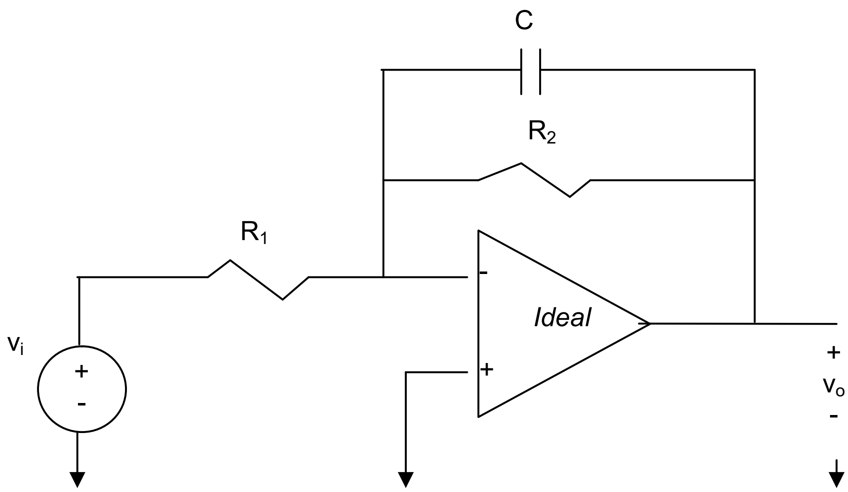
Fundamentals of Electrical Circuits - Chapter 13

1S. Using the following circuit:

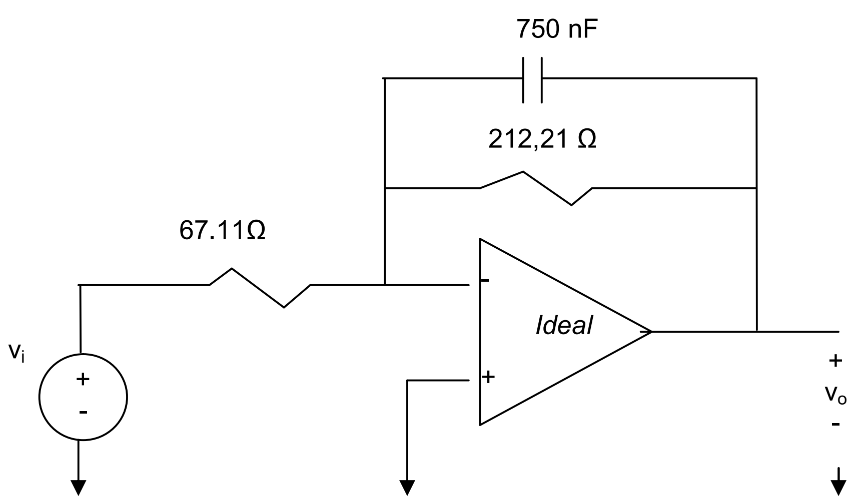


a) Design a low pass filter with a passband gain of 10 dB and a cutoff frequency of 1 kHz. Assume a 750 nF capacitor is available.
b) Draw the circuit diagram and label all components.

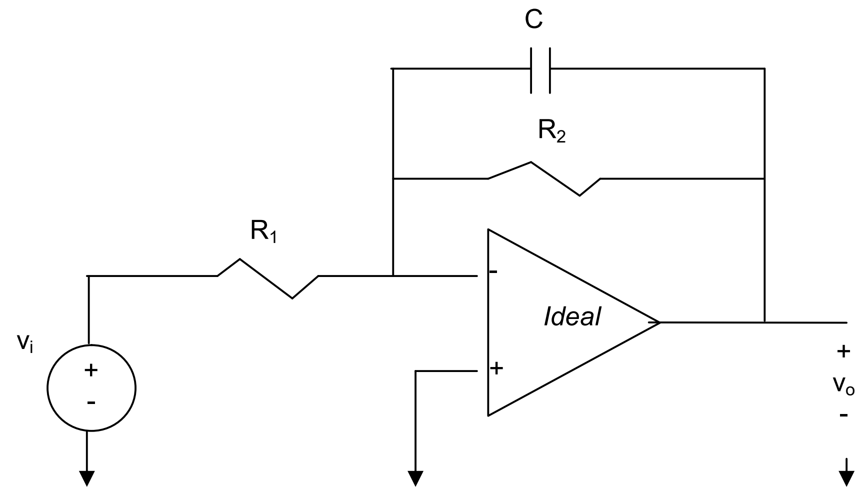
Solution:

a) Gain = 10 dB = 20 log K → K = $10^{0.5}$ = 3.16
For a low Pass Filter:
$K = R_2 / R_1 \rightarrow R_1 = R_2 / 3.16$
$W_c = 1 / (R_2 C) \rightarrow R_2 = 1 / ((2000\pi)(750\times10^{-9})) = 212.21\ \Omega$
$R_1 = 212.21 / 3.16 = 67.15\ \Omega$

b)



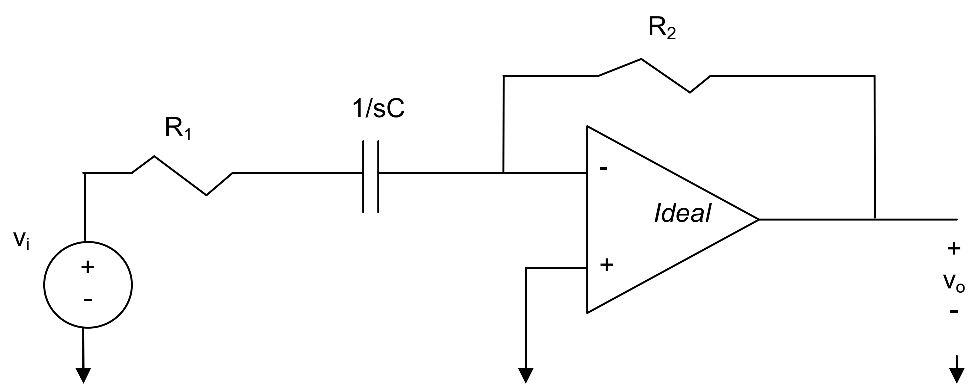
1U. Using the following circuit:



a) Design a low pass filter with a passband gain of 40 dB and a cutoff frequency of 5 kHz. Assume only a 0.5 uF capacitor is available.
b) Draw the circuit diagram and label all components.

Solution:

2S. Using the following circuit:

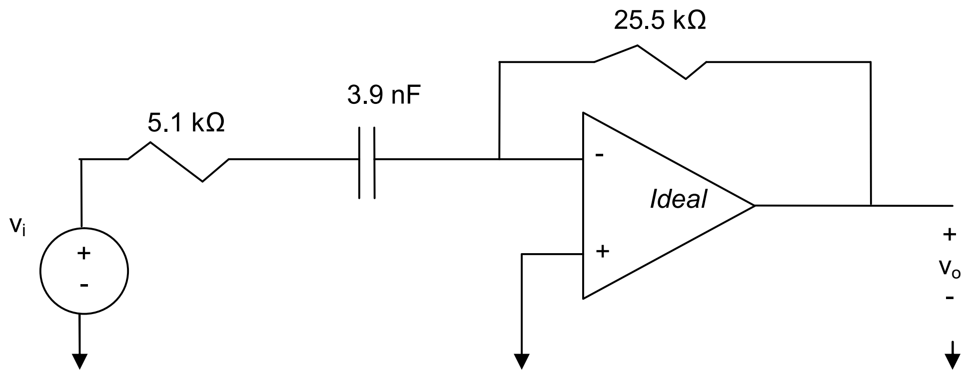


a) Design a high pass filter with a cutoff frequency of 8 kHz and a passband gain of 14 dB. Use a 3.9 nF capacitor in the design.
b) Draw a circuit diagram of the filter and label all the components.

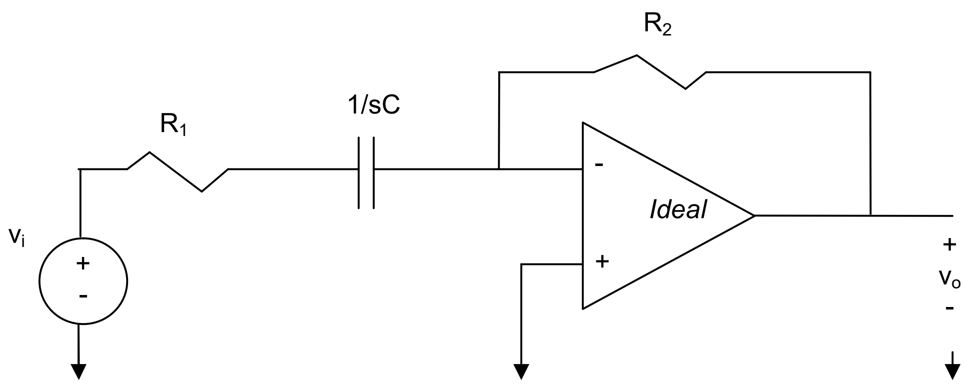
Solution:

a) Gain = 14 dB = 20 log K → $K = 10^{(14/20)} = 5.01$
For a high Pass Filter:
$K = R_2 / R_1 \rightarrow R_2 = 5.01R_1$
$W_c = 1 / (R_1C) \rightarrow R_1 = 1 / ((16000\pi)(3.9\times10^{-9})) = 5101.12\ \Omega$
$R_2 = (5101.12)(5.01) = 25{,}556.6\ \Omega$

b)



2U. Using the following circuit:



a) Design a high pass filter with a cutoff frequency of 25 kHz and a passband gain of 60 dB. Use only a 2.5 pF capacitor in the design.
b) Draw a circuit diagram of the filter and label all the components.

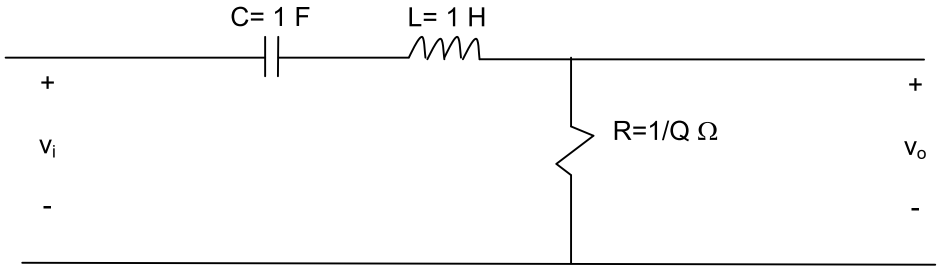
Solution:

3S. The voltage transfer function of the prototype bandpass filter for the following circuit is

$$H(s) = \frac{(\frac{1}{Q})s}{s^2 + (\frac{1}{Q})s + 1}.$$

If the circuit is scaled in both magnitude and frequency, the scaled transfer function is

$$H(s) = \frac{(\frac{1}{Q})(\frac{s}{k_f})}{(\frac{s}{k_f})^2 + (\frac{1}{Q})\frac{s}{k_f} + 1}$$



a) Specify the component values for the above prototype passive bandpass filter if the quality factor of the filter is 20.
b) Specify the component values for the band pass filter described above if the quality factor is 20; the center, or resonant frequency is 40 krad/s; and the impedance at resonance is 5 k Ω.
c) Draw a circuit diagram of the scaled filter and label all the components.

Solution:

a) L = 1 H , C = 1 F, R = 1/Q =1/20 = 0.05 Ω

b) At resonance the Capacitor and Inductor cancel each others effect such that H(s) =1 in other words Zeq = R' = 5000 therefore:

Magnitude Scaling Factor, $K_m = R'/R = 5000/0.05 = 100{,}000$
Frequency Scaling Factor, $K_f = w_o' / w_o$
We have $w_o' = 40{,}000$ rad/s but need to find w_o

Since we know the above band pass filter's transfer function is:

$$H(s) = \frac{\beta s}{s^2 + \beta s + w_o^{\,2}}$$

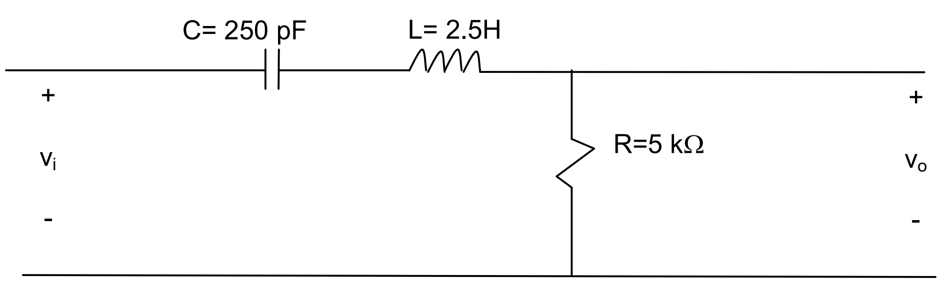
$$\beta = \frac{w_o}{Q} \Rightarrow H(s) = \frac{\frac{w_o}{Q} s}{s^2 + \frac{w_o}{Q} s + w_o^{\,2}}$$

Comparing this equation with the one given in the problem state, we can conclude that:
$w_o = 1$ Therfore:

Frequency Scaling Factor, $K_f = w_o' / w_o = 40{,}000$

$R' = K_m R = (0.05)(100{,}000) = 5$ KΩ
$L' = (K_m/K_f) L = 100{,}000/40{,}000 = 2.5$ H
$C' = C / (K_m K_f) = 1/(40{,}000)(100{,}000) = 250$ pF

c)

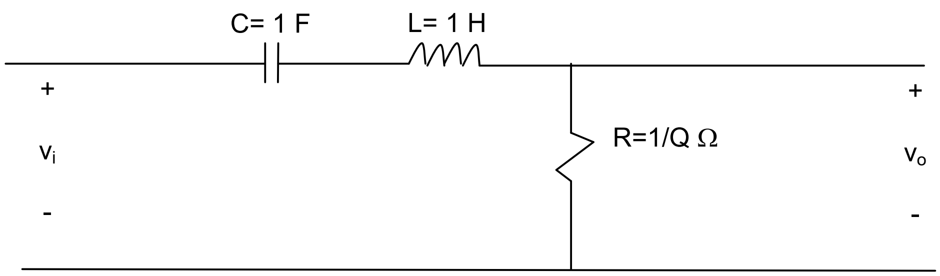


3U. The voltage transfer function of the prototype bandpass filter for the following circuit is

$$H(s) = \frac{(\frac{1}{Q})s}{s^2 + (\frac{1}{Q})s + 1}.$$

If the circuit is scaled in both magnitude and frequency, the scaled transfer function is

$$H(s) = \frac{(\frac{1}{Q})(\frac{s}{k_f})}{(\frac{s}{k_f})^2 + (\frac{1}{Q})\frac{s}{k_f} + 1}$$



a) Specify the component values for the above prototype passive bandpass filter if the quality factor of the filter is 60.
b) Specify the component values for the band pass filter described above if the quality factor is 60; the center, or resonant frequency is 50 krad/s; and the impedance at resonance is 10 k Ω.
c) Draw a circuit diagram of the scaled filter and label all the components.

Solution:

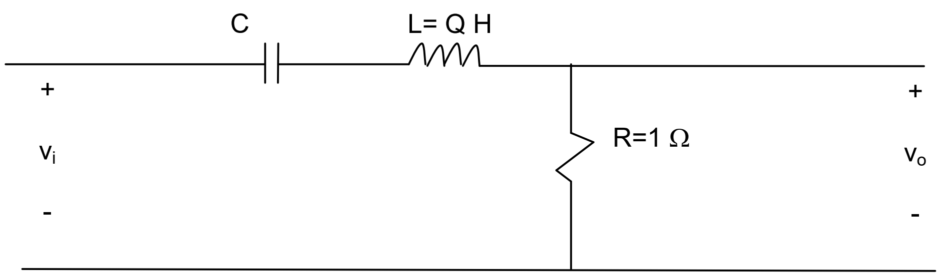
4S. The voltage transfer function of the prototype bandpass filter for the following circuit is

$$H(s) = \frac{(\frac{1}{Q})s}{s^2 + (\frac{1}{Q})s + 1}.$$

If the circuit is scaled in both magnitude and frequency, the scaled transfer function is

$$H(s) = \frac{(\frac{1}{Q})(\frac{s}{k_f})}{(\frac{s}{k_f})^2 + (\frac{1}{Q})\frac{s}{k_f} + 1}$$

and its w_o = 1 rad/sec.



a) What is the value of C in the prototype filter circuit?
b) What is the transfer function of the proto-type filter?
c) Use the alternative prototype circuit just described to design a passive bandpass filter that has a quality factor of 16, a center frequency of 25 krad/s, and an impedance of 10kΩ at resonance.
d) Draw a diagram of the scaled filter and label all the components.
e) Use the relationship shown to write the transfer function of the scaled circuit.

Solution:

a)
bandpass filter → $w_o^2 = 1/LC$ → $1 = 1/LC$ → $C = 1/L$ → $C = 1/Q$

b) Since we know the above band pass filter's transfer function is:

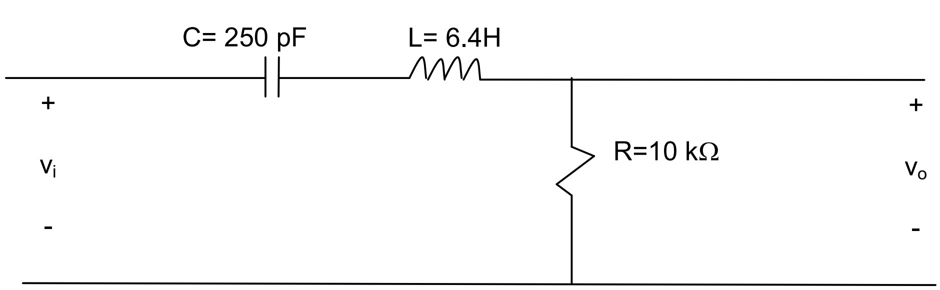
$$H(s) = \frac{\beta s}{s^2 + \beta s + w_o^{\,2}}$$

$$\beta = \frac{w_o}{Q} = \frac{1}{Q} \Rightarrow H(s) = \frac{\frac{1}{Q}s}{s^2 + \frac{1}{Q}s + 1}$$

c) R=1 Ω, R'=10 kΩ, Q=16
L=Q = 16 H, C = 1/L = 0.0625 F
Frequency Scaling Factor, $K_f = w_o' / w_o = 25{,}000$
Magnitude Scaling Factor, $K_m = R'/R = 10{,}000/1 = 10{,}000$

$R' = K_m R = 10$ kΩ
$L' = (K_m/K_f) L = (10{,}000/25{,}000)(16) = 6.4$ H
$C' = C / (K_m K_f) = 1/(10{,}000)(25{,}000) = 250$ pF

d)



e)

$$H(s) = \frac{(\frac{w_o}{Q})s}{s^2 + (\frac{w_o}{Q})s + w_o^{\ 2}} = \frac{(\frac{25000}{16})s}{s^2 + (\frac{25000}{16})s + 25000^2}$$

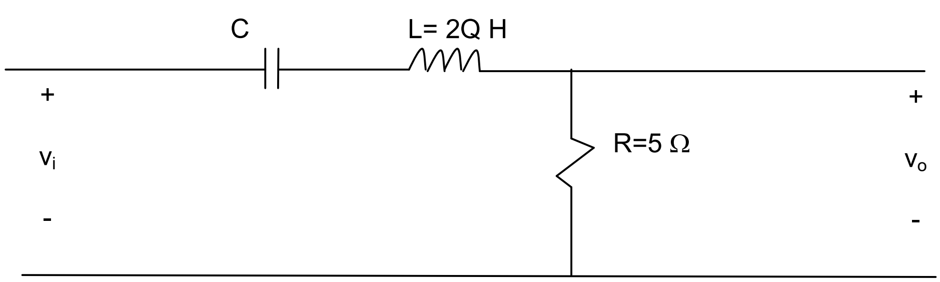
4U. The voltage transfer function of the prototype bandpass filter for the following circuit is

$$H(s) = \frac{(\frac{1}{Q})s}{s^2 + (\frac{1}{Q})s + 1}.$$

If the circuit is scaled in both magnitude and frequency, the scaled transfer function is

$$H(s) = \frac{(\frac{1}{Q})(\frac{s}{k_f})}{(\frac{s}{k_f})^2 + (\frac{1}{Q})\frac{s}{k_f} + 1}$$

and its w_o = 10 rad/sec.



a) What is the value of C in the prototype filter circuit?
b) What is the transfer function of the proto-type filter?
c) Use the alternative prototype circuit just described to design a passive bandpass filter that has a quality factor of 40, a center frequency of 50 krad/s, and an impedance of 10kΩ at resonance.
d) Draw a diagram of the scaled filter and label all the components.
e) Use the relationship shown to write the transfer function of the scaled circuit.

Solution:

5S. Design a unity-gain bandpass filter, using a cascade connection, to give a center frequency of 200 Hz and a bandwidth of 1000 Hz. Use 5 uF capacitors. Specify f_{c1}, f_{c2}, R_L, and R_H,.

Solution:

K = 1
$w_o = 2\pi f_o = 400\pi$
$\beta = 2\pi\,(1000) = 2000\pi$

To find the cut off frequency w_{c1} and w_{c2} user the following two equations:
$\beta = w_{c2} - w_{c1} = 2000\pi$
$w_o^{\ 2} = w_{c1}\, w_{c2} = (400\pi)^2$

Find w_{c2} in-term of w_{c1} from equation 2 and plug it back into equation 1

$w_{c1}^2 + 2000\pi\, w_{c1} - (400\pi)^2 = 0$

$$w_{c1} = -1000\pi \pm \sqrt{10^6\Pi^2 - 4(400\Pi)^2} = -1000\pi(1 \pm \sqrt{1.16})$$

$$\textit{Since negative does not make sense the only valid answer is } w_{c1} = 242.01\ rad/s$$

$$\textit{Therefore } w_{c2} = 2000\Pi + w_{c1} = 6525.19\ rad/s$$

which means f_{c1} = 38.52 Hz and f_{c1} = 1038.52 Hz

Since this is a two stage Bandpass filter we know:
The low pass filter Stage

$$w_{c2} = \frac{1}{R_L C_L} = 6525.19 \Rightarrow R_H = \frac{1}{(6525.19)(5x10^{-6})} = 30.65\Omega$$

The high pass filter Stage

$$w_{c1} = \frac{1}{R_H C_H} = 242.01 \Rightarrow R_L = \frac{1}{(242.01)(5x10^{-6})} = 826.43\Omega$$

5U. Design a unity-gain bandpass filter, using a cascade connection, to give a center frequency of 400 kHz and a bandwidth of 600 kHz. Use 10 pF capacitors. Specify f_{c1}, f_{c2}, R_L, and R_H,.

Solution:

6S. Design a parallel bandreject filter with a center frequency of 1000 rad/s, a bandwidth of 4000 rad/s, and a passband gain of 6. Use 0.2 uF capacitors and specify all resistor values.

Solution:

K = 6
w_o = 1000 rad/s
β = 4000 rad/s
C= 0.2 uF

To find the cut off frequency w_{c1} and w_{c2} user the following two equations:
$\beta = w_{c2} - w_{c1} = 4000$
$w_o^2 = w_{c1}\, w_{c2} = 10^6$

Find w_{c2} in-term of w_{c1} from equation 2 and plug it back into equation 1

$w_{c1}^2 + 4000\, w_{c1} - 10^6 = 0$

$$w_{c1} = -2000 \pm 1000\sqrt{5}$$

$$\textit{Since negative does not make sense the only valid answer is } w_{c1} = 236.07\ rad/s$$

$$\textit{Therefore } w_{c2} = 4{,}000 + w_{c1} = 4236.07\ rad/s$$

Since this is a two stage Bandreject filter we know:
The low pass filter Stage

$$w_{c1} = \frac{1}{R_L C_L} = 236.07 \Rightarrow R_L = \frac{1}{(236.07)(2x10^{-7})} = 21{,}180.2\ \Omega$$

The high pass filter Stage

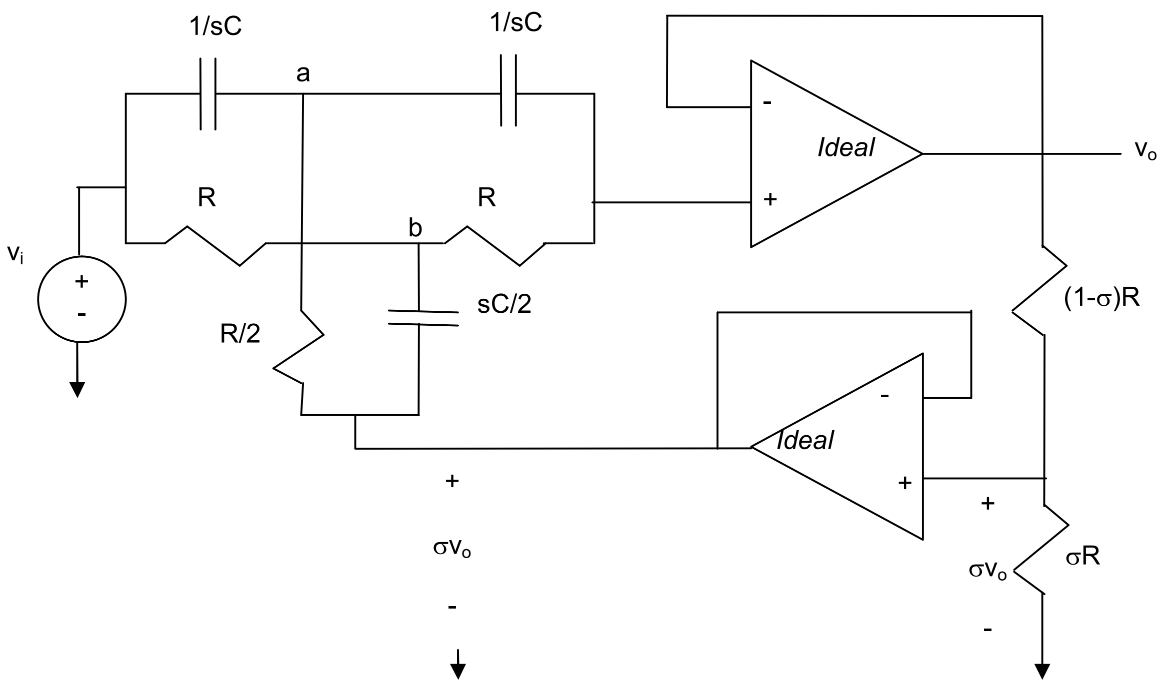
$$w_{c2} = \frac{1}{R_H C_H} = 4236.07 \Rightarrow R_H = \frac{1}{(4236.07)(2x10^{-7})} = 1180.34\ \Omega$$

K = R_f / R_i = 6 → if R_i = 10 kΩ then R_f = 60 kΩ

6U. Design a parallel bandreject filter with a center frequency of 20 M rad/s, a bandwidth of 4 Mrad/s, and a passband gain of 20. Use 1 nF capacitors and specify all resistor values.

Solution:

7S. Using the following circuit:



a) design a narrow-band bandreject filter having a center frequency of 1 kHz and a quality factor of 20. Base the design on C=15 nF.
b) Draw the circuit diagram of the filter and label all component values on the diagram.
c) What is the scaled transfer function of the filter?

$$H(s) = \frac{s^2 + w_o^{\ 2}}{s^2 + \beta s + w_o^{\ 2}} \quad where\ w_o^{\ 2} = \frac{1}{R^2 C^2}, \quad \beta = \frac{4(1-\sigma)}{RC}, \quad \sigma = 1 - \frac{1}{4Q}$$

Solution:

a) Given: w_o = 2000π; Q = 20 and C=15 nF

$$w_o = \frac{1}{RC} \Rightarrow R = \frac{1}{2000\pi(15x10^{-9})} = 10610.3\ \Omega$$

$$\sigma = 1 - \frac{1}{4Q} = 1 - 1/80 = 0.9875 \Rightarrow (1-\sigma)R = 133\ \Omega \quad \& \quad \sigma R = 10478\ \Omega$$

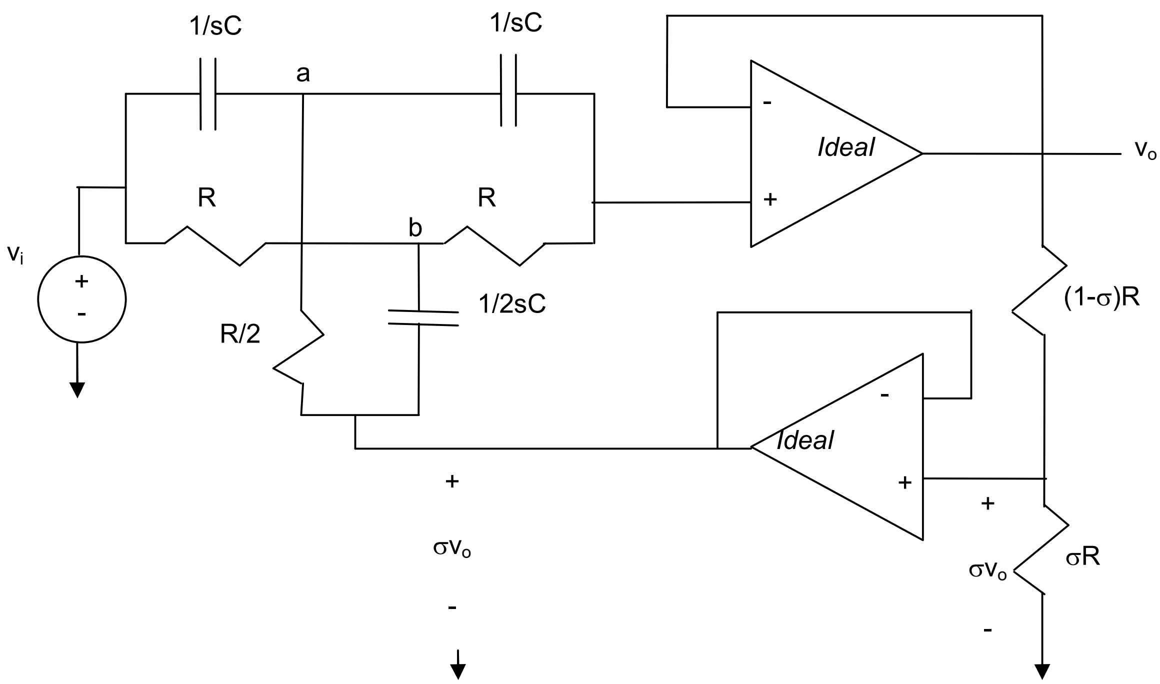
b) Just plug in the value in the above circuit

c) To get to scaled transfer function:
$K_f = w_o'/w_o = 2000\pi / 1 = 2000\pi \rightarrow w_o = w_o'/2000\pi$

$$\beta = \frac{4(1-\sigma)}{RC}, = \frac{4(1-0.9875)}{1/2000\pi} = 100\pi$$

$$H(s) = \frac{(s/2000\pi)^2 + 1}{(s/2000\pi)^2 + 100\pi(s/2000\pi) + 1} = \frac{s^2 + 4x10^6\pi^2}{s^2 + 2x10^5\pi^2 s + 4x10^6\pi^2}$$

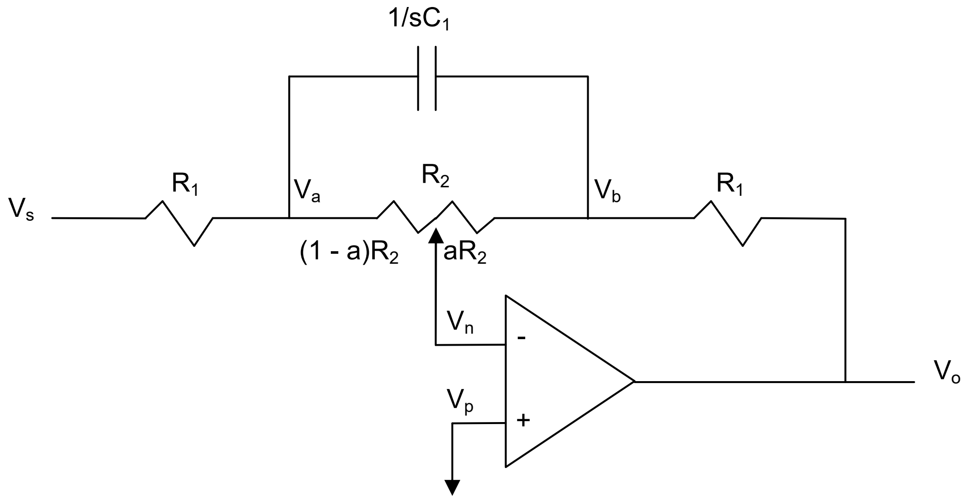
7U. Using the following circuit:



a) design a narrow-band bandreject filter having a center frequency of 5 kHz and a quality factor of 20. Use only C=10 nF in your design.
b) Draw the circuit diagram of the filter and label all component values on the diagram.
c) What is the scaled transfer function of the filter?

$$H(s) = \frac{s^2 + w_o^2}{s^2 + \beta s + w_o^2} \quad where\ w_o^2 = \frac{1}{R^2C^2}, \quad \beta = \frac{4(1-\sigma)}{RC}, \quad \sigma = 1 - \frac{1}{4Q}$$

8S. Using the following circuit



design a volume control circuit to give a maximum gain of 20 dB and a gain of 17 dB at a frequency of 40 Hz. Use an 11.1 kΩ resistor and a 100 kΩ potentiometer. Test your design by calculating the maximum gain at w=0 and the gain at w=1/R_2C_1 using the selected values of R_1, R_2, and C_1.

$$Given: H(s)=\frac{Vo}{Vs}=\frac{-(R_1+\alpha R_2+R_1R_2C_1s)}{R_1+(1-\alpha)R_2+R_1R_2C_1s}$$, w_c=1/aR_2C_1 “Low Pass Filter”

Note: to derive the above transfer function, write KCL for nodes Vn, Va and Vb (Assume Ideal Op Amp)

Solution:

for a low pass filter at w = 0 we have Maximum gain and also a =1 we get a gain of 20 dB →

$$|H(jw)|_{\max}=\frac{R_1+R_2}{R_1}$$

$$20\log\left(\frac{R_1+R_2}{R_1}\right)=20dB \Rightarrow \frac{R_1+R_2}{R_1}=10 \Rightarrow R_2=9R_1$$

Picking R_1 = 11.1 kΩ → R_2 = 99.9 kΩ

note that 17 dB is 3 dB lower than max which means 40 hz is the cut off frequency therefore w_c=1/R_2C_1 → C_1 = 1/((100x10^3)(80π) = 39.79 nF

Testing Phase

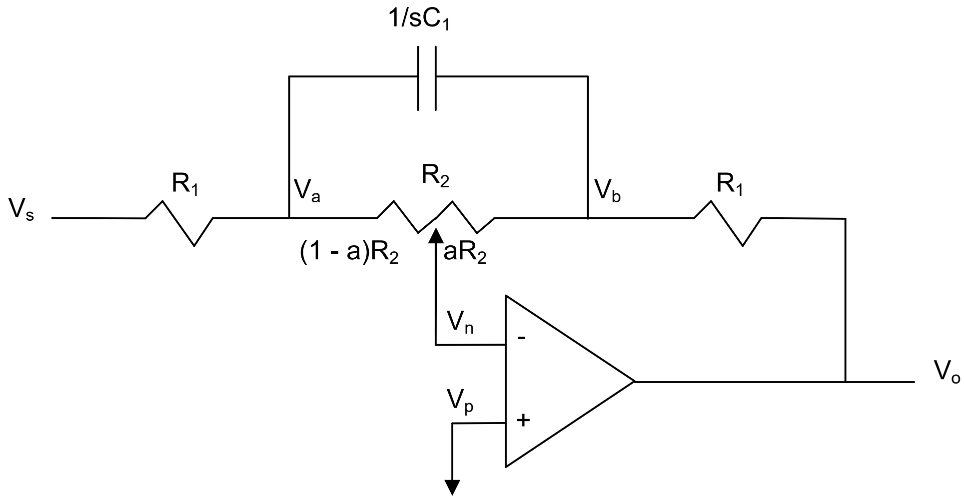
Using the standard parts and given values → C=39 nF and R1 = 11.1 and R2 = 100 K

at max (w=0, a=1) → $20\log|H(jw)|_{\max}=20\log\left(\frac{11.1+100}{11.1}\right)=20.01dB$

w= 1/R2C1 = 1/(10^5x39x10^{-9}) = 256.41 rad/s & a=1

$$20\log|H(j256.41)|=20\log\left|\frac{-(R_1+\alpha R_2+R_1R_2C_1j256.41)}{R_1+(1-\alpha)R_2+R_1R_2C_1j256.41}\right|=20\log(7.11)=17.04dB$$

8U. Using the following circuit



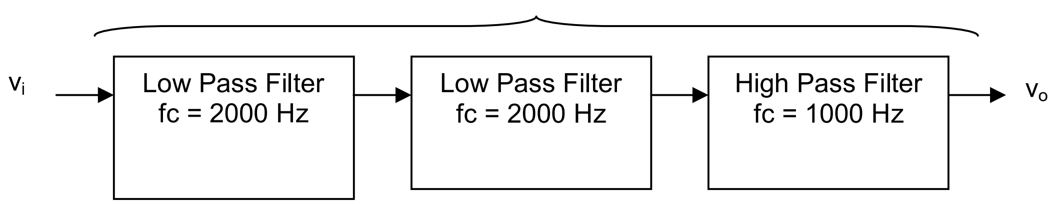
design a bass volume control circuit that has a maximum gain of 13.98 dB that drops off 3 dB at 50 Hz.

$Given: H(s) = \frac{Vo}{Vs} = \frac{-(R_1 + \alpha R_2 + R_1 R_2 C_1 s)}{R_1 + (1-\alpha)R_2 + R_1 R_2 C_1 s}$, $w_c = 1/R_2C_1$ "Low Pass Filter"

Note: to derive the above transfer function, write KCL for nodes Vn, Va and Vb (Assume Ideal Op Amp)

Solution:

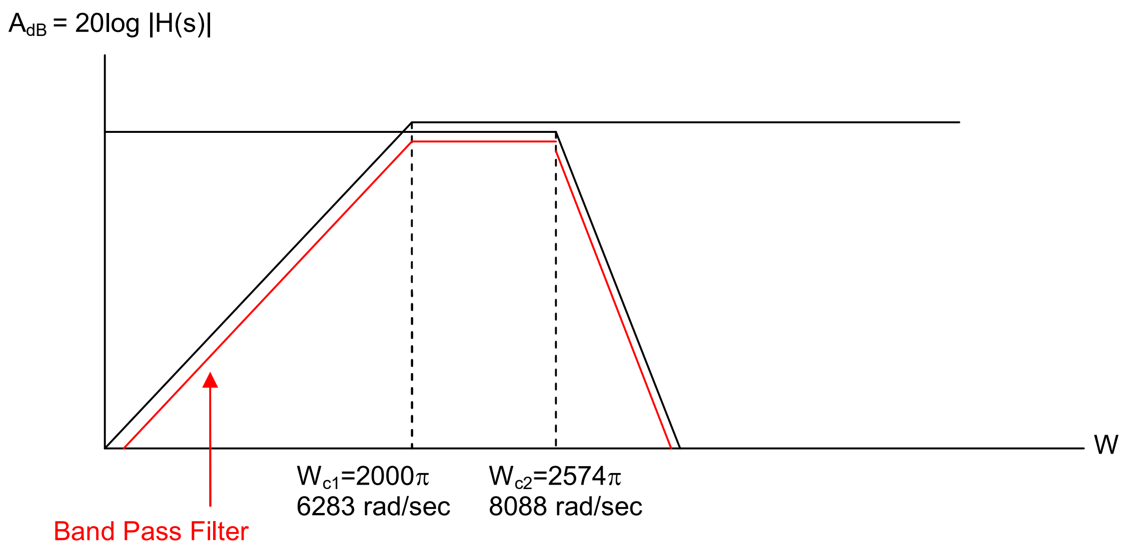
9S. What are the filter type and cut off frequencies (w_{c1} & w_{c2}) of the system represented by the following block diagram? Explain your answers.



Solution:

n=2 stage low pass filter → $w_{LCn} = \left(\sqrt{\sqrt[n]{2} - 1}\right) Wc_1 = \left(\sqrt{\sqrt[2]{2} - 1}\right)(2000)(2\pi) = 2574\pi$

n=1 stage high pass filter → $w_{Hc} = (1000)(2\pi) = 2000\pi$



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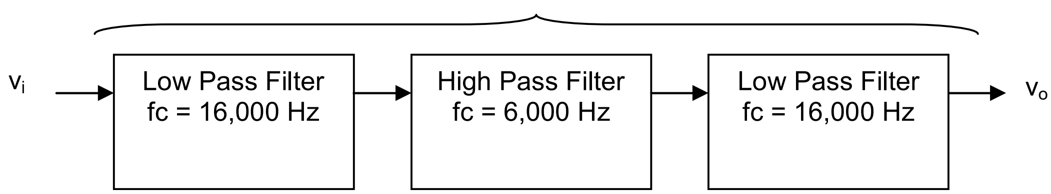

Solution: