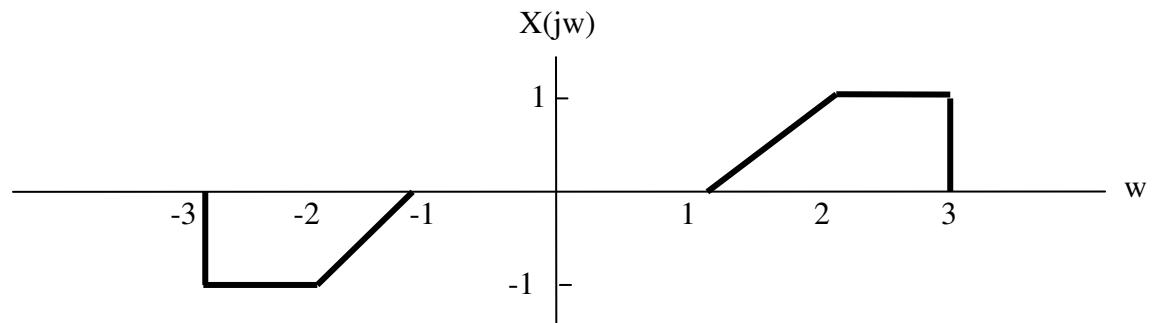
b)
$$X(jw) = \cos(4w + \pi/3)$$

c) $X(jw)$ as given by the following magnitude and phase plots



d)
$$X(jw) = 2[\delta(w-1) - \delta(w+1)] + 3[\delta(w-2\pi) + \delta(w+2\pi)]$$

e) $X(jw)$ as shown in the following figure



Solution:

a)

$$X(jw) = \frac{2 \sin[3(w - 2\pi)]}{(w - 2\pi)}$$

Inverse Fourier Transform Table

$$X_1(jw) = \frac{2 \sin w T_1}{w} \xrightarrow{F^{-1}\{\cdot\}} 1 \text{ for } |t| < T_1 \text{ otherwise } x_1(t) = 0$$

$$X_2(jw) = j(w - w_0) \xrightarrow{F^{-1}\{\cdot\}} e^{jw_0 t} x_1(t)$$

therefore

$$\begin{aligned} x(t) &= e^{j2\pi t} & |t| < 3 \\ &0 & \text{Otherwise} \end{aligned}$$

b)

$$X(jw) = \cos(4w + \pi/3) = \frac{1}{2} \{e^{j(4w + \pi/3)} + e^{-j(4w + \pi/3)}\} = \frac{1}{2} e^{j4w} e^{j\pi/3} + \frac{1}{2} e^{-j4w} e^{-j\pi/3}$$

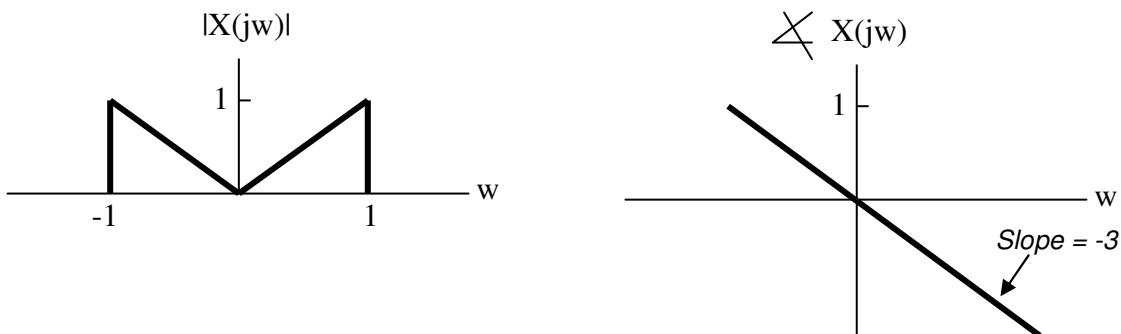
Inverse Fourier Transform Table

$$e^{-jwt_0} \xrightarrow{F^{-1}\{\cdot\}} 2\pi\delta(t - t_0)$$

Therefore

$$x(t) = \frac{1}{2} e^{j\pi/3} \delta(t + 4) + \frac{1}{2} e^{-j\pi/3} \delta(t - 4)$$

c)



$$X(jw) = |X(jw)| e^{j\{\text{phase}\{X(jw)\}\}}$$

$$\text{Inv. F.T.} \Rightarrow x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) e^{jw_0 t} dw = \frac{1}{2\pi} \int_{-1}^0 -we^{j(-3w)} e^{jw_0 t} dw + \frac{1}{2\pi} \int_0^1 we^{j(-3w)} e^{jw_0 t} dw$$

$$x(t) = \frac{1}{\pi} \left[\frac{\sin(t-3)}{t-3} + \frac{\cos(t-3)-1}{(t-3)^2} \right]$$

d)

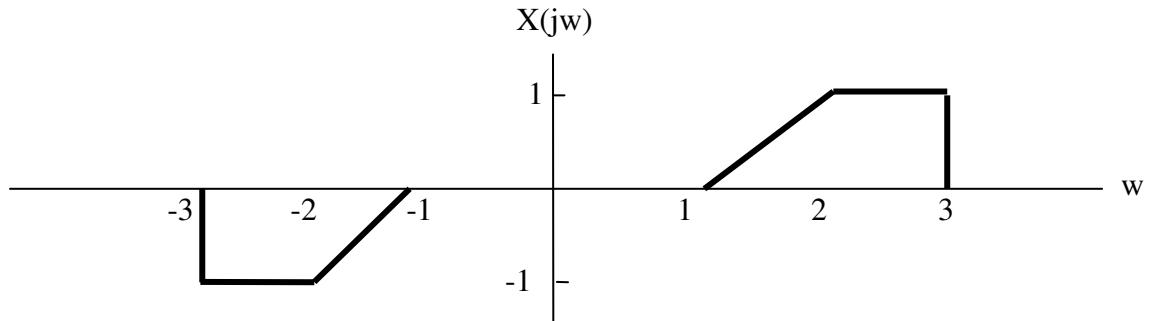
$$X(jw) = 2[\delta(w-1) - \delta(w+1)] + 3[\delta(w-2\pi) + \delta(w+2\pi)]$$

Inv.. F.T. Table \Rightarrow

$$2\pi\delta(w-w_0) \xrightarrow{\text{Inv.FT}\{\cdot\}} e^{jw_0 t}$$

$$x(t) = \frac{2}{2\pi} [e^{jt} - e^{-jt}] + \frac{3}{2\pi} [e^{j2\pi t} + e^{-j2\pi t}] = \frac{2j}{\pi} \sin t + \frac{3}{\pi} \cos(2\pi t)$$

e)



$$\text{Inv. F.T.} \Rightarrow x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) e^{jw_0 t} dw$$

$$x(t) = \frac{1}{2\pi} \int_{-3}^{-2} -e^{jw_0 t} dw + \frac{1}{2\pi} \int_{-2}^{-1} (1+t)e^{jw_0 t} dw + \frac{1}{2\pi} \int_{-1}^2 (-1+t)e^{jw_0 t} dw + \frac{1}{2\pi} \int_2^3 e^{jw_0 t} dw$$

$$x(t) = \frac{\cos 3t}{j\pi} + \frac{\sin t - \sin 2t}{j\pi^2}$$

8U. Determine the continuous-time signal corresponding to each of the following transforms.

a) $X(jw) = \frac{3\sin[5(w-5\pi)]}{(w-5\pi)}$

b) $X(jw) = \sin(2w - 2\pi/3)$

c) $X(jw) = 5[\delta(-w-3) - \delta(-w+3)] + 9[\delta(w-21\pi) + \delta(w+21\pi)]$

Solution:

9S. Compute Fourier transform for the following signals:

$$x(t) = \sum_{k=0}^{\infty} a^k \delta(t - kT) \quad \text{for } |a| < 1$$

Solution:

$$x(t) = \sum_{k=0}^{\infty} a^k \delta(t - kT) \quad \text{for } |a| < 1$$

$$X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jw t} dt = \int_{-\infty}^{\infty} \sum_{k=0}^{\infty} a^k \delta(t - kT) e^{-jw t} dt$$

Note : $\delta(t - kT) = 1$ for $t = KT$ otherwise $\delta(t - kT) = 0$ therefore :

$$X(jw) = \sum_{k=0}^{\infty} a^k e^{-jwkT} = \sum_{k=0}^{\infty} (a^1 e^{-jwT})^k$$

Apply finite sum $\Rightarrow \sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$ when $|a| < 1$

$$X(jw) = \frac{1}{1 - ae^{-jwT}}$$

9U. Compute Fourier transform for the following signals:

$$x(t) = 5 \sum_{k=0}^{\infty} \left(\frac{a}{2}\right)^k \delta(t - kT) \quad \text{for } |a| < 2$$

Solution: