ENGR 253 LAB #6 - Discrete Fourier Transform (DFT) and Inverse DFT

Objective
Utilizing Fourier transform methods to analyze the original, the transformed and the approximated signals.

Resources
- Signals & Systems textbook by Oppenheim and Willsky
- Windows running MATLAB release 14 or later
- USB hard disk or other removable drives
  {note: Lab computer data is lost after reboot}
- Course Lecture Material

Background
1) Fourier Transform for aperiodic and periodic signals

   - Continuous-time

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}\,dt \xrightarrow{Approximateion\ \tau = dt \to 0} \lim_{\tau \to 0} \sum_{n=-\infty}^{\infty} x(n\tau)e^{-jw_0 n\tau}\tau$$

For a signal that is non-zero only in the range 0 < t ≤ T and T=Nτ where N is an integer, X=τ*fft(x) may be used to approximate X(jw). Each element will be calculated using the following:

$$X(k+1) = \tau \sum_{n=-\infty}^{N-1} x(n\tau)e^{-jw_k n\tau} \quad for \quad 0 \le k \le N$$

$$X(k+1) \approx X(jw_k) \quad for \quad w_k = \frac{2\pi k}{N\tau} \quad for \quad 0 \le k \le \frac{N}{2}$$

$$= \frac{2\pi k}{N\tau} - \frac{2\pi}{\tau} \quad for \quad \frac{N}{2} + 1 \le k \le N$$

N is assumed to be even and fft returns positive frequency sample before the negative frequency samples. Function fftshift() rearrange the samples such that they are placed in the vector from the most negative to most positive frequency.

iff() function performs the inverse fourier transform function.

   - Discrete-time

$$X(e^{jw}) = \sum_{n=<N>} x[n]e^{-jwn} \quad Fourier\ Transform$$

$$x[n] = \frac{1}{2\pi} \cdot \int_{2\pi} X(e^{jw})e^{jwn}\,dw \quad Fourier\ Inverse\ Transform$$

MATLAB functions, fft() & ifft() are designed to implement the above transformation equations. For a value of N that is less than number of samples in x or X, the functions may be used as shown below:

```
X = fft(x, N)       % N-point Fourier Transform of x, padded with zeros if X has less
                    %   than N points and truncated if it has more
x =  ifft(X,N)      % N-point inverse Fourier Transform of X,  returns the n-point
                    %   inverse DFT of vector X.
```
Note: For any X, ifft(fft(X)) equals X to within roundoff error

2) Determination of approximation error
   Typically approximation error, goodness of an approximation, is determined by the size of energy difference between the approximated signal and the original signal. For this experiment use the difference in total energy over the nonzero period as measure of approximation error:

$$E = \sum_{k=<N>} x[n]^2 - \sum_{k=<N_1>} \overline{x[n]}^2 \quad where$$
$$x[n] = orignial\ signal\ (N\ terms)$$
$$\overline{x[n]} = approximated\ signal(N_1\ terms)$$

3) Sample Data Files
Standard MATLAB installation contains data files that may be used as the input for experiments. These files are typically saved in the following directory:

C:\Program Files\MATLAB704\toolbox\matlab\audiovideo

The files are also available on the course website. The following sequence of MATLAB commands load data file containing music, play the file content as sound and plot the data:

```
load handel.mat              % load the handel.mat data file into matrix y
sound(y(100:18000),8192)     % output the value of y(100:18000) as an sound value
                             % with sample rate of 8192 (default)
                             % Sound function only accepts real values so user magnitude of signal.
stem(y)                      % plot value stored in y
```

Experiment #1

Load file handel.mat and plot its data for the range $100 \le n < 18{,}000$. Also play the values stored in the file handel.mat as sound. Explain the content that you hear.

Experiment #2

Using MATLABS fft() function, determine and plot the Fourier Transform of the function that has its sample values stored in the handel.mat for the range $100 \le n < 18{,}000$.
Hint: In this case N = 17,901.

Experiment #3

Using fft() and ifft() find the Fourier Transform and then reconstruct the signal for each of the following cases:

a) 10,000-term approximation
b) 16,000-term approximation
c) 17,901-term approximation

For each case, plot the Fourier Transform, the corresponding approximated signal and the original signal. Explain your observations for each case.

Hint: A 1 by 3 subplot is helpful in comparing the original and approximated signals for each case.

Experiment #4

For each approximation level in Experiment #3, determine the percent of error (E) and the effect of number of terms on error size and reproduction of the music (your perceptions).

Experiment #5

a) What number of terms from handel.mat is required to reproduce the signal to perfection based on your own hearing?

b) Using MATLAB ability to produce sound, determine your hearing range. How does your hearing range impacts your answers to part a and b. Explain your process and your conclusion.
Hint: A healthy young person who has not been exposed to excessively loud sounds will typically be able to hear sounds from 20 to 20 kHz.

Report Requirements

Reports must be prepared individually even if the experiments are performed as a team. All reports must be computer printed (Formulas and Diagrams may be hand drawn) and at minimum include:

For each Experiment

a) A clear problem statement; specifying items given and to be found.
b) Theory or process used.
c) Resulting circuits, calculation, tables, timing diagram, schematic and other relevant results.

For the report as a whole

a) Cover sheet with your name, class, lab, completion date and team members' names.
b) Lessons Learned from the experiments.
c) A new experiment and expected results which provide additional opportunity to practice the concepts in this lab.