

Signals & Systems - Chapter 2

1S. Let $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$ and $h[n] = 2\delta[n+1] + 2\delta[n-1]$

Compute and plot each of the following convolutions:

a) $y_1[n] = x[n] * h[n]$

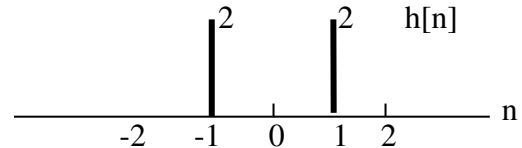
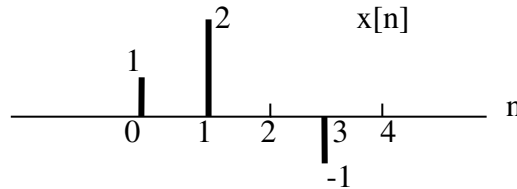
b) $y_2[n] = x[n+2] * h[n]$

c) $y_3[n] = x[n] * h[n+2]$

Solution:

a)

Theory—We have $y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$



$$y_1[n] = h[-1]x[n+1] + h[1]x[n-1] = 2x[n+1] + 2x[n-1]$$

$$y_1[n] = 2\delta[n+1] + 4\delta[n] - 2\delta[n-2] + 2\delta[n-1] + 4\delta[n-2] - 2\delta[n-4]$$

$$y_1[n] = -2\delta[n-4] + 2\delta[n-2] + 2\delta[n-1] + 4\delta[n] + 2\delta[n+1]$$

b) Note $x[n] \rightarrow x[n+2]$ which is a shift of +2

Since we have linear time invariant system

$$y_2[n] = x[n+2] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[(n+2)-k] \rightarrow y_2[n] = y_1[n+2] \text{ Therefore}$$

$$y_2[n] = -2\delta[n-2] + 2\delta[n] + 2\delta[n+1] + 4\delta[n+2] + 2\delta[n+3]$$

c) Note $h[n] \rightarrow h[n+2]$ which is a shift of +2

Since we have linear time invariant system

$$y_3[n] = x[n] * h[n+2] = \sum_{k=-\infty}^{\infty} h[k+2]x[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n+2-k] \rightarrow y_3[n] = y_1[n+2] \text{ Therefore}$$

$$y_3[n] = -2\delta[n-2] + 2\delta[n] + 2\delta[n+1] + 4\delta[n+2] + 2\delta[n+3]$$

1U. Let $x[n] = \delta[-n] + 2\delta[n+1] - \delta[n-4]$ and $h[n] = \delta[n+2] + 3\delta[n+1]$

Compute and plot each of the following convolutions:

a) $y_1[n] = x[n] * h[n]$

b) $y_2[n] = x[n-2] * h[n]$

c) $y_3[n] = x[n] * h[n-2]$

Solution:

2S. Consider an input $x[n]$ and a unit impulse response $h[n]$ given by

$$x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2]$$

$$h[n] = u[n+2]$$

Determine and plot the output $y[n] = x[n] * h[n]$.

Solution:

a)

Theory—We have $y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{k-2} u[k-2]u[n+2-k]$$

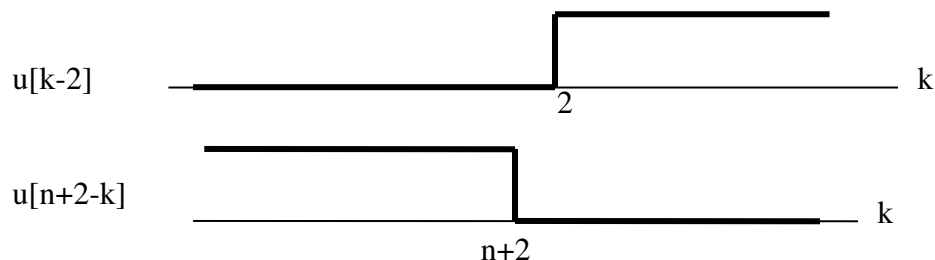
taking a look at plot of the two step function we see that

$u[k-2] = 1$ when $k-2 \geq 0$ or $k \geq 2$

0 otherwise

$u[n+2-k] = 1$ when $n+2-k \geq 0$ or $k \leq n+2$

0 otherwise



There are two conditions:

1) when $n+2 < 2 \rightarrow y[n] = 0$ for $n < 0$

2) when $n+2 \geq 2 \rightarrow y[n] = \sum_{k=2}^{n+2} \left(\frac{1}{2}\right)^{k-2} = \sum_{k=0}^n \left(\frac{1}{2}\right)^k$ for $n \geq 0$

Apply finite sum formula $\rightarrow \sum_{k=0}^n a^k = \frac{1-a^{n+1}}{1-a}$ for $n \geq 0$ & $0 < |a| < 1$

$$y[n] = \frac{1-(1/2)^{n+1}}{1-1/2} u[n] = 2[1-(1/2)^{n+1}]u[n]$$

2U. Consider an input $x[n]$ and a unit impulse response $h[n]$ given by

$$x[n] = \left(\frac{1}{4}\right)^{n-3} u[n+2]$$

$$h[n] = 2u[n-2]$$

Determine and plot the output $y[n] = x[n] * h[n]$.

Solution:

3S. Compute and plot $y[n] = x[n] * h[n]$, where

$$x[n] = 1 \quad \text{for } 3 \leq n \leq 8$$

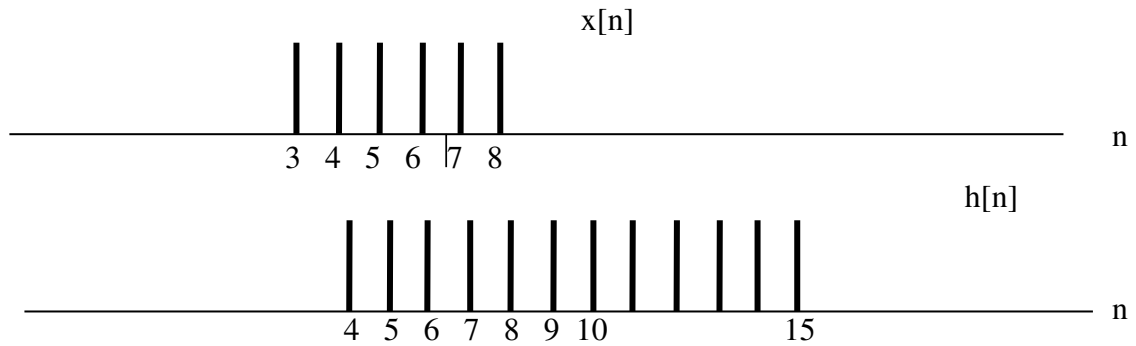
$$0 \quad \text{otherwise}$$

$$h[n] = 1 \quad \text{for } 4 \leq n \leq 15$$

$$0 \quad \text{otherwise}$$

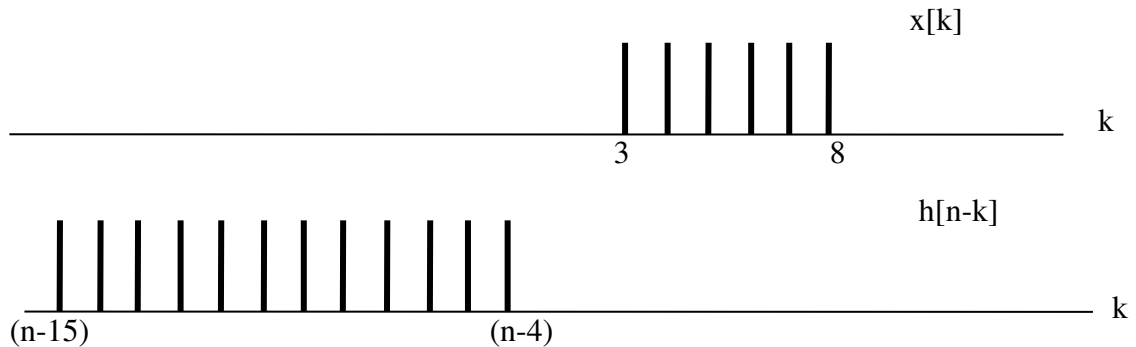
Solution:

a)



$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Let's take the graphic path to find out the intervals of interest --- first need to plot $h[n-k]$



Look for segment where the signal have the same amount of overlap

for $3 \leq n-4 \leq 8$ or $7 \leq n \leq 12$

$$y[n] = n - 6$$

Partial front-end overlap

for $8 \leq n-4$ & $n-15 \leq 3$ or $12 \leq n \leq 18$

$$y[n] = 6$$

Full overlap

for $3 \leq n-15 \leq 8$ or $18 \leq n \leq 23$

$$y[n] = 24 - n$$

Partial back-end overlap

otherwise

$$y[n] = 0$$

or no overlap.

3U. Compute and plot $y[n] = x[n] * h[n]$, where

$$x[n] = 1 \quad \text{for } 2 \leq n \leq 5$$

0 otherwise

$$h[n] = 1 \quad \text{for } -3 \leq n \leq 8$$

0 otherwise

Solution:

4S. Compute and plot the convolution $y[n] = x[n] * h[n]$, where

$$x[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1] \quad \text{and} \quad h[n] = u[n-1]$$

Solution:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{3}\right)^{-k} u[-k-1]u[n-k-1]$$

$$u[-k-1] = 1 \text{ for } -k-1 \geq 0 \rightarrow k \leq -1$$

$$0 \text{ otherwise}$$

Condition 1 – when $k \leq -1$ the summation is not 0 & the equation for $y[n]$ can be written as:

$$y[n] = \sum_{k=-\infty}^{-1} \left(\frac{1}{3}\right)^{-k} u[n-k-1]$$

$$u[n-k-1] = 1 \text{ for } n-k-1 \geq 0 \rightarrow k \leq n-1$$

$$0 \text{ otherwise}$$

Condition 2 – when $k \leq n-1$ the summation is not 0 & the equation for $y[n]$ can be written as:

$$y[n] = \sum_{k=-\infty}^{n-1} \left(\frac{1}{3}\right)^{-k}$$

Considering Condition 1 and 2, we have two intervals for output:

$$\text{For } n \geq 0 \text{ the above equation reduces } \rightarrow y[n] = \sum_{k=-\infty}^{-1} \left(\frac{1}{3}\right)^{-k}$$

$$\text{For } n < 0 \text{ the above equation reduces } \rightarrow y[n] = \sum_{k=-\infty}^{n-1} \left(\frac{1}{3}\right)^{-k}$$

These equations can be further simplified by applying the Infinite Geometric Sum Series.

4U. Compute and plot the convolution $y[n] = x[n] * h[n]$, where

$$x[n] = \left(\frac{1}{8}\right)^{-n} u[n+1] \text{ and } h[n] = u[n-5]$$

Solution:

5S. Compute the convolution $y[n] = x[n] * h[n]$ of the following pairs of signals:

$$x[n] = \alpha^n u[n]$$

a) $h[n] = \beta^n u[n]$

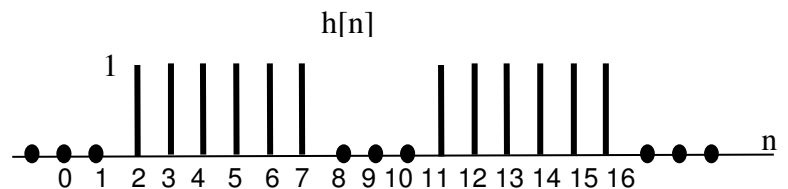
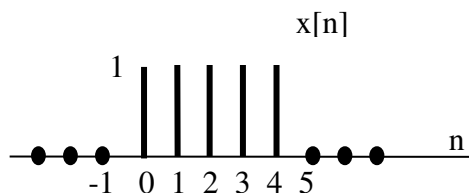
$$\text{when } \alpha \neq \beta$$

b) $x[n] = h[n] = \alpha^n u[n]$

c) $x[n] = \left(-\frac{1}{2}\right)^n u[n-4]$

$$h[n] = 4^n u[2-n]$$

d) $x[n]$ and $h[n]$ are as in following figure



Hint: first draw $x(t)$ and $h(t)$. reflect $h(t)$ about $x=0$ and then walk the signal to find the limits.

Solution:

a)

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[n] = \sum_{k=0}^n a^k \beta^{n-k} \quad \text{for } n \geq 0$$

$$y[n] = \beta^n \sum_{k=0}^n (a/\beta)^k \quad \text{for } n \geq 0$$

or apply inifinit series $\sum_{k=0}^n a^k = \frac{1-a^{n+1}}{1-a}$

$$y[n] = \left[\frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} \right] u[n] \quad \text{for } \alpha \neq \beta$$

b)

Note, this part is the same as "part a" except for the fact that $a=\beta$

$$y[n] = \alpha^n \left[\sum_{k=0}^n (1)^k \right] u[n] = (n+1)a^n u[n]$$

c)

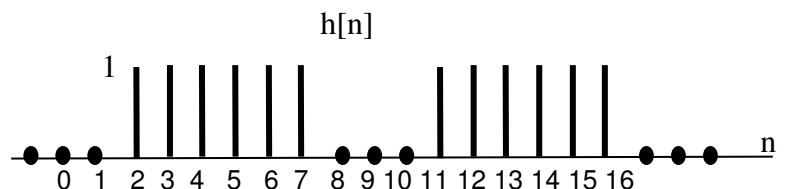
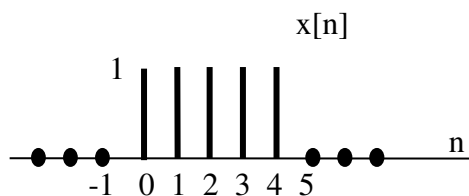
$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[n] = \sum_{k=4}^{\infty} (-1/2)^k (4)^{n-k} = (4)^n \sum_{k=4}^{\infty} (-1/8)^k = (4)^n \left[\sum_{k=0}^{\infty} (-1/8)^k - \sum_{k=0}^3 (-1/8)^k \right] \quad \text{for } n \leq 6$$

$$y[n] = \sum_{k=n}^{\infty} (-1/2)^k (4)^{n-k} = (4)^n \sum_{k=n}^{\infty} (-1/8)^k = (4)^n \left[\sum_{k=0}^{\infty} (-1/8)^k - \sum_{k=0}^{n-1} (-1/8)^k \right] \quad \text{for } n > 6$$

To Further Simplify apply inifinit series $\sum_{k=0}^n a^k = \frac{1-a^{n+1}}{1-a}$ since $a = -1/8 \Rightarrow \sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$

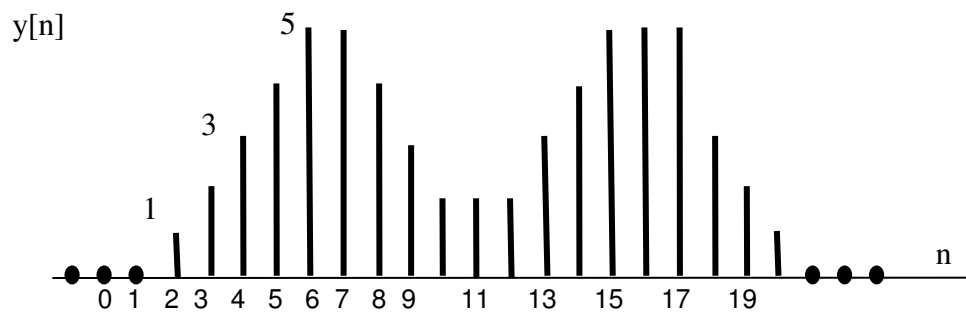
d)



$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[n] = x[0]h[n-0] + x[1]h[n-1] + x[2]h[n-2] + x[3]h[n-3] + x[4]h[n-4]$$

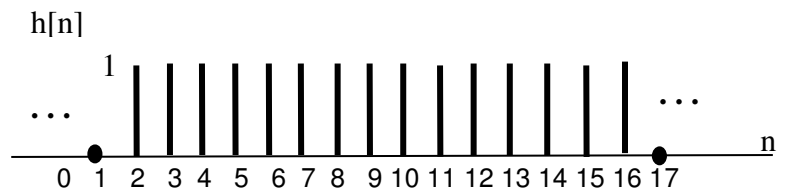
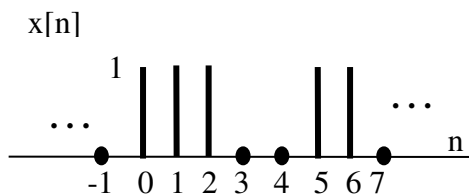
$$y[n] = h[n] + h[n-1] + h[n-2] + h[n-3] + h[n-4]$$



5U. Compute the convolution $y[n] = x[n] * h[n]$ of the following pairs of signals:

a) $x[n] = \left(\frac{1}{3}\right)^n u[n+2]$
 $h[n] = 8^n u[3+n]$

b)

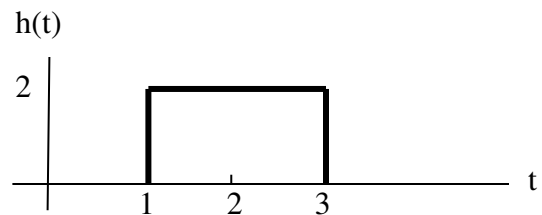
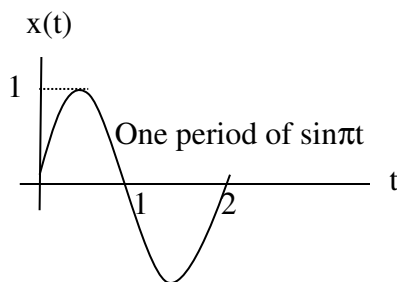


6S. For each of the following pairs of waveforms, use the convolution integral to find response $y(t)$ of the LTI system with impulse response $h(t)$ and $x(t)$. Sketch your results.

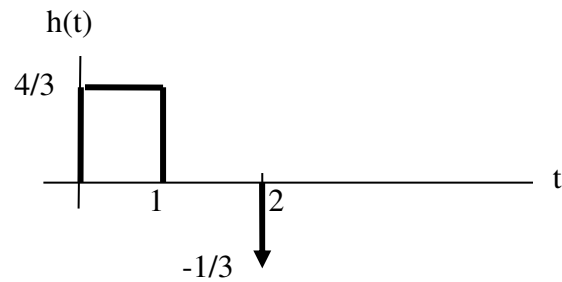
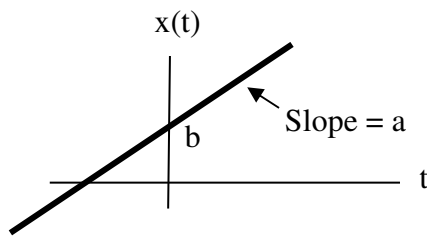
a) $x(t) = e^{-\alpha t} u(t)$
 $h(t) = e^{-\beta t} u(t)$ (Do this both when $\alpha \neq \beta$ and $\alpha = \beta$.)

b) $x(t) = u(t) - 2u(t-2) + u(t-5)$ and $h(t) = e^{2t}u(1-t)$

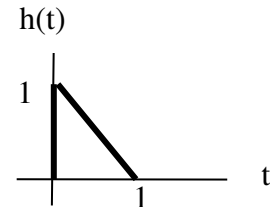
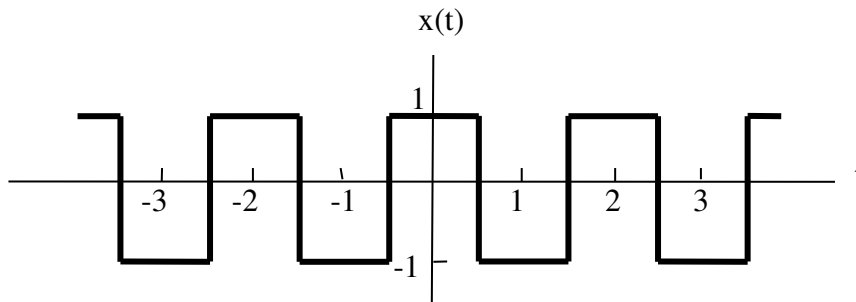
c) $x(t)$ and $h(t)$ shown below:



d) $x(t)$ and $h(t)$ shown below:



e) $x(t)$ and $h(t)$ shown below:



Hint: first draw $x(t)$ and $h(t)$. reflect $h(t)$ about $x=0$ and then walk the signal to find the limits.

Solutions:

a) The desired Convolution is:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$y(t) = 0 \text{ (no overlap) for } t \geq 0$$

$$y(t) = \int_0^t e^{-\alpha\tau} e^{-\beta(t-\tau)} d\tau \quad \text{for } t \geq 0$$

Therefore

$$y(t) = e^{-\beta t} \left[\int_0^t e^{(\beta-\alpha)\tau} d\tau \right] u(t)$$

$$\text{for } \alpha = \beta \Rightarrow y(t) = e^{-\beta t} \left[\int_0^t d\tau \right] u(t) = te^{-\beta t} u(t)$$

$$\text{for } \alpha \neq \beta \Rightarrow y(t) = \frac{e^{-\beta t} [e^{(\beta-\alpha)t} - 1]}{\beta - \alpha}$$

b) The desired Convolution is:

First draw $x(t)$ and $h(-t)$ to find the number of unique overlap sections.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Considering the intervals :

$$t-1 < 0 \rightarrow t < 1 \rightarrow y(t) = 0$$

$$1 \leq t < 3 \rightarrow y(t) = \int_0^{t-1} e^{2(t-\tau)} d\tau$$

$$3 \leq t \leq 6 \rightarrow y(t) = \int_0^2 e^{2(t-\tau)} d\tau - \int_2^{t-1} e^{2(t-\tau)} d\tau$$

$$t > 6 \rightarrow y(t) = 0 \quad \text{for } 6 < t$$

Therefore

$$t < 1 \rightarrow y(t) = 0$$

$$1 \leq t < 3 \rightarrow y(t) = e^{2t} \left[-\frac{1}{2} e^{-2\tau} \right]_0^{t-1} = -\frac{1}{2} e^{2t} [e^{-2(t-1)} - 1]$$

$$\text{for } 3 \leq t \leq 6 \quad y(t) = e^{2t} \left[-\frac{1}{2} e^{-2\tau} \right]_0^2 + \frac{1}{2} e^{-2\tau} \Big|_2^{t-1} = \frac{1}{2} e^{2t} [-(e^{-2} - 1) + (e^{-2(t-1)} - e^{-2})]$$

$$t > 6 \rightarrow y(t) = 0$$

c) The desired Convolution is:

First draw $x(t)$ and $h(-t)$ to find the number of unique overlap sections.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_0^2 \sin(\pi\tau)h(t-\tau)d\tau$$

Considering the overlap types :

$$y(t) = 0 \quad \text{for } t \leq 1$$

$$y(t) = (2/\pi)[1 - \cos\{\pi(t-1)\}] \quad \text{for } 1 < t < 3$$

$$y(t) = (2/\pi)[1 - \cos\{\pi(t-3)\} - 1] \quad \text{for } 3 < t < 5$$

$$y(t) = 0 \quad \text{for } 5 < t$$

d) The desired Convolution is:

$$\text{let } h(t) = h_1(t) - (1/3)\delta(t-2) \quad \text{where } h_1(t) = \begin{cases} 4/3 & \text{for } 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

so we can use the system linearity to write:

$$y(t) = h(t)*x(t) = h_1(t)*x(t) - (1/3)x(t-2)$$

we have

$$h_1(t) * x(t) = \int_{t-1}^t \frac{4}{3}(a\tau + b)d\tau = \frac{4}{3} \left[\left(\frac{1}{2} a t^2 - \frac{1}{2} a(t-1)^2 + b t - b(t-1) \right) \right]$$

Therefore

$$y(t) = \frac{4}{3} \left[\left(\frac{1}{2} a t^2 - \frac{1}{2} a(t-1)^2 + b t - b(t-1) \right) \right] - \frac{1}{3} \left[\left(\frac{1}{3} a(t-2) + b \right) \right]$$

e) The desired Convolution is:

for LTI system when the input is periodic, it implies that output $y(t)$ is also periodic so determine one period. But remember to include effect of other periods on calculation of single period output.

So we will take the period $-1/2 < t < 3/2$

$x(t)=1$ for $-1/2 < t < 1/2$

0 for $1/2 < t < 3/2$

$h(t) = t+1$ for $0 < t < 1$

= 0 otherwise

First draw $x(t)$ and $h(-t)$ to find the number of unique overlap sections.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$y(t) = \int_{t-1}^{-1/2} (t-\tau-1)d\tau - \int_{-1/2}^t (t-\tau-1)d\tau = -t^2 + t + 1/4 \quad \text{for } -1/2 < t < 1/2$$

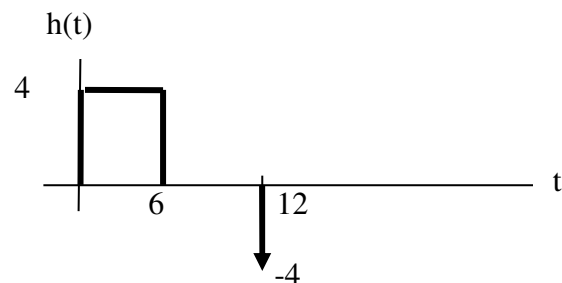
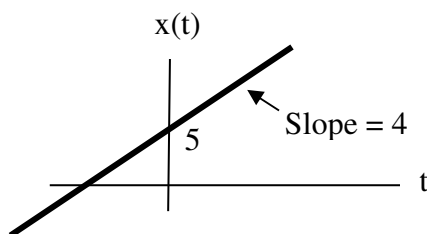
$$y(t) = - \int_{t-1}^{1/2} (t-\tau-1)d\tau + \int_{1/2}^t (t-\tau-1)d\tau = t^2 - 3t + 7/4 \quad \text{for } 1/2 < t < 3/2$$

Note: Output period is also 2.

6U. For each of the following pairs of waveforms, use the convolution integral to find response $y(t)$ of the LTI system with impulse response $h(t)$ and $x(t)$. Sketch your results.

a) $x(t) = 3u(t) - 5u(t+2) + 3u(t+3)$ and $h(t) = e^{-2t}u(-2+t)$

b) $x(t)$ and $h(t)$ shown below:



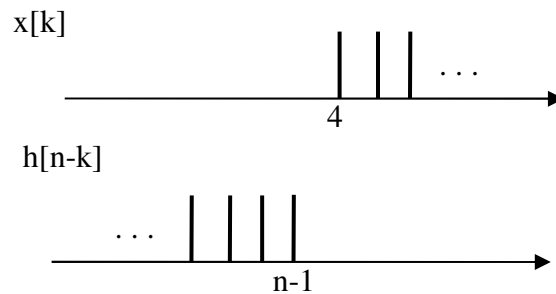
Solutions:

7S. Compute the convolution $y[n]=x[n]*h[n]$ of the following pair of signals:

$$x[n] = a^n u[n-4]$$

$$h[n] = b^n u[n-1] \quad \text{where } 0 < a < b$$

Solution:



$$x[n] = a^n u[n-4]$$

$$h[n] = b^n u[n-1]$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$$y[n] = 0 \quad \text{for } n-1 < 4 \Rightarrow n < 5$$

$$y[n] = \sum_{k=4}^{n-1} a^k \beta^{n-k} \quad \text{for } n \geq 5$$

$$y[n] = \beta^n \sum_{k=0}^{n-5} (a/\beta)^k \quad \text{for } n \geq 0$$

7U. Compute the convolution $y[n]=x[n]*h[n]$ of the following pair of signals:

$$x[n] = \left(\frac{1}{3}\right)^n u[n-2]$$

$$h[n] = \left(\frac{1}{2}\right)^n u[n+3]$$

Solution:

8S. The following are the impulse response of discrete-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.

a) $h[n] = (1/5)^n u[n]$

b) $h[n] = (0.8)^n u[n+2]$

c) $h[n] = (1/2)^n u[-n]$

d) $h[n] = (5)^n u[3-n]$

e) $h[n] = (-1/2)^n u[n] + (1.01)^n u[n-1]$

f) $h[n] = (-1/2)^n u[n] + (1.01)^n u[1-n]$

g) $h[n] = n (1/3)^n u[n-1]$

Solution:

Notes \rightarrow "System is causal if $h[n]=0$ for $n<0$ --- System is stable if $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$ "

a) $h[n] = (1/5)^n u[n]$

$h[n]=0$ for $n<0$ therefore system is causal

$$\sum_{n=0}^{\infty} (1/5)^n = \frac{1}{1 - (1/5)} = 5/4 < \infty \text{ therefore the System is stable}$$

b) $h[n] = (0.8)^n u[n + 2]$

$h[n] \neq 0$ for $n < 0$ therefore system is non-causal

$$\sum_{n=-2}^{\infty} (0.8)^n = \sum_{n=0}^{\infty} (0.8)^{n-2} = \frac{(0.8)^{-2}}{1 - (0.8)} < \infty \text{ therefore the System is stable}$$

c) $h[n] = (1/2)^n u[-n]$

$h[n] \neq 0$ for $n < 0$ therefore system is non-causal

$$\sum_{n=-\infty}^0 (1/2)^n = \infty \text{ therefore the System is not stable}$$

d) $h[n] = (5)^n u[3 - n]$

$h[n] \neq 0$ for $n < 0$ therefore system is non-causal

$$\sum_{n=-\infty}^3 (5)^n = \sum_{n=0}^{\infty} (1/5)^n - \sum_{n=-3}^0 (1/5)^n = \frac{1}{1 - (1/5)} - (1/5)^{-3} - (1/5)^{-2} - (1/5)^{-1} - (1/5)^0 = 625/4 < \infty$$

therefore the System is stable

e) $h[n] = (-1/2)^n u[n] + (1.01)^n u[n - 1]$

$h[n] = 0$ for $n < 0$ therefore system is causal

$$\sum_{n=0}^{\infty} |(-1/2)^n| + \sum_{n=1}^{\infty} (1.01)^n = \frac{1}{1 - (1/2)} + \infty = \infty \text{ therefore the System is not stable}$$

f) $h[n] = (-1/2)^n u[n] + (1.01)^n u[1 - n]$

$h[n] \neq 0$ for $n < 0$ therefore system is non-causal

$$\sum_{n=0}^{\infty} |(-1/2)^n| + \sum_{n=-\infty}^1 (1.01)^n = \frac{1}{1 - (1/2)} + \sum_{n=0}^{\infty} (1.01)^{-n} - 1.01 = 2/3 + \frac{1}{1 - (1/1.01)} - 1.01 < \infty$$

therefore the System is stable

g) $h[n] = (1/3)^n u[n - 1]$

$h[n] = 0$ for $n < 0$ therefore system is causal

$$\sum_{n=1}^{\infty} n(1/3)^n = \frac{1}{1 - (1/3)} = 3/4 = 0.75 < \infty \text{ therefore the System is stable}$$

8U. The following are the impulse response of discrete-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.

a) $h[n] = (1/5)^n u[-n]$

b) $h[n] = (0.8)^n u[n - 2]$

c) $h[n] = (1/2)^n u[n]$

d) $h[n] = (5)^n u[3 + n]$

e) $h[n] = n(1/3)^n u[n + 1]$

Solution:

9S. The following are the impulse responses of continuous-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.

a) $h(t) = e^{-4t} u(t - 2)$

b) $h(t) = e^{-6t} u(3 - t)$

c) $h(t) = e^{-2t} u(t + 50)$

d) $h(t) = e^{2t} u(-1 - t)$

e) $h(t) = e^{-6|t|}$

f) $h(t) = t e^{-t} u(t)$

g) $h(t) = (2 e^{-t} - e^{(t-1000)/100}) u(t)$

Notes \rightarrow "System is causal if $h(t)=0$ for $t<0$ --- System is stable if $\int_{-\infty}^{\infty} |h(t)| dt$ "

Solution:

a) $h(t) = e^{-4t} u(t-2)$

$h(t)=0$ for $t<0$ therefore System is causal

$$\int_2^{\infty} |e^{-4t}| dt = -1/4(e^{-\infty} - e^{-8}) = 1/4(e^{-8}) < \infty \text{ therefore System is stable}$$

b) $h(t) = e^{-6t} u(3-t)$

$h(t) \neq 0$ for $t<0$ therefore System is not causal

$$\int_{-\infty}^3 |e^{-6t}| dt = 1/6(e^{-18} - e^{-\infty}) = \infty \text{ therefore System is unstable}$$

c) $h(t) = e^{-2t} u(t+50)$

$h(t) \neq 0$ for $t<0$ therefore System is not causal

$$\int_{-50}^{\infty} |e^{-2t}| dt = -1/2(e^{-\infty} - e^{100}) = 1/2e^{100} < \infty \text{ therefore System is stable}$$

d) $h(t) = e^{2t} u(-1-t)$

$h(t) \neq 0$ for $t<0$ therefore System is not causal

$$\int_{-\infty}^{-1} |e^{2t}| dt = 1/2(e^{-2} - e^{-\infty}) = e^{-2}/2 < \infty \text{ therefore System is stable}$$

e) $h(t) = e^{-6|t|}$

$h(t) \neq 0$ for $t<0$ therefore System is not causal

$$2 \int_0^{\infty} |e^{-6|t|}| dt = -1/3(e^{-\infty} - e^0) = 1/3 < \infty \text{ therefore System is stable}$$

f) $h(t) = t e^{-t} u(t)$

$h(t) = 0$ for $t<0$ therefore System is causal

$$\int_0^{\infty} |t e^{-t}| dt = 1 < \infty \text{ therefore System is stable (Use integral by part to solve)}$$

g) $h(t) = (2 e^{-t} - e^{(t-1000)/100}) u(t)$

$h(t) = 0$ for $t<0$ therefore System is causal

$$\int_0^{\infty} |2 e^{-t} - e^{(t-1000)/100}| dt = \infty \text{ therefore System is unstable}$$

9U. The following are the impulse responses of continuous-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.

a) $h(t) = e^{-3t} u(t+4)$

b) $h(t) = e^{-5t} u(13-t)$

c) $h(t) = e^{-2t} u(-t+10)$

d) $h(t) = e^{2t} u(-3-t)$

e) $h(t) = e^{-10|t|}$

Solution:

10S. $h[n] = (-1/2)^n u[n] + (1.01)^n u[n - 1]$ is the impulse of response of a system. Determine if the system is Causal and/or Stable

Solution:

$$h[n] = (-1/2)^n u[n] + (1.01)^n u[n - 1]$$

Notes \rightarrow "System is causal if $h[n]=0$ for $n<0$ --- System is stable if $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$ "

$h[n]=0$ for $n<0$ therefore system is causal

$$\sum_{n=0}^{\infty} |(-1/2)^n| + \sum_{n=1}^{\infty} (1.01)^n = \frac{1}{1 - (1/2)} + \infty = \infty \text{ therefore the System is not stable}$$

10U. $h[n] = (-1/8)^n u[n + 1] + (1.21)^n u[n]$ is the impulse of response of a system. Determine if the system is Causal and/or Stable

Solution: