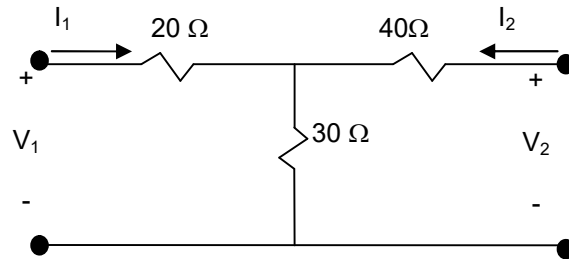


Fundamentals of Electrical Circuits - Chapter 14

1S. Find the z parameter for the following circuit.



Solution:

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

where :

$$z_{11} = \frac{V_1}{I_1} \Omega \quad \text{and} \quad z_{21} = \frac{V_2}{I_1} \Omega \quad \text{when } I_2 = 0$$

$$z_{22} = \frac{V_2}{I_2} \Omega \quad \text{and} \quad z_{12} = \frac{V_1}{I_2} \Omega \quad \text{when } I_1 = 0$$

Step 1) Let $I_2 = 0$ find $Z_{11} = V_1/I_1$, $Z_{21} = V_2/I_1$

$$Z_{11} = V_1/I_1 = V_1/(V_1 / (20 + 30)) = 50 \Omega$$

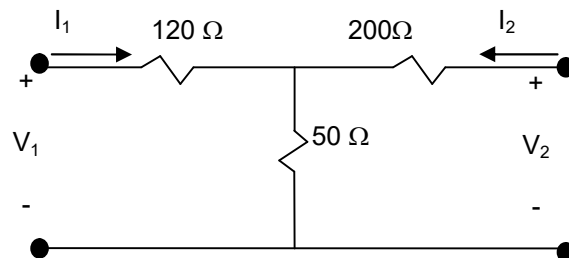
$$Z_{21} = V_2/I_1 = 30 \Omega$$

Step 2) Let $I_1 = 0$ find $Z_{22} = V_2/I_2$, $Z_{12} = V_1/I_2$

$$Z_{22} = V_2/I_2 = V_2/(V_2 / (40 + 30)) = 70 \Omega$$

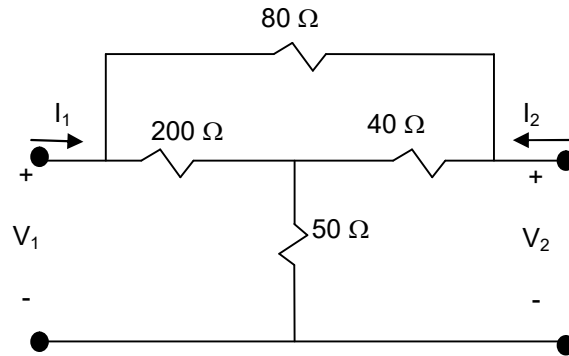
$$Z_{12} = V_1/I_2 = 30 \Omega$$

1U. Find the z parameter for the following circuit.



Solution:

2S. Find the g parameters for the circuit shown below:



Solution:

We have →

$$I_1 = g_{11}V_1 + g_{12}I_2$$

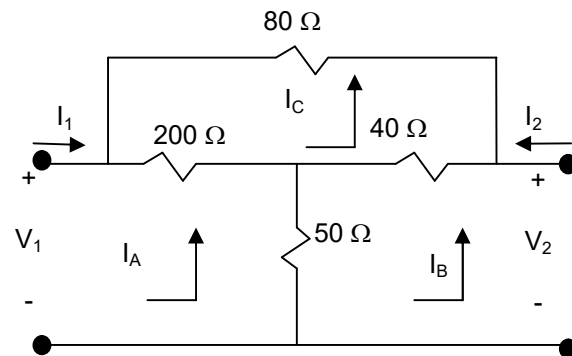
$$V_2 = g_{21}V_1 + g_{22}I_2$$

where :

$$g_{11} = \frac{I_1}{V_1} \text{ S and } g_{21} = \frac{V_2}{V_1} \text{ when } I_2 = 0$$

$$g_{22} = \frac{V_2}{I_2} \Omega \text{ and } g_{12} = \frac{I_1}{I_2} \text{ when } V_1 = 0$$

Step1) When $I_2 = 0$



Write the KVL for each of the three meshes

$$\text{Messh\#1} \Rightarrow +V_1 + 50(I_A - I_B) + 200(I_A - I_C) = 0$$

$$\text{Messh\#2} \Rightarrow 40(I_B - I_C) + 50(I_B - I_A) - V_2 = 0$$

$$\text{Messh\#3} \Rightarrow 200(I_C - I_A) + 40(I_C - I_B) + 80I_C = 0$$

Simplify above three equation and apply $I_2 = 0$

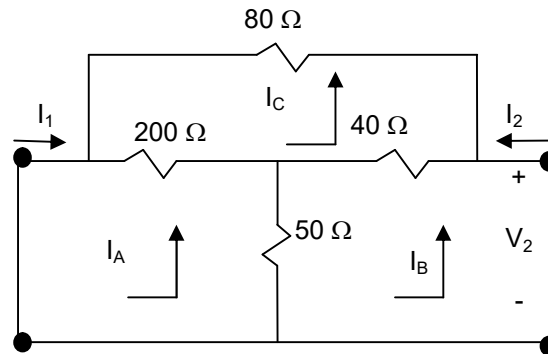
$$\text{Messh\#1} \Rightarrow +V_1 + 250I_A - 200I_C = 0 \Rightarrow V_1 = -125I_A$$

$$\text{Messh\#2} \Rightarrow -V_2 - 40I_C - 50I_A = 0 \Rightarrow V_2 = -75I_A$$

$$\text{Messh\#3} \Rightarrow 320I_C - 200I_A = 0 \Rightarrow I_C = (20/32)I_A$$

Therefore $g_{11} = I_1/V_1 = 1/125 \text{ S}$ and $g_{21} = V_2/V_1 = 75/125 \text{ S}$

Step2) When $V_1 = 0 \rightarrow g_{22} = V_2/I_2$ and $g_{12} = I_1/I_2$



Write the KVL for each of the three meshes

$$\text{Mesh\# A} \rightarrow 50(I_A - I_B) + 200(I_A - I_C) = 0$$

$$\text{Mesh\# B} \rightarrow 40(I_B - I_C) + 50(I_B - I_A) - V_2 = 0$$

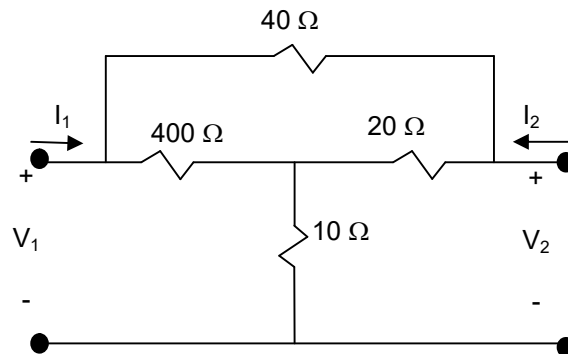
$$\text{Mesh\# C} \rightarrow 200(I_C - I_A) + 40(I_C - I_B) + 80I_C = 0$$

$$\text{Note: } I_A = -I_1, \quad I_B = I_2$$

Solve to find :

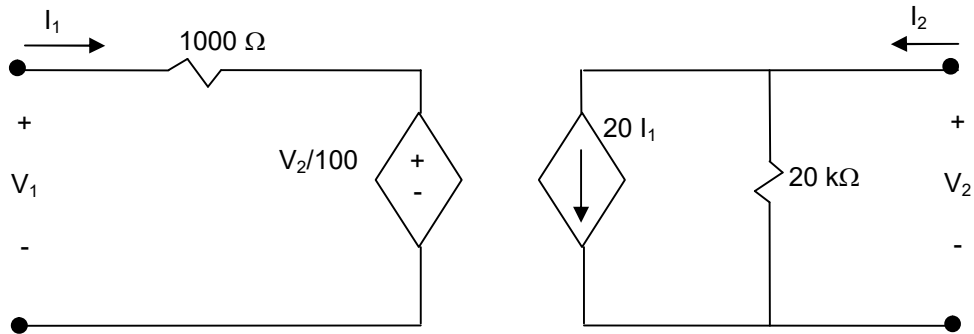
$$g_{22} = \frac{V_2}{I_2} = 40\Omega \quad g_{12} = \frac{I_1}{I_2} = -0.6$$

2U. Find the g parameters for the circuit shown below:



Solution:

3S. Find the h parameter for the following circuit



Solution:

$$V_1 = h_{11}I_1 + h_{12}V_2$$

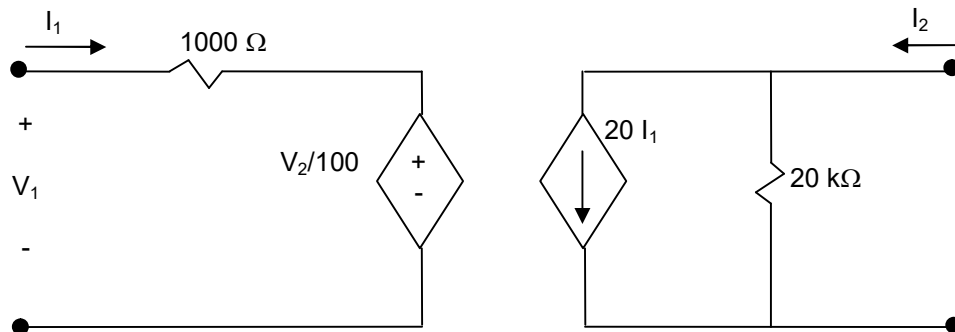
$$I_2 = h_{21}I_1 + h_{22}V_2$$

where :

$$h_{11} = \frac{V_1}{I_1} \Omega \quad \text{and} \quad h_{21} = \frac{I_2}{I_1} \quad \text{when } V_2 = 0$$

$$h_{12} = \frac{V_1}{V_2} \quad \text{and} \quad h_{22} = \frac{I_2}{V_2} S \quad \text{when } I_1 = 0$$

Step 1) Let $V_2 = 0$

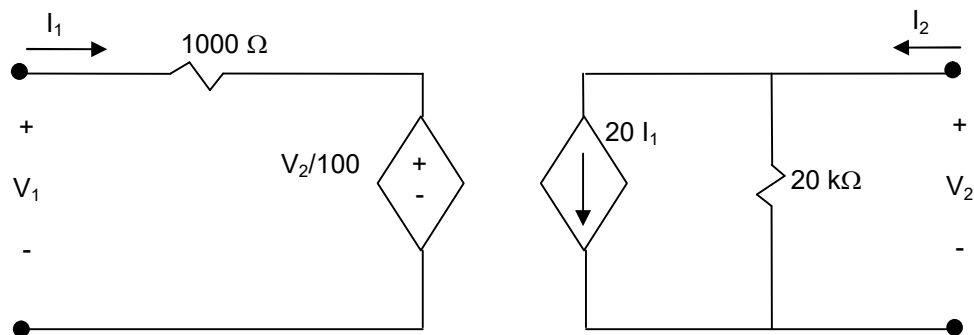


KVL on the right side $\rightarrow I_2 = 20 I_1 \rightarrow h_{21} = I_2 / I_1 = 20$

KVL on the left hand side $\rightarrow -V_1 + 1000I_1 + V_2/100 = 0$ (note $V_2=0$) $\rightarrow -V_1 + 1000I_1 = 0$

Replace I_2 with I_1 from first equation $\rightarrow -V_1 + 1000I_1 = 0 \rightarrow h_{11} = V_1 / I_1 = 1000 \Omega$

Step 2) Let $I_1 = 0$

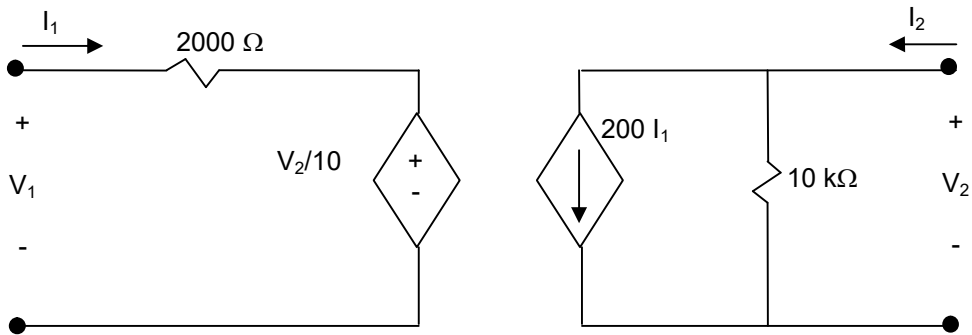


KCL on the right side $\rightarrow 20 I_1 + V_2/20,000 - I_2 = 0$ (note $I_1 = 0$) $\rightarrow$

$$V_2/20,000 = I_2 \rightarrow h_{22} = I_2 / V_2 = 1/20,000$$

KVL on the left hand side ($I_1 = 0$) $\rightarrow V_1 = +10^{-2}V_2 \rightarrow h_{12} = V_1 / V_2 = 10^{-2}$

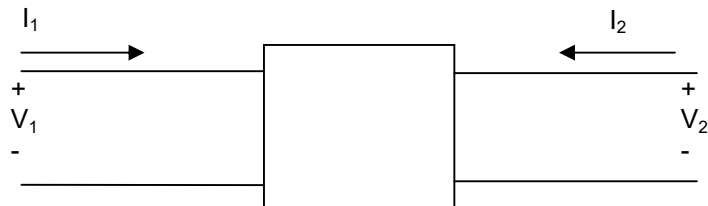
3U. Find the h parameter for the following circuit



Solution:

4S. The following direct-current measurements were made on the two-port network shown below. Calculate the g parameters for the shown network.

Port 2 Open	Port 2 Short-Circuit
$V_1 = 20 \text{ mV}$	$I_1 = 200 \text{ uA}$
$V_2 = -5 \text{ V}$	$I_2 = 50 \text{ uA}$
$I_1 = 0.25 \text{ uA}$	$V_1 = 10 \text{ V}$



Solution:

we have

$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

: where :

$$g_{11} = \frac{I_1}{V_1} \text{ S} \quad \text{and} \quad g_{21} = \frac{V_2}{V_1} \quad \text{when } I_2 = 0$$

$$g_{22} = \frac{V_2}{I_2} \Omega \quad \text{and} \quad g_{12} = \frac{I_1}{I_2} \quad \text{when } V_1 = 0$$

$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

where $I_2 = 0$ (Port2 Open):

$$g_{11} = \frac{I_1}{V_1} = \frac{0.25\mu A}{20mV} = 12.5\mu S$$

$$g_{21} = \frac{V_2}{V_1} = \frac{-5}{20mV} = -250$$

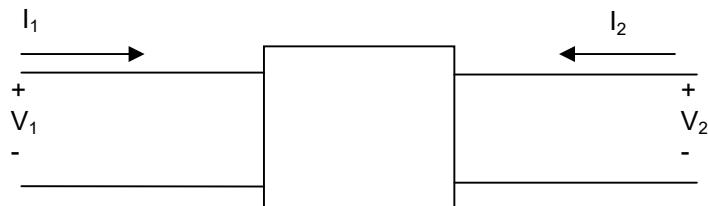
where $V_2 = 0$ (Port2 Short):

$$V_2 = g_{21}V_1 + g_{22}I_2 \Rightarrow 0 = (-250)10 + g_{22}50 \times 10^{-6} \Rightarrow g_{22} = 50M\Omega$$

$$I_1 = g_{11}V_1 + g_{12}I_2 \Rightarrow 200 \times 10^{-6} = (125 \times 10^{-6})10 + g_{12}(50 \times 10^{-6}) \Rightarrow g_{12} = 1.5$$

4U. The following direct-current measurements were made on the two-port network shown below. Calculate the g parameters for the shown network.

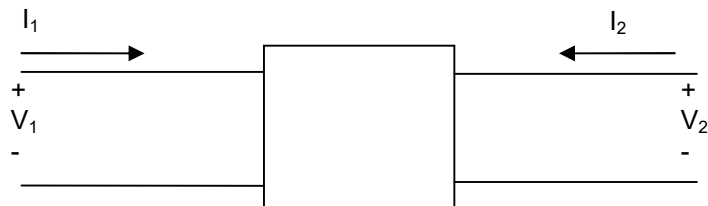
Port 1 Open	Port 1 Short-Circuit
$V_1 = 10\text{ mV}$	$I_1 = 400\text{ }\mu\text{A}$
$V_2 = -10\text{ V}$	$I_2 = 25\text{ }\mu\text{A}$
$I_2 = 0.5\text{ }\mu\text{A}$	$V_2 = 5\text{ V}$



Solution:

5S. Using the following measurements and network:

Port 2 Open	Port 2 Short-Circuit
$V_1 = 20\text{ mV}$	$I_1 = 200\text{ }\mu\text{A}$
$V_2 = -5\text{ V}$	$I_2 = 50\text{ }\mu\text{A}$
$I_1 = 0.25\text{ }\mu\text{A}$	$V_1 = 10\text{ V}$



Find the y parameter for the network

Solution:

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

where :

$$y_{11} = \frac{I_1}{V_1} S \quad \text{and} \quad y_{21} = \frac{I_2}{V_1} S \quad \text{when } V_2 = 0$$

$$y_{22} = \frac{I_2}{V_2} S \quad \text{and} \quad y_{12} = \frac{I_1}{V_2} S \quad \text{when } V_1 = 0$$

when Port 2 Short ($V_2 = 0$):

$$y_{11} = \frac{I_1}{V_1} = \frac{200 \times 10^{-6}}{10} = 20 \mu S$$

$$y_{21} = \frac{I_2}{V_1} = \frac{50 \times 10^{-6}}{10} = 5 \mu S$$

when Port 2 Open ($I_2 = 0$)

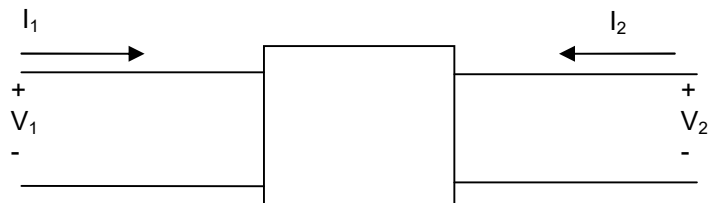
$$I_2 = y_{21}V_1 + y_{22}V_2 \Rightarrow 0 = (5 \times 10^{-6})(20 \times 10^{-3}) + y_{22}(-5) \Rightarrow y_{22} = 20 nS$$

$$I_1 = y_{11}V_1 + y_{12}V_2 \Rightarrow 0.25 \times 10^{-6} = (20 \times 10^{-6})(20 \times 10^{-3}) + y_{12}(-5) \Rightarrow y_{12} = 30 nS$$

$$y = \begin{bmatrix} 20 \mu S & 30 nS \\ 5 \mu S & 20 nS \end{bmatrix}$$

5U. Using the following measurements and network:

Port 1 Open	Port 1 Short-Circuit
$V_1 = 100 \text{ mV}$	$I_1 = 20 \text{ uA}$
$V_2 = -10 \text{ V}$	$I_2 = 40 \text{ uA}$
$I_2 = 0.125 \text{ uA}$	$V_2 = 5 \text{ V}$

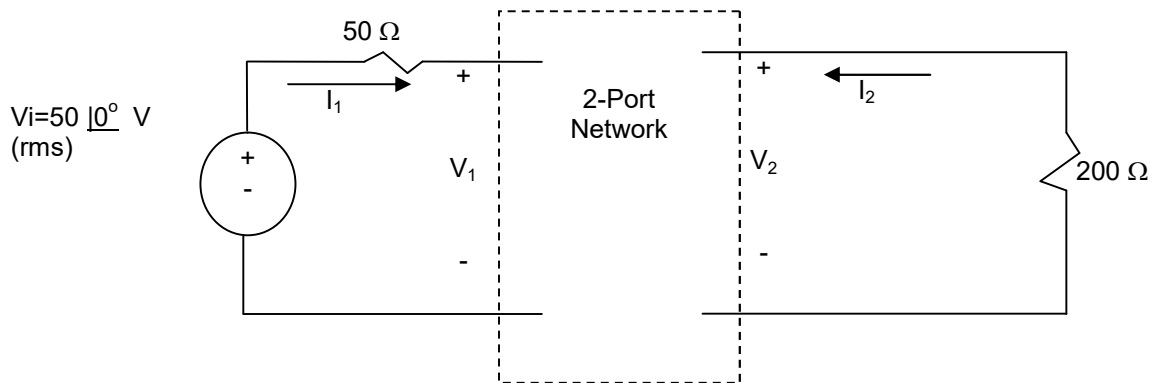


Find the y parameter for the network

6S. The Z parameters of the network in the following circuit are:

$$Z_{11} = 25 \Omega, \quad Z_{12} = 1000 \Omega, \quad Z_{21} = 200 \Omega, \quad Z_{22} = 400 \Omega$$

Find the ratio of magnitude of effective output power to that supplied by the ideal voltage source.



Solution:

a)

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

where :

$$z_{11} = \frac{V_1}{I_1} \Omega \quad \text{and} \quad z_{21} = \frac{V_2}{I_1} \Omega \quad \text{when } I_2 = 0$$

$$z_{22} = \frac{V_2}{I_2} \Omega \quad \text{and} \quad z_{12} = \frac{V_1}{I_2} \Omega \quad \text{when } I_1 = 0$$

Plug in the given parameters

$$V_1 = 25I_1 + 1000I_2$$

$$V_2 = 200I_1 + 400I_2$$

apply kvl to input and output loops

$$V_1 = 50 - 50I_1$$

$$V_2 = -200I_2$$

reorganize in order to solve it using matrix operation

$$-V_1 + 0V_2 + 25I_1 + 1000I_2 = 0$$

$$0V_1 - V_2 + 200I_1 + 400I_2 = 0$$

$$V_1 + 0V_2 + 50I_1 + 0I_2 = 50$$

$$0V_1 + V_2 + 0I_1 + 200I_2 = 0$$

Solving the above 4 unknowns:

$$V_1 = 59.68 \text{ V}, \quad V_2 = -12.9 = 12.90 \angle 180^\circ, \text{ V}, \quad I_1 = -0.19 = 0.19 \angle 180^\circ \text{ A}, \quad I_2 = 0.06 \text{ A}$$

$$P_{i \text{ rms}} = (V)(I) = (59.68)(0.19) = 11.3 \text{ W}$$

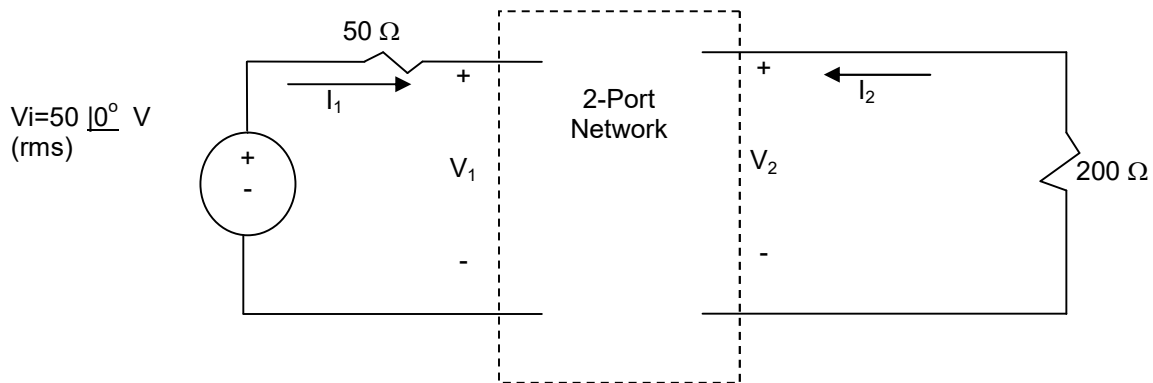
$$P_{o \text{ rms}} = (V)(I) = (12.90)(0.06) = 0.77 \text{ W}$$

$$P_o / P_i = 0.77 / 11.3 = 0.068$$

6U. The Z parameters of the network in the following circuit are:

$$Z_{11} = 50 \text{ k}\Omega, \quad Z_{12} = 2000 \text{ k}\Omega, \quad Z_{21} = 400 \text{ k}\Omega, \quad Z_{22} = 200 \text{ k}\Omega$$

Find the ratio of magnitude of effective output power to that supplied by the ideal voltage source.



Solution:

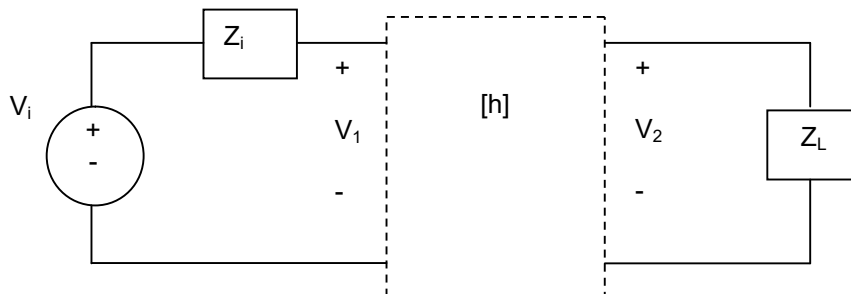
7S. The h parameter for the following two-port power amplifier circuit are:

$$h_{11} = 100\Omega, h_{12} = 10^{-4}, h_{21} = 50, h_{22} = 50 \text{ mS}.$$

The internal impedance of the source is $500 + j0 \Omega$ and the load impedance is $100 + j0 \text{ k}\Omega$. The ideal voltage source output is.

$$v_i = 424.3 \cos 40,000t \text{ mV}$$

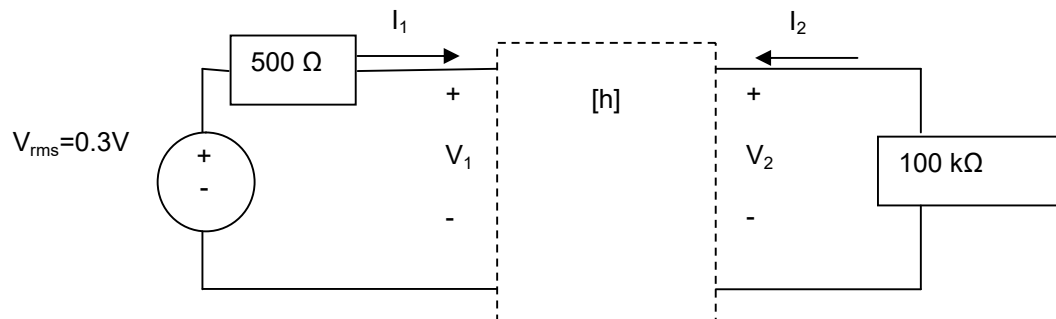
Find the rms value of V_2 .



Solution:

We have

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{424.3}{\sqrt{2}} = 300 \text{ mV}$$



Write Kvl at Port1 loop $\rightarrow -0.3 + 500i_1 + V_1 = 0$ eq (1)

Write Kvl at Port2 loop $\rightarrow 10^6 i_2 + V_2 = 0$ eq (2)

from the following h parameters and values given:

$$V_1 = h_{11}I_1 + h_{12}V_2 \rightarrow V_1 = 100I_1 + 10^{-4}V_2 \quad eq(3)$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \rightarrow I_2 = 50I_1 + 0.05V_2 \quad eq(4)$$

Use equations 1-4 to find $V_2 = 500 \text{ V}$

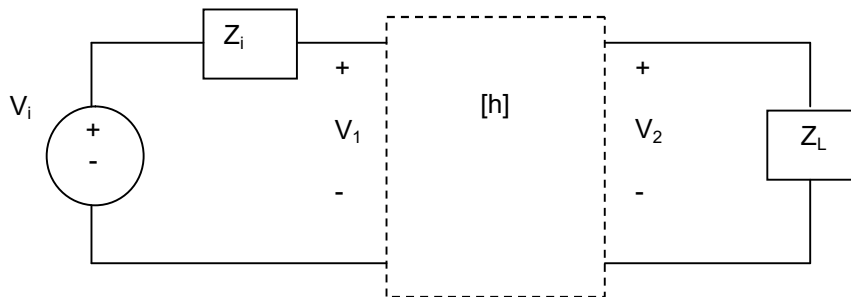
7U. The h parameter for the following two-port power amplifier circuit are:

$$h_{11} = 200\Omega, \quad h_{12} = 10^{-3}, \quad h_{21} = 10, \quad h_{22} = 25 \text{ mS}.$$

The internal impedance of the source is $400 + j0 \Omega$ and the load impedance is $200 + j0 \text{ k}\Omega$. The ideal voltage source output is.

$$v_i = 300 \cos 25,000t \text{ mV}$$

Find the rms value of V_2 .



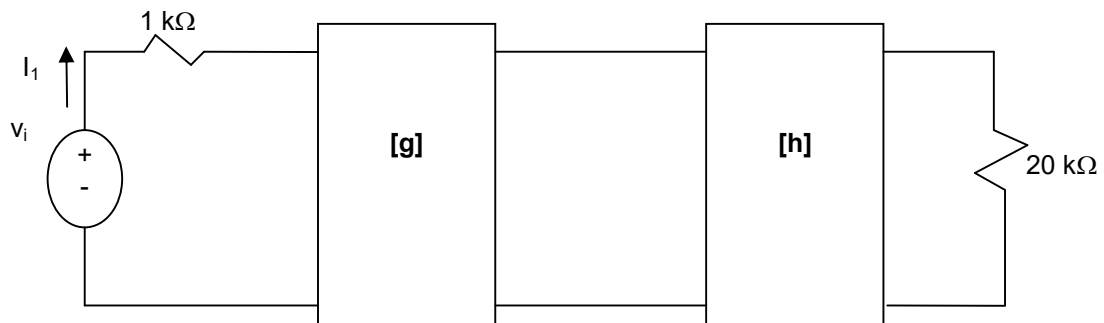
Solution:

8S. The g and h parameters for the resistive two-ports in the following circuit are given by

$$g_{11} = .025 \text{ S}, \quad g_{12} = 12, \quad g_{21} = 100, \quad g_{22} = 50 \text{ k}\Omega.$$

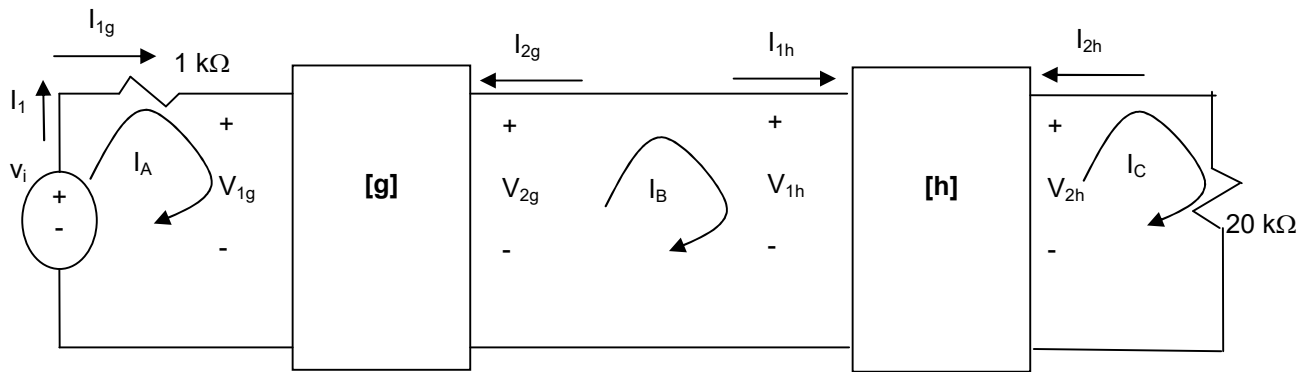
$$h_{11} = 4 \text{ k}\Omega, \quad h_{12} = -1, \quad h_{21} = -5, \quad h_{22} = 8 \text{ mS}.$$

Calculate i_1 if $v_i = 10.39 \text{ V dc}$.



Solution:

We have:



For g 2 – port – Circuit

$$I_1 = g_{11}V_1 + g_{12}I_2 \Rightarrow I_{1g} = 0.025V_{1g} + 12I_{2g}$$

$$V_2 = g_{21}V_1 + g_{22}I_2 \Rightarrow V_{2g} = 100V_{1g} + 50,000I_{2g}$$

For h 2 – port – Circuit

$$V_1 = h_{11}I_1 + h_{12}V_2 \Rightarrow V_{1h} = 4,000I_{1h} - V_{2h}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \Rightarrow I_{2h} = -5I_{1h} + 0.008V_{2h}$$

For the 3 loops

$$-10.39 + 1,000I_{1g} + V_{1g} = 0$$

$$V_{2g} = V_{1h} \quad \& \quad I_{2g} = -I_{1h}$$

$$-20,000I_{2h} - V_{2h} = 0$$

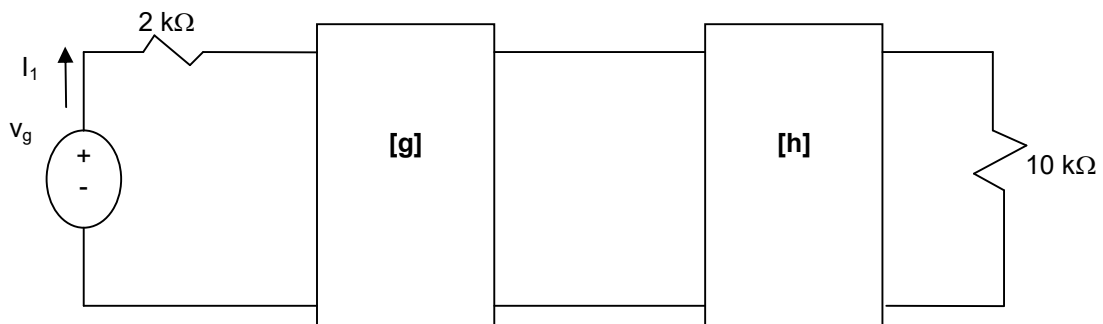
Solve the above 8 equations and 8 unknowns to find the value of $I_1 = I_{1g} = 7.49 \text{ mA}$

8U. The g and h parameters for the resistive two-ports in the following circuit are given by

$$g_{11} = .125 \text{ S}, \quad g_{12} = 10, \quad g_{21} = 200, \quad g_{22} = 100 \text{ k}\Omega.$$

$$h_{11} = 25 \text{ k}\Omega, \quad h_{12} = -10, \quad h_{21} = -20, \quad h_{22} = 4 \text{ mS}.$$

Calculate i_1 if $v_j = 21 \text{ V dc}$.



Solution: