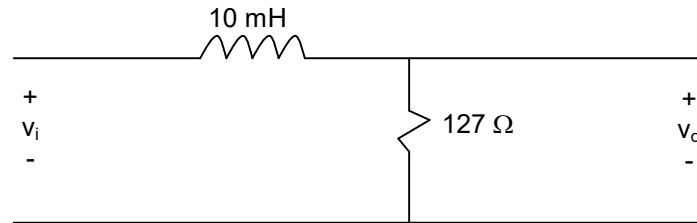


Fundamentals of Electrical Circuits - Chapter 12

1S. The following diagram is an RL filter:



- Find the cutoff frequency in hertz
- Calculate $H(j\omega)$ at ω_c , $0.2\omega_c$ and $5\omega_c$.
- If $v_i = 10 \cos \omega t$ V, write the steady-state expression for v_o when $\omega = \omega_c$, $\omega = 0.2\omega_c$ and $\omega = 5\omega_c$.

Solution:

a)

RL Low Pass filter $\rightarrow \omega_c = R/L = 127 / (10 \times 10^{-3}) = 12.7$ Krad/s
 $f_c = \omega_c / 2\pi = 2021.27$ Hz.

b)

$$\text{Transfer function is } H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\omega_c}{s + \omega_c} = \frac{12700}{s + 12700}$$

$$H(s = j\omega_c) = \frac{12700}{s + 12700} = \frac{12700}{j12700 + 12700} = \frac{12700}{\sqrt{12700^2 + 12700^2} \angle \tan^{-1} 1} = 0.707 \angle -45^\circ$$

$$H(s = j0.2\omega_c) = \frac{12700}{j(0.2)12700 + 12700} = 0.981 \angle -11.31^\circ$$

$$H(s = j5\omega_c) = \frac{12700}{j(5)12700 + 12700} = 0.196 \angle -78.69^\circ$$

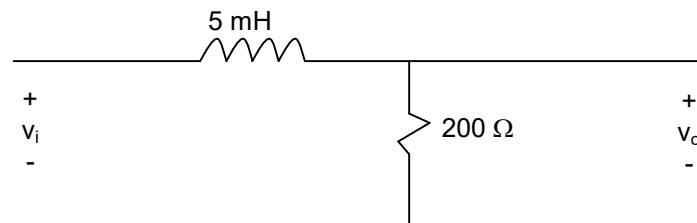
c)

$$v_o(t) |_{\omega_c} = (0.707)(10)\cos(\omega_c t - 45^\circ) = 7.07 \cos(12700t - 45^\circ) \text{ V}$$

$$v_o(t) |_{0.2\omega_c} = (0.981)(10)\cos(0.2\omega_c t - 11.31^\circ) = 9.81 \cos(2540t - 11.31^\circ) \text{ V}$$

$$v_o(t) |_{5\omega_c} = (0.196)(10)\cos(5\omega_c t - 78.69^\circ) = 1.96 \cos(63500t - 78.69^\circ) \text{ V}$$

1U. The following diagram is an RL filter:

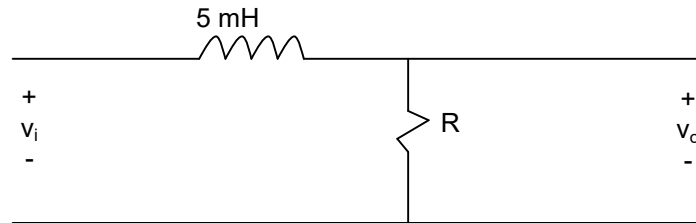


- Find the cutoff frequency in hertz
- Calculate $H(j\omega)$ at $0.5\omega_c$, ω_c , and $10\omega_c$.
- If $v_i = 20 \cos \omega t$ V, write the steady-state expression for v_o when $\omega = \omega_c$, $\omega = 0.5\omega_c$ and $\omega = 10\omega_c$.

Solution:

- 2S. Use a 5 mH inductor to design a low-pass , RL, Passive filter with a cutoff frequency of 1 kHz.
- Specify the value of the resistor.
 - A load having a resistance of $270\ \Omega$ is connected across the output terminal of the filter. What is the corner, or cutoff, frequency of the loaded filter in hertz?

Solution:



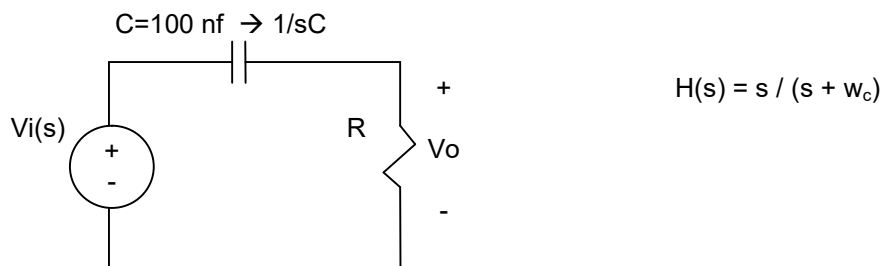
- $f_c = 1000\text{ Hz} \rightarrow \omega_c = 2\pi f_c = 2000\pi\text{ rad/s}$
 we have $\omega_c = R/L \rightarrow R = L \omega_c = (0.005)(2000\pi) = 31.42\ \Omega$
- $R_{eqv} = 31.42 \parallel 270 = 28.14\ \Omega$
 we have $\omega_c = R/L = (28.14) / (0.005) = 5628\text{ rad/s}$
 $f_c = \omega_c / 2\pi = 5628 / 2\pi = 895.77\text{ Hz}$

- 2U. Use a 10 mH inductor to design a low-pass , RL, Passive filter with a cutoff frequency of 10 kHz.
- Specify the value of the resistor.
 - A load having a resistance of $500\ \Omega$ is connected across the output terminal of the filter. What is the corner, or cutoff, frequency of the loaded filter in hertz?

Solution:

- 3S. Using a 100 nF capacitor, design a high-pass passive filter with a cutoff frequency of 300 Hz.
- Specify the value of R in k Ω .
 - A 47 k Ω resistor is connected across the output terminals of the filter. What is the cutoff frequency, in hertz, of the loaded filter?

Solution:



- $\omega_c = 1/RC \rightarrow 2\pi (300) = 1/(R \times 100 \times 10^{-9}) \rightarrow R = 5305.16\ \Omega$
- $R_{eqv} = 5305.16 \parallel 47000 = 4767.08\ \Omega$
 $\omega_c = 1/RC = 1 / (4767.08 \times 100 \times 10^{-9}) = 2097.7\text{ rad/s}$
 $f_c = \omega_c / 2\pi = 2097.7 / 2\pi = 333.86\text{ Hz}$

- 3U. Using a 25 nF capacitor, design a high-pass passive filter with a cutoff frequency of 100 Hz.

- Specify the value of R in k Ω .
- A 20 k Ω resistor is connected across the output terminals of the filter. What is the cutoff frequency, in

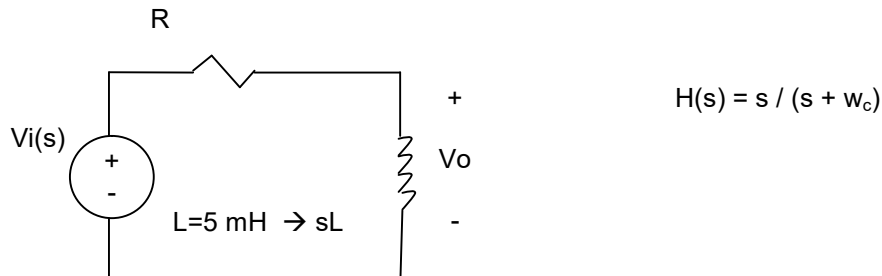
hertz, of the loaded filter?

Solution:

4S. Using a 5 mH inductor, design a high-pass, RL, passive filter with a cutoff frequency of 25 krad/s.

- Specify the value of the resistance.
- Assume the filter is connected to a pure resistive load. The cutoff frequency is not to drop below 24 krad/s. What is the smallest load resistor that can be connected across the output terminals of the filter?

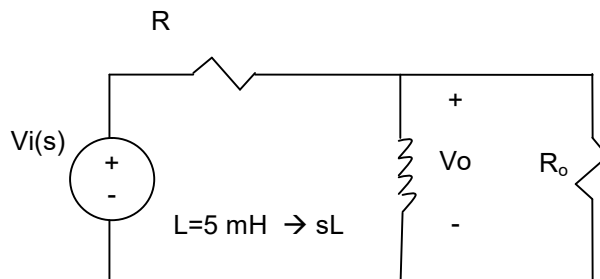
Solution:



a)

$$w_c = R/L \rightarrow R = w_c L = (25000)(0.005) = 125 \Omega$$

b)



$$H_{loaded}(S) = \frac{V_o}{V_1} = \frac{Z_{eq}}{R + Z_{eq}} = \frac{\frac{1}{\frac{1}{sL} + \frac{1}{R_o}}}{R + \frac{1}{\frac{1}{sL} + \frac{1}{R_o}}} = \frac{(sL)(R)}{R(sL + R_o) + sLR_o} = \frac{jwLR}{RR_o + jwL(R + R_o)}$$

$$|H_{loaded}(S)| = \frac{wLR}{\sqrt{w^2 L^2 (R + R_o)^2 + R^2 R_o^2}}$$

$$\text{When } |H(s)| = 1/\sqrt{2} \Rightarrow w = w_c$$

$$w_{C-loaded} \Rightarrow \frac{(wLR)^2}{w^2 L^2 (R + R_o)^2 + R^2 R_o^2} = \frac{1}{2} \Rightarrow w^2 L^2 R^2 - w^2 L^2 R_o^2 - R^2 R_o^2 - 2w^2 L^2 R_o R = 0$$

$$w_{C-loaded} = \sqrt{\frac{R^2 R_o^2}{L^2 R^2 - L^2 R_o^2 - 2L^2 R R_o}} = \frac{R}{L} \sqrt{\frac{R_o^2}{R^2 - R_o^2 - 2R R_o}}$$

$$w_{C-loaded} = w_{C-unloaded} \sqrt{\frac{R_o^2}{R^2 - R_o^2 - 2R R_o}}$$

$$\frac{w_{C-loaded}}{w_{C-unloaded}} = \sqrt{\frac{R_o^2}{R_o^2 - R^2 - 2R R_o}} = \frac{24000}{25000} > 0.96 \quad \& \quad R = 125 \Omega$$

$$\sqrt{\frac{R_o^2}{R_o^2 - (125)^2 - 250R_o}} > 0.96$$

$$R_o > 301.7 \Omega$$

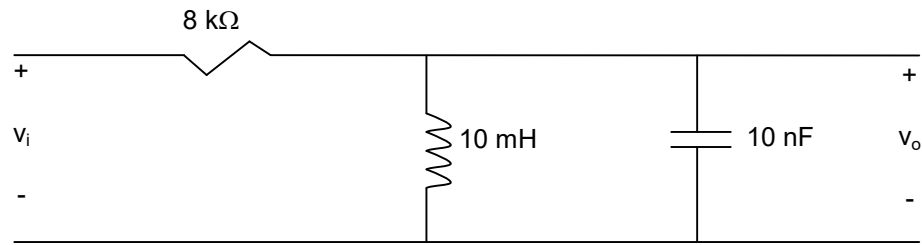
4U. Using a 10 mH inductor, design a high-pass, RL, passive filter with a cutoff frequency of 10 krads/s.

- Specify the value of the resistance.
- Assume the filter is connected to a pure resistive load. The cutoff frequency is not to drop below 8 krad/s. What is the smallest load resistor that can be connected across the output terminals of the filter?

Solution:

5A. For the bandpass filter shown below find

- w_o
- f_o
- Q
- w_{c1}
- f_{c1}
- w_{c2}
- f_{c2}
- β



Solution:

a)

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(10 \times 10^{-3})(10 \times 10^{-9})}} = 10^5 \text{ rad/s}$$

b)

$$f_0 = \frac{\omega_0}{2\pi} = \frac{10^5}{2\pi} = 15920 \text{ Hz}$$

c)

$$Q = \frac{\omega_0}{\beta} = RC\omega_0 = (8 \times 10^3)(10 \times 10^{-9})(10^5) = 8$$

d)

$$\omega_{c1} = \omega_0 \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = 93.95 \text{ krad/s}$$

e)

$$f_{c1} = \frac{\omega_{c1}}{2\pi} = 14.96 \text{ kHz}$$

f)

$$\omega_{c2} = \omega_0 \left[+\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = 106.45 \text{ krad/s}$$

g)

$$f_{c2} = \frac{\omega_{c2}}{2\pi} = 16.94 \text{ kHz}$$

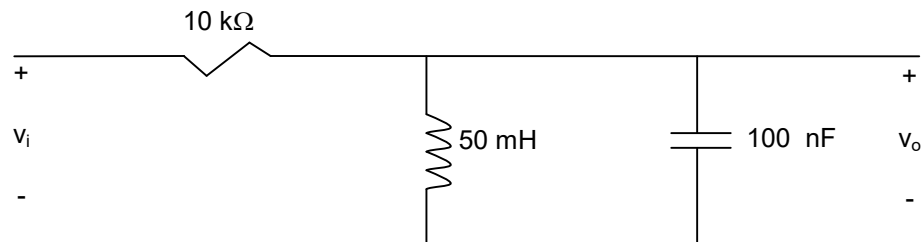
h)

$$\beta = \frac{\omega_0}{Q} = \frac{10^5}{8} = 12.5 \text{ krad/s} \quad \text{or} \quad 1.99 \text{ kHz} = \frac{f_0}{Q}$$

5U. For the bandpass filter shown below find

- a) ω_0
- b) f_0
- c) Q
- d) ω_{c1}
- e) f_{c1}

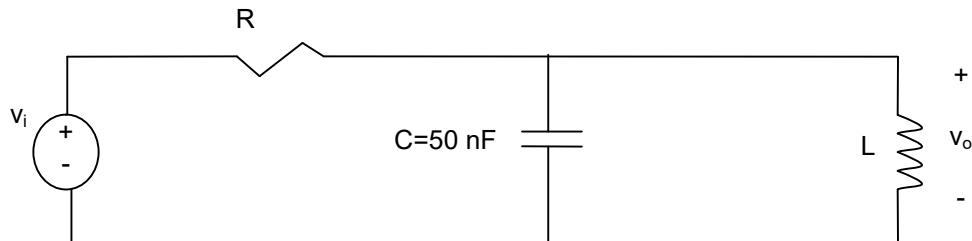
- f) ω_{c2}
 g) f_{c2}
 h) β



Solution:

6S. Using a 50 nF capacitor in the following bandpass circuit, design a filter with a quality factor of 5 and a center frequency of 20 krad/s.

- a) Specify the numerical values of R and L.
 b) Calculate the upper and lower cutoff frequencies in kilohertz.
 c) Calculate the bandwidth in hertz.



Solution:

a)

Given: $Q=5$, $\omega_o = 20 \text{ krad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} \rightarrow L = \frac{1}{\omega_o^2 C} = \frac{1}{(2 \times 10^4)^2 (50 \times 10^{-9})} = 50 \text{ mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{1/RC} \rightarrow R = \frac{Q}{\omega_o C} = \frac{5}{(2 \times 10^4)(50 \times 10^{-9})} = 5 \text{ k}\Omega$$

b)

$$\omega_{c1} = \omega_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q} \right)^2} \right] = 18.10 \text{ krad/s}$$

$$f_{c1} = \frac{\omega_{c1}}{2\pi} = 2.88 \text{ kHz}$$

$$\omega_{c2} = \omega_o \left[+\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q} \right)^2} \right] = 22.10 \text{ krad/s}$$

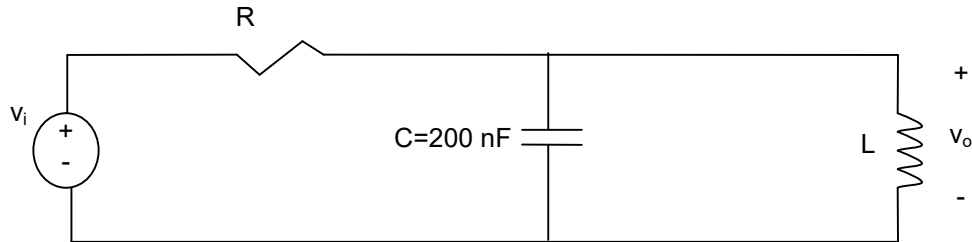
$$f_{c2} = \frac{\omega_{c2}}{2\pi} = 3.52 \text{ kHz}$$

c)

$$\beta = \frac{\omega_0}{Q} = 4000 \text{ rad/s} \quad \text{or} \quad 636.62 \text{ Hz}$$

6U. Using a 200 nF capacitor in the following bandpass circuit, design a filter with a quality factor of 10 and a center frequency of 5 krad/s.

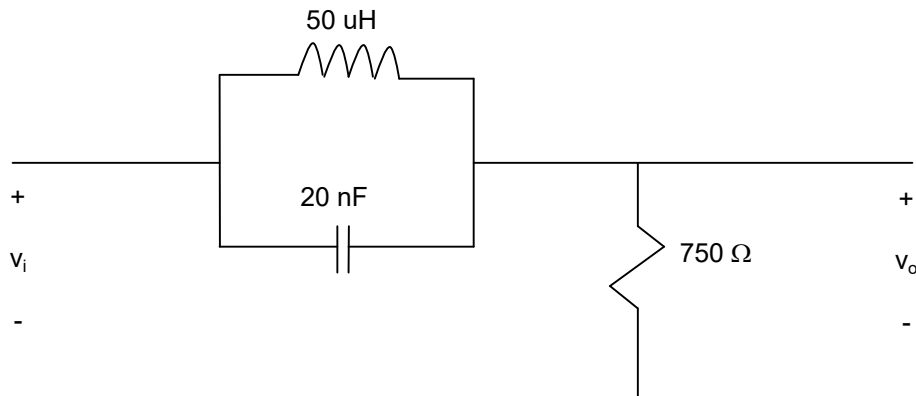
- Specify the numerical values of R and L.
- Calculate the upper and lower cutoff frequencies in kilohertz.
- Calculate the bandwidth in hertz.



Solution:

7S. For the bandreject filter in the following figure, calculate

- ω_0
- f_0
- Q
- ω_{c1}
- f_{c1}
- ω_{c2}
- f_{c2}
- β in kilohertz



Solution:

a)

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(50 \times 10^{-6})(20 \times 10^{-9})}} = 10^6 \text{ rad/s}$$

b)

$$f_0 = \frac{\omega_0}{2\pi} = \frac{10^6}{2\pi} = 159.15 \text{ kHz}$$

c)

$$Q = \frac{\omega_o}{\beta} = RC\omega_o = (750)(20 \times 10^{-9})(10^6) = 15$$

d)

$$\omega_{c1} = \omega_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q} \right)^2} \right] = 967.22 \text{ krad / s}$$

e)

$$f_{c1} = \frac{\omega_{c1}}{2\pi} = 153.94 \text{ kHz}$$

f)

$$\omega_{c2} = \omega_o \left[+\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q} \right)^2} \right] = 1.03 \text{ Mrad / s}$$

g)

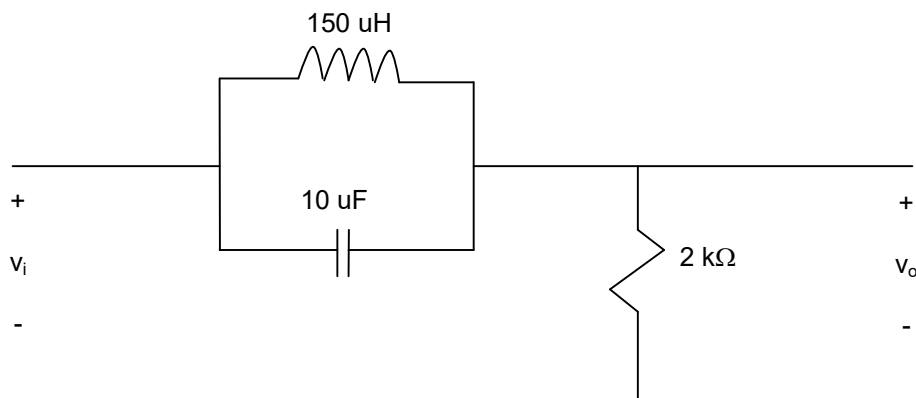
$$f_{c2} = \frac{\omega_{c2}}{2\pi} = 164.55 \text{ kHz}$$

h)

$$\beta = f_{c2} - f_{c1} = 164.55 - 153.94 = 10.61 \text{ kHz}$$

7U. For the bandreject filter in the following figure, calculate

- ω_o
- f_o
- Q
- ω_{c1}
- f_{c1}
- ω_{c2}
- f_{c2}
- β in kilohertz

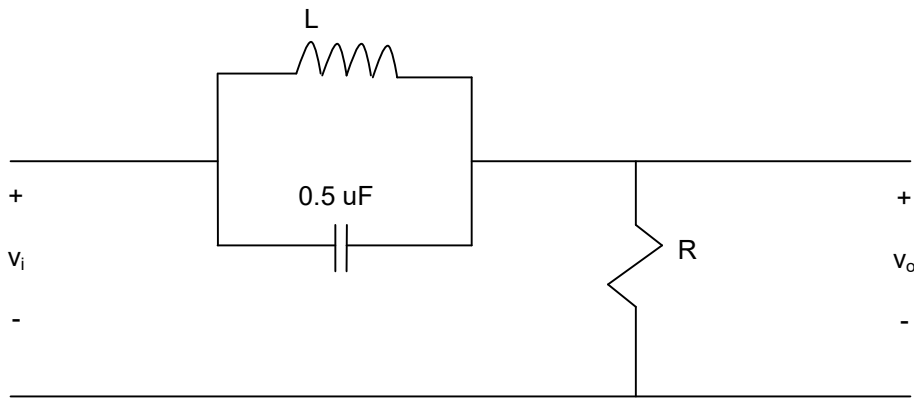


Solution:

8S. Use a $0.5 \mu\text{F}$ capacitor to design a bandreject filter shown below. The filter has a center frequency of 4 kHz and a quality factor of 5.

- Specify the numerical values of R and L .

- b) Calculate the upper and lower corner, or cutoff, frequencies in kilohertz.
 c) Calculate the filter bandwidth in hertz.



Solution:

a)

$$f_o = 4 \text{ krad/s} \rightarrow \omega_o = 2\pi f_o = 8\pi \text{ krad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} \rightarrow L = \frac{1}{\omega_o^2 C} = \frac{1}{(8000\pi)^2 (0.5 \times 10^{-6})} = 3.17 \text{ mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{1/RC} \rightarrow R = \frac{Q}{\omega_o C} = \frac{5}{(8000\pi)(0.5 \times 10^{-6})} = 397.89 \Omega$$

b)

$$f_{c1} = f_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = 3.62 \text{ kHz}$$

$$f_{c2} = f_o \left[+\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = 4.42 \text{ kHz}$$

c)

$$\beta = f_{c2} - f_{c1} = 800 \text{ Hz}$$

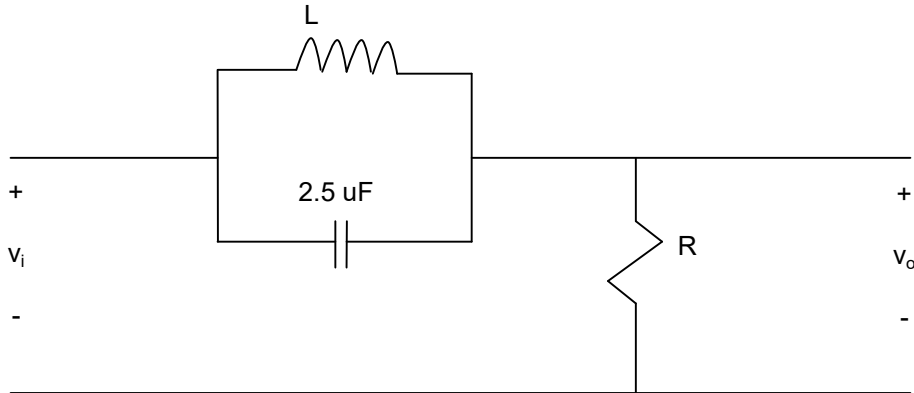
or

$$\beta = \frac{f_o}{Q} = 800 \text{ Hz}$$

8U. Use a $2.5 \mu F$ capacitor to design a bandreject filter shown below. The filter has a center frequency of 10 kHz and a quality factor of 10.

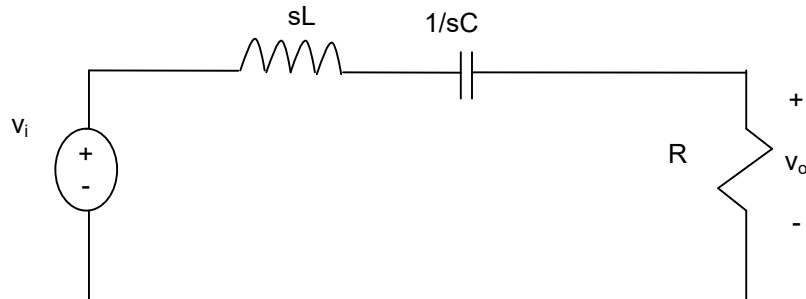
- a) Specify the numerical values of R and L .
 b) Calculate the upper and lower corner, or cutoff, frequencies in kilohertz.

c) Calculate the filter bandwidth in hertz.



Solution:

9S. Design a series RLC bandpass filter for detecting the low-frequency tone generated by pushing a telephone button shown below:



$$H(s) = \frac{(R/L)s}{s^2 + (R/L)s + 1/LC} \quad \omega_o = \sqrt{1/LC} \quad \beta = R/L$$

The Frequency generated by telephone buttons (Dual Tone Multiple Frequency – DTMF) shown below:

1	ABC 2	DEF 3	697 Hz	← Low Frequency Group
GHI 4	JKL 5	MNO 6	770 Hz	
PRS 7	TUV 8	WXY 9	852 Hz	
*	OPER 0	#	941 Hz	
1209 Hz 1336 Hz 1477 Hz			High-Frequency Group	

- Calculate the values of L and C that place the cutoff frequencies at the edges of the DTMF low-frequency band. Note that the resistance in standard telephone circuits is always $R=600 \Omega$.
- What is the output amplitude of this circuit at each of the low-band frequencies, relative to the peak amplitude of the bandpass filter?
- What is the output amplitude of this circuit at the lowest of the high-band frequencies?

Solution:

a)

Set the cutoff frequencies (f_{c1} and f_{c2}) at the edges of the low frequency group, therefore:

$$\omega_{c1} = 2\pi(697) = 4379.38 \text{ rad/s}$$

$$\omega_{c2} = 2\pi(941) = 5912.48 \text{ rad/s}$$

Thus

$$\beta = \omega_{c2} - \omega_{c1} = 1533.10 \text{ rad/s}$$

$$\beta = R/L \rightarrow L = \frac{R}{\beta} = \frac{600}{1533.10} = 0.39H$$

$$\omega_o^2 = (\omega_{c1} * \omega_{c2})$$

$$\omega_o = \sqrt{\frac{1}{LC}} \rightarrow C = \frac{1}{(\omega_o)^2 L} = \frac{1}{(5912.48)(4379.38)(0.39)} = 0.1 \mu F$$

b) At the outmost two frequencies in the low-frequency group ($\omega_{c1} = 697 \text{ Hz}$ and $\omega_{c2} = 941 \text{ Hz}$) the amplitudes are:

$$|V_{697 \text{ Hz}}| = |V_{697 \text{ Hz}}| = \frac{|V_{peak}|}{\sqrt{2}} = 0.707 |V_{peak}|$$

The non cut-off frequency need to be calculated by using the following equations:

$$|V| = |V_{peakz}| \parallel H(j\omega) = |V_{peakz}| \frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + (\omega\beta)^2}}$$

Therefore

$$|V_{770 \text{ Hz}}| = |V_{peakz}| \frac{(4838.05)(1533.10)}{\sqrt{(5088.52^2 - 4838.05^2)^2 + ((4838.05)(1533.10))^2}} = 0.948 |V_{peakz}|$$

$$|V_{852 \text{ Hz}}| = |V_{peakz}| \frac{(5353.27)(1533.10)}{\sqrt{(5088.52^2 - 5353.27^2)^2 + ((5353.27)(1533.10))^2}} = 0.948 |V_{peakz}|$$

It is not a coincidence that these two magnitudes are the same. The frequencies in both bands of the DTMF system were carefully chosen to produce this type of predictable behavior with linear filters. In other words, the frequencies were chosen to be equally far apart with respect to the response produced by a linear filter. Most musical scales consist of tones designed with this same property – note intervals are selected to place the notes equally far apart. That is why the DTMF tones remind us of musical notes! Unlike musical scales, DTMF frequencies were selected to be harmonically unrelated, to lower the risk of misidentifying a tone's frequency if the circuit elements are not perfectly linear.

c) The high-band frequency closest to the low-frequency band is 1209 Hz. The amplitude of a tone with this frequency is:

$$|V_{1209 \text{ Hz}}| = |V_{peakz}| \frac{(7596.37)(1533.10)}{\sqrt{(5088.52^2 - 7596.37^2)^2 + ((7596.37)(1533.10))^2}} = 0.344 |V_{peakz}|$$

This is less than one half the amplitude of the signals with the low-band cutoff frequencies, ensuring adequate separation of the bands.

9U. Design a series RLC bandreject filter for eliminating the low-frequency tones (697-941 Hz) generated by pushing a telephone button shown below:

1	ABC 2	DEF 3	697 Hz	← Low Frequency Group
GHI 4	JKL 5	MNO 6	770 Hz	
PRS 7	TUV 8	WXY 9	852 Hz	
*	OPER 0	#	941 Hz	
1209 Hz	1336 Hz	1477 Hz	High-Frequency Group	

Solution: