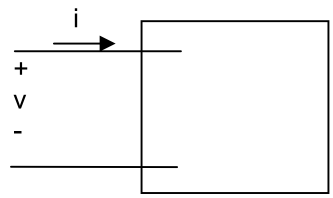
Fundamentals of Electrical Circuits - Chapter 10

1S. The following sets of values for v and I pertain to the circuit seen in following figure. For each set of values calculate P & Q and state whether the circuit inside the box is absorbing or delivering (1) average power and (2) magnetizing vars.

a) v = 100 cos(wt + 50°) V, i=10 cos(wt + 15°) A.
b) v = 40 cos(wt - 15°) V, i=20 cos(wt + 60°) A.
c) v = 400 cos(wt + 30°) V, i=10 sin(wt + 240°) A.
d) v = 200 sin(wt + 250°) V, i=5 cos(wt + 40°) A.



Solution:
We have

$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$ If Positive is absorbing; if negative is generating (Passive Convention)

$Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i)$

a)
$$P = \frac{1}{2}(100)(10)\cos(50-15) = 409.58\,W \quad (abs)$$
$$Q = \frac{1}{2}(100)(10)\sin(50-15) = 286.79\,VAR \quad (abs)$$

b)
$$P = \frac{1}{2}(40)(20)\cos(-15-60) = 103.52\ \ W \quad (abs)$$
$$Q = \frac{1}{2}(40)(10)\sin(-15-60) = -386.37\ \ VAR \quad (gen)$$

c) v = 400 cos(wt + 30°) V
i=10 sin(wt + 240°) A → I = 10 cos (wt + 240 -90) = 10 cos(wt +150)

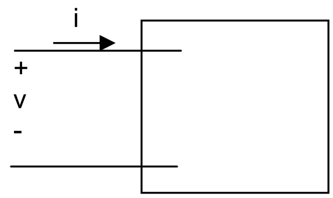
$$P = \frac{1}{2}(400)(10)\cos(30-150) = -1{,}000\,W \quad (gen)$$
$$Q = \frac{1}{2}(400)(10)\sin(30-150) = -1{,}732.05\,VAR \quad (gen)$$

d) v = 200 sin(wt + 250°) = 200 cos (wt +250 – 90) = 200 cos (wt + 160)
i=5 cos(wt + 40°) A.

$$P = \frac{1}{2}(200)(5)\cos(160-40) = -250\ \ W \quad (generating)$$
$$Q = \frac{1}{2}(200)(5)\sin(160-40) = +433.01\ \ VAR \quad (Consu\min g)$$

1U. The following sets of values for v and I pertain to the circuit seen in following figure. For each set of values calculate P & Q and state whether the circuit inside the box is absorbing or delivering (1) average power and (2) magnetizing vars.

a) $v = 100\cos(wt + 90^{o})$ V, $i=20\sin(wt - 30^{o})$ A.
b) $v = 20\sin(wt - 45^{o})$ V, $i=60\cos(wt + 30^{o})$ A.
c) $v = 45\cos(wt -120^{o})$ V, $i=15\sin(wt + 240^{o})$ A.
d) $v = 100\cos(wt + 250^{o})$ V, $i=5\cos(wt + 40^{o})$ A.



2S. You are drying your hair, sitting under a sunlamp and watching a basketball game on television. At the same time your housemate is vacuuming the rug in your air-conditioned house. All these appliances are supplied from a 120 V branch circuit protected by a 15 A circuit breaker with the following power requirements:

Product	Average Power usage (Watts)
Hair dryer	600
Television (tub-type)	240
Vacuum Cleaner	630
Sunlamp	279
Air condition (room)	860

Use the above information to answer the following questions:

a) Will the breaker interrupt the game?
b) Will you be able to watch television if you turns off the sunlamp and turn off the vacuum cleaner?

Solution:

a) Total Power being used = P{hair dryer) + P{sunlamp} + P{Television} + P{Vacuum} + P{Air Condition}
= 600+240+630+279+860 = 2609 W
I_{eff} = Total Power / V_{eff} = 2609 / 120 = 21.74 A
Since I_{eff} needed is more than Circuit Breaker (15 A) would allow, the game will be interrupted.

b) Total Power being used = P{hair dryer) + P{Television} + P{Air Condition}
= 600+240+860 = 1700 W
I_{eff} = Total Power / V_{eff} = 1700 / 120 = 14.2 A
Since I_{eff} needed is less than Circuit Breaker (15 A) would allow, the game will not be interrupted.

2U. You are working on your computer while the television is one, and heating your dinner in the microwave. At the same time your housemate is vacuuming the rug in your air-conditioned house. All these appliances are supplied from a 120 V branch circuit protected by a 20 A circuit breaker with the following power requirements:

Product	Average Power usage (Watts)
Microwave	2000
Television	120
Vacuum Cleaner	700
Light	150
Air condition	1500
Computer System	160

Use the above information to answer the following questions:

a) Will you exceed the circuit breaker's capacity and loose all your unsaved work on the computer?
b) Will you be able to continue working on computer if you turns off the microwave and vacuum cleaner?

Solution:

3S. A personal Computer with a monitor and keyboard requires 40 W at 115 V (rms).
a) Calculate the rms value of the current carried by its power cord.
b) A laser printer for the personal computer in (a) is rated at 90 W at 115 V(rms). If this printer is plugged into the same wall outlet as the computer what is the rms value of the current drawn from the outlet.

Solution:

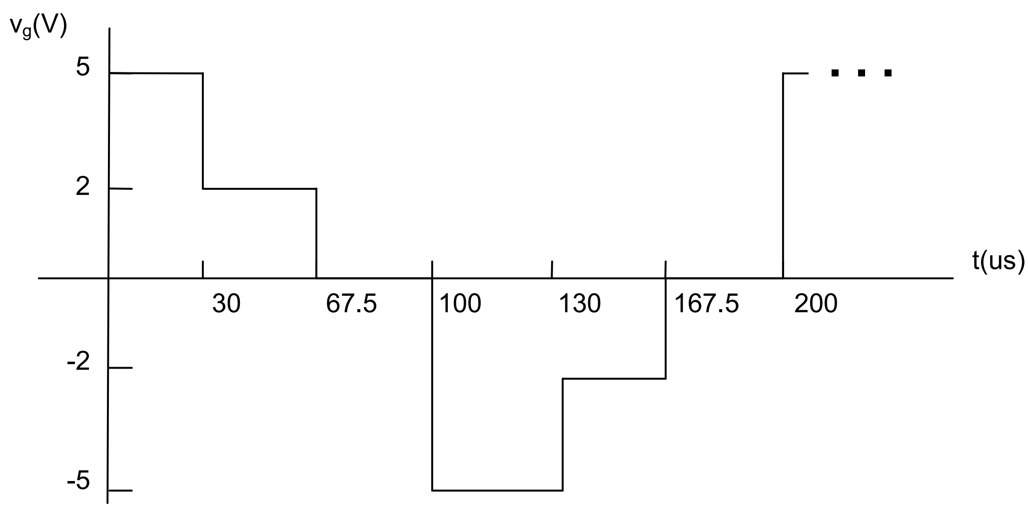
a) $I_{eff\ PC} = P / V_{eff} = 40 / 115 = .35$ A

b) $I_{eff\ printer} = P / V_{eff} = 90 / 115 = .78$ A
$Ieff\ outlet = I_{eff\ PC} + I_{eff\ printer} = .35 + .78 = 1.13$ A

3U. A Smart phone chrager uses maximum of 100 mW at 115 V (rms).
a) Calculate the rms value of the current carried by its power cord.
b) A bluetooth-enabled laser printer is connected to the same outlet while the smart phone is being charged. Printer is rated at 90 W at 115 V(rms). What is the rms value of the current drawn from the outlet.

Solution:

4S. For the following Signal:



a) Find the rms value of the periodic voltage.
b) If the voltage is applied to the terminals of a 2.25 Ohm resistor, what is the average power dissipated by the resistor.

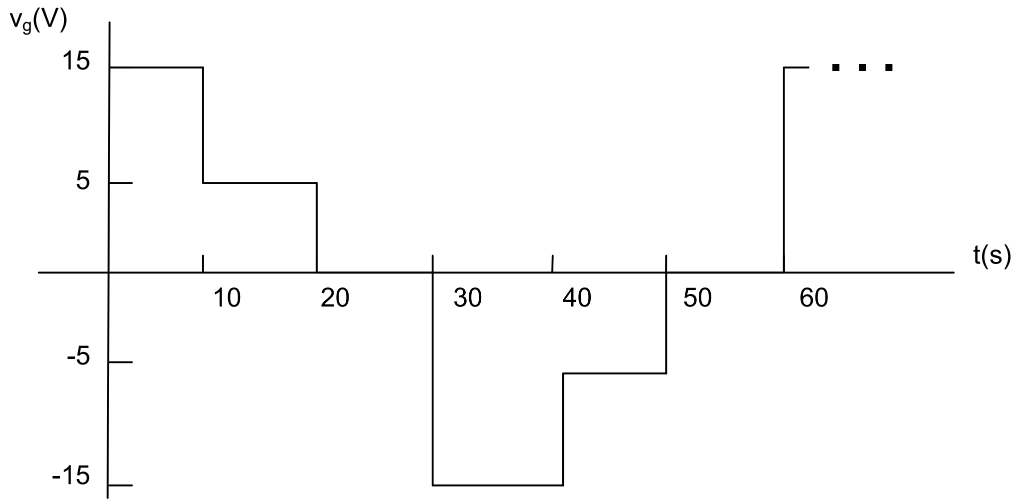
Solution:

a)

$$Vrms = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)dt} = \sqrt{\frac{2}{200}\left\{\int_0^{30} 5^2\,dt + \int_{30}^{67.5} 2^2\,dt + 0\right\}} = \sqrt{\frac{2}{200}\{25(30-0)+4(67.5-30)+0\}} = 3\,V$$

b) $P = V^2_{eff}/R = (3)^2/2.25 = 4$ W

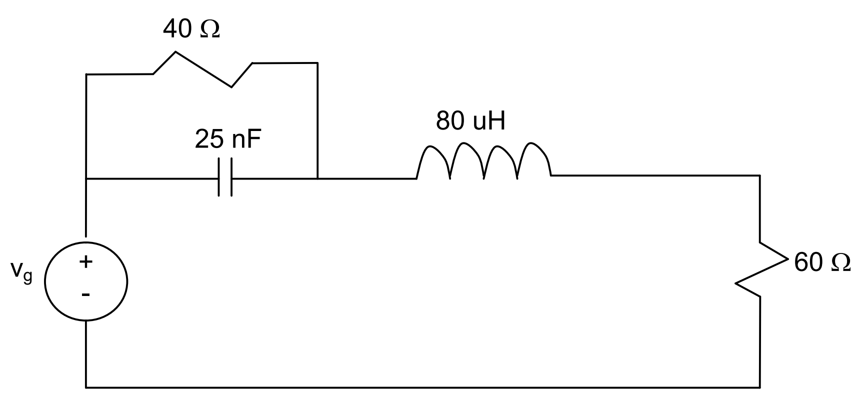
4U. For the following Signal:



a) Find the rms value of the periodic voltage.
b) If the voltage is applied to the terminals of a 10 Ohm resistor, what is the average power dissipated by the resistor.

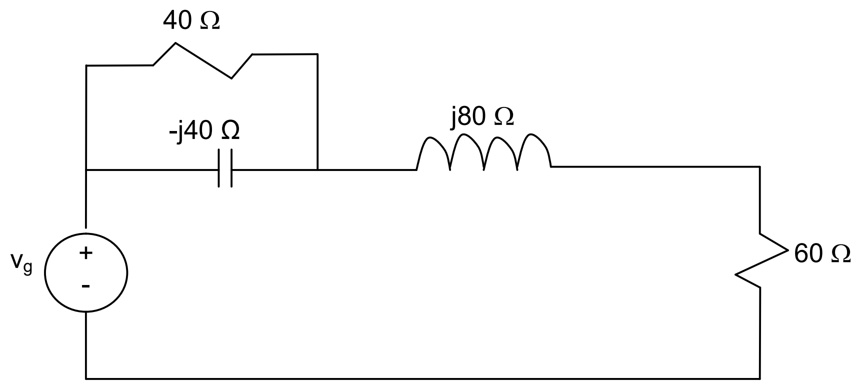
Solution:

5S. The following Circuit is powered by v_g = 40 cos10^6t V.



a) Find the average power, the reactive power and the apparent power supplied by the voltage source in the following circuit (Steady State).
b) Check your answer in (a) by showing magnitudes of P_{dev} and Σ P_{abs}. are equal.
c) Check your answer in (a) by showing magnitudes of Q_{dev} and Σ Q_{abs}.are equal.

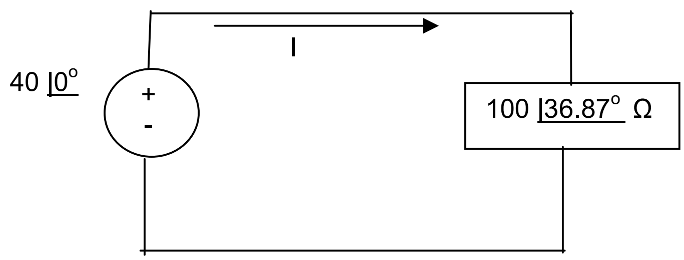
Solution:
a)
Step 1) Redraw the circuit in Frequency domain (R→R, C→-j/wC, L → jwL where w = 106)



Step 2) Find

Zeq seen by the supply = (40 || -j40) + j80 + 60 = 80 + j60 = $\sqrt{80^2+60^2} \mid \underline{\tan^{-1} 60/80}$

Zeq = 100$\underline{|36.87^o}$ Ω



$$I = \frac{V}{Z} = \frac{40 \mid \underline{0^o}}{100 \mid \underline{36.87^o}} = .4 \mid \underline{-36.87^o}$$

$$P = -\frac{1}{2}(40)(.4)\cos(0-(-36.87)) = -6.4\, W(del)$$

$$Q = -\frac{1}{2}(40)(.4)\sin(0-(-36.87)) = -4.8\, VAR(del)$$

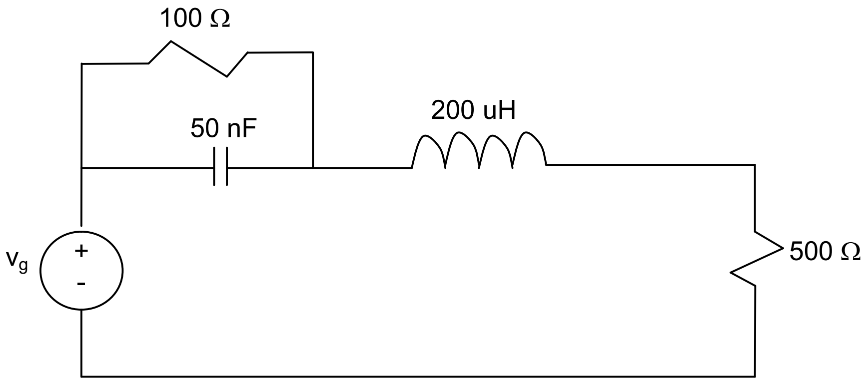
Complex Power = S = P + jQ = 6.4 + j4.8 VA

Apparent Power = |S| = $\sqrt{6.4^2+4.8^2} = 8\, VA$

b & c) Now the absorbed side:

S =$|I_{eff}|^2$Z = (1/2) $|I_m|^2$ Z = $(.4^2)$(80 + j60) = 6.4 + j 4.8 = P + jQ → P=6.4 W; Q=4.8 VA

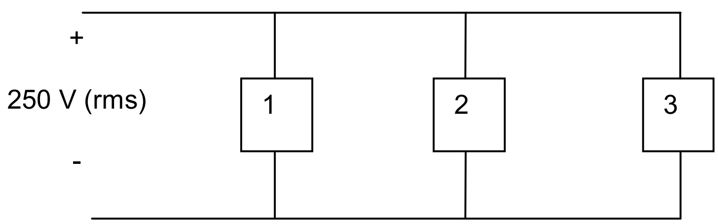
5U. The following Circuit is powered by v_g = 60 cos2000πt V.



a) Find the average power, the reactive power and the apparent power supplied by the voltage source in the following circuit if.
b) Check your answer in (a) by showing magnitudes of P_{dev} and Σ P_{abs}. are equal.
c) Check your answer in (a) by showing magnitudes of Q_{dev} and Σ Q_{abs}.are equal.

Solution:

6S. Three loads are connected in parallel across a 250 V (rms) line, as shown in the following figure. Load 1 absorbs 16 kW and 18 kVAR. Load 2 absorbs 10 kVA at 0.6 pF lead. Load 3 absorbs 8 kW at unity power factor.



a) Find the impedance that is equivalent to three parallel loads.
b) Find the power factor of the equivalent load as seen from the line's input terminals.

Solution:
a) Given:
V_{rms} =V_{eff} = 250 V
S1 = 16 + j18 kVA
|S2| = 10 kVA at pf2 = cos (θv – θi) = 0.6 → |sin(θv – θi)| = $\sqrt{1-(0.6)^2}$ =0.8
since it PF leads (current leads θv – θi <0) → the rpf = sin(θv – θi)= - 0.8
{Note: used sin2x + cos2x = 1 to find rpf}
P2=|S2| cos (θv – θi) = 10 * .6 = 6 kW
Q2= |S2| sin (θv – θi) = 10 *(- .8) = -8 kVAR
S2 = P + JQ = 6 - j8 kVA

|S3| = 8 kW at pf =1 → rpf = 0
S3 = 8 kVA

S_{total} = S1 + S2 + S3 = 30 + j10 kVA
using

$$S = \frac{|V_{eff}|^2}{Z^*} \Rightarrow Z^* = \frac{|V_{eff}|^2}{S}$$

$$Z^* = \frac{250^2}{1000(30+j10)} = \frac{62.5}{\sqrt{30^2+10^2}\ |\underline{\tan^{-1}(10/30)}} = \frac{62.5}{31.62\,|\underline{18.43^o}} = 1.98\,|\underline{-18.43^o} = 1.89 - j.63$$

$$Z = 1.88 + j.63 = 1.98\,|\underline{18.43^o}$$

b) S_{total} = 30 + j10 kVA
Power factory angle = (θv – θi) = $\tan^{-1}$ (10/30) =18.43°
Power factor = cos (power factor angle = cos (18.43°) = .9487
Lagging since (θv – θi)>0 meaning or Current lags the voltage

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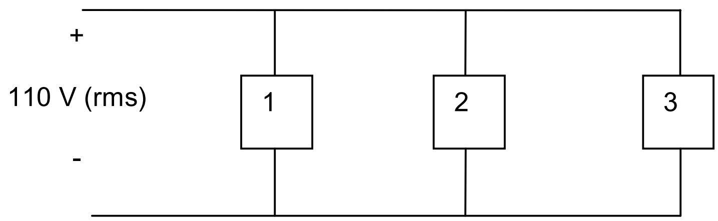
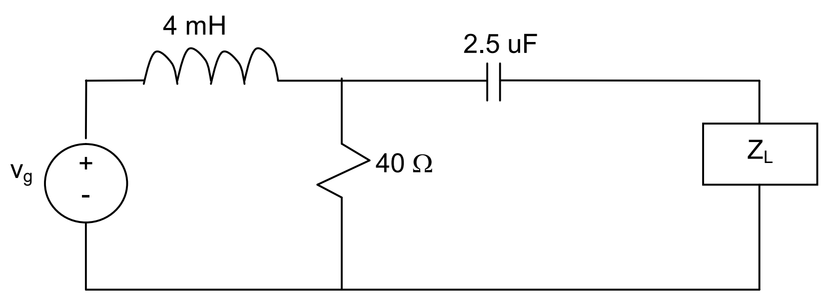
6U. Three loads are connected in parallel across a 110 V (rms) line, as shown in the following figure. Load 1 absorbs 5 kW and 25 kVAR. Load 2 absorbs 20 kVA at 0.5 pf lagging. Load 3 absorbs 15 kW at unity power factor.



a) Find the impedance that is equivalent to three parallel loads.
b) Find the power factor of the equivalent load as seen from the line's input terminals.

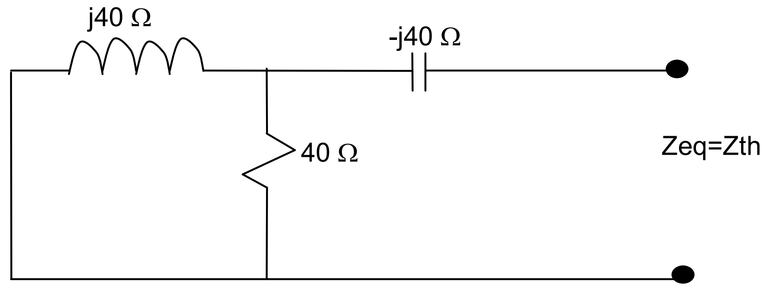
Solution:

7S. a) Determine the load impedance for the following circuit that will result in maximum average power being transferred to the load if w = 10 krad/s.



b) Determine the maximum average power if vg = 120 cos 10,000t V.

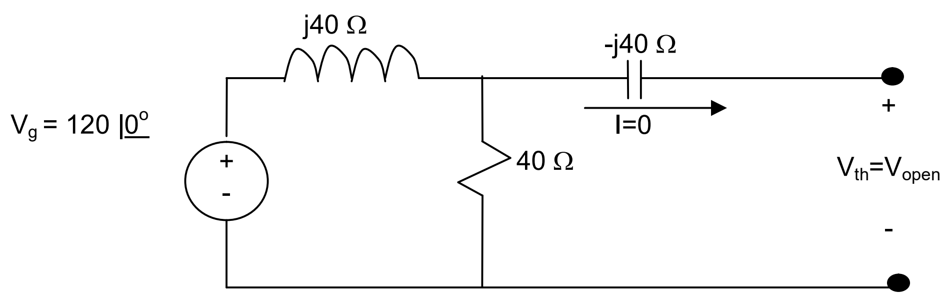
Solution:
a) First need to find the thevenin equivalent with respect to the load (Z_L)
Step 1) Redraw the circuit in Frequency domain (R→R, C→-j/wC, L → jwL where w = 10^4)
Step 2) Deactivate the supply (v=0 or short) to find the Zeq =Zth



Zth = (40 || j40) –j40 = 20 -j20 = 28.28 $\underline{|-45^o}$ Ω

Maximum power is transferred when Z_L = Zth* = 20 + j20 = 28.28 $\underline{|45^o}$ Ω

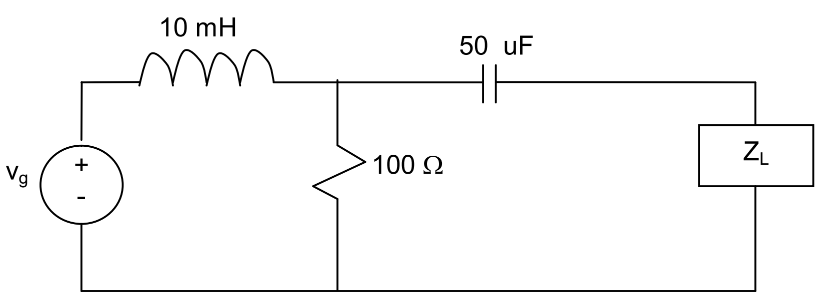
b) Now we have to first find the find Vopen = Vth



V_{th} = {120/(40 + j40)} 40 = 480 / (4 + j4) = 480 / (5.65 $\underline{|45^o}$) = 84.96 $\underline{|-45}$

Pmax = $(Veff)^2/(4R_L)$=(1/8) ($V^2{}_m$)/(R_L) = (1/8)$(84.96)^2$/(20) = 45.11 W

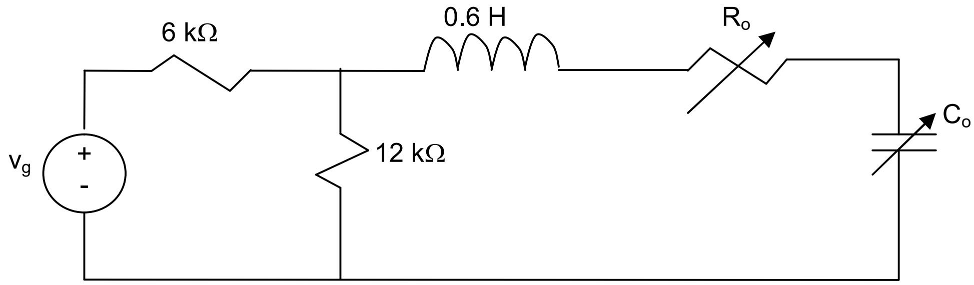
7U. a) Determine the load impedance for the following circuit that will result in maximum average power being transferred to the load if w = 20 krad/s.



b) Determine the maximum average power if vg = 50 cos 20,000t V.

Solution:

8S. The peak amplitude of the sinusoidal voltage source in the circuit shown in following circuit is 180 V, and its frequency is 5000 rad/sec. The load resistor can be varied from 0 to 4000Ω, and the load capacitor can be varied from 0.1 to 0.5 uF.



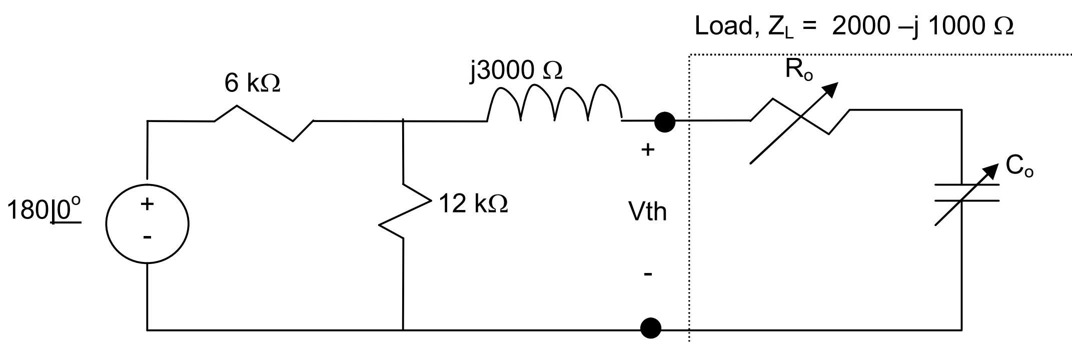
a) Calculate the average power delivered to the load when R_o = 2000 Ω and C_o = 0.2 uF.
b) Determine the settings of R_o and C_o that will result in the most average power being transferred to R_o.
c) What is the most average power in (b)? Is it greater then power in (a)?
d) If there are no constraints on R_o and C_o, what is the maximum average power that can be delivered to a load?
e) What are the values of Ro and Co for the condition of (d)?
f) Is the average power calculated in (d) larger than that calculated in (c)?

Solution:

a) First need to find the thevenin equivalent with respect to the load (Z_L)

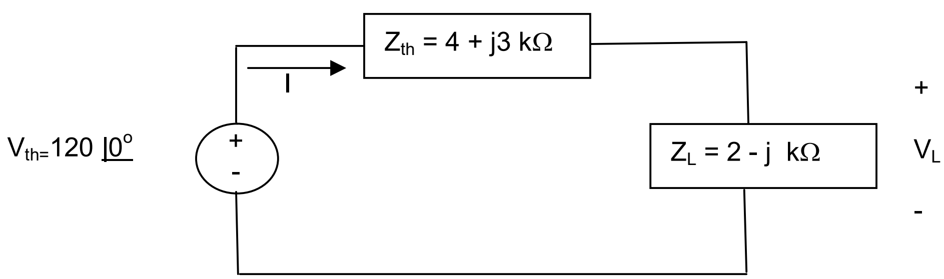
Step 1) Redraw the circuit in Frequency domain (R→R, C→-j/wC, L → jwL where w = 5,000)

Step 2) Deactivate the supply (v=0 or short) to find the Zeq =Zth



Z_{th} = (6 || 12) + j3 = 4 + j3 kΩ = $5\underline{|36.87^o}$ kΩ

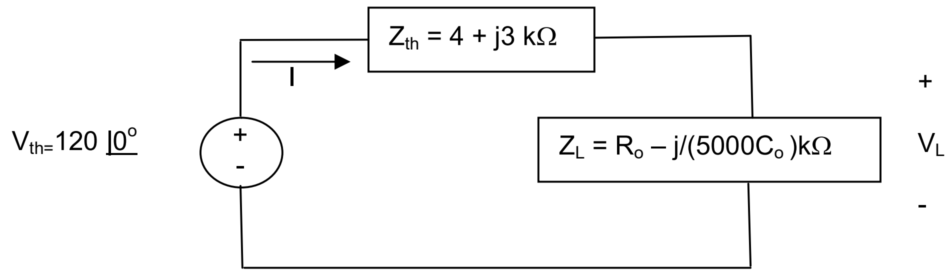
V_{th} = $\left(\frac{180}{6+12}\right)12 = 120\,\underline{|\,0^o}\ V$



I = $(120)\left(\frac{1}{4+j3+2-j}\right) = \frac{120}{6+2j} = \frac{120}{\sqrt{36+4}\,\underline{|\tan^{-1}2/6}} = 18.97\,\underline{|-18.43^o}\ mA$

P = average power = $|I_{eff}|^2R_L$ = $(1/2)|I_m|^2R_L$ = $(1/2)(18.97)^2(2000)$ = 360 W

b) We can leverage last section since the Thevenin equivalent will be the same only the load is different



The condition for maximum power transfer is $Z_{th} = Z_L^*$ therefore
4 + j3 = R_o + j/5000C_o → Ro = 4 kΩ
&
1/(5000)C_o = 3 kΩ → Co = 66.67 nf (Violates 0.1-0.5 uF restriction)

c) The restrictions that the load resistor can be varied from 0 to 4000Ω, and the load capacitor can be varied from 0.1 to 0.5 uF Leads to setting :
Z_L = 4000 –j/(5000)(.1E-6) → Z_L = 4 –j2

$$I = (120)\left(\frac{1}{4+j3+4-j2}\right)=\frac{120}{8+1j}=\frac{120}{\sqrt{64+1}\,\underline{|\tan^{-1}1/8}}=14.88\,\underline{|-7.13^o}\ mA$$

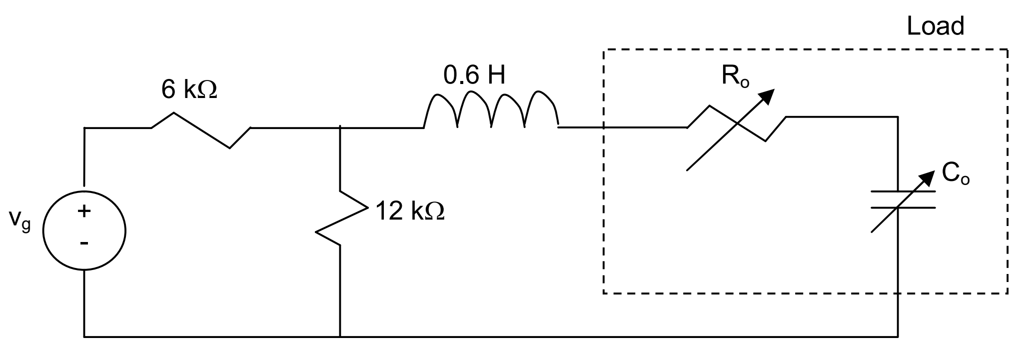
P = average power = $|I_{eff}|^2R_L$ = (1/2)$|I_m|^2R_L$ = $(1/2)(14.88)^2(4000)$ = 443 W which is greater than 360 W in part (a)

d) When there are no constraint we can use the values from b to find maximum average power transfer
Pmax = $(1/8)(V^2_{m\text{-}th}/R_L)=(1/8)(120^2)(1/4000)$ = 450 W

e) Same as derived in part b.

f) Yes.450 W > 443 W as expected.

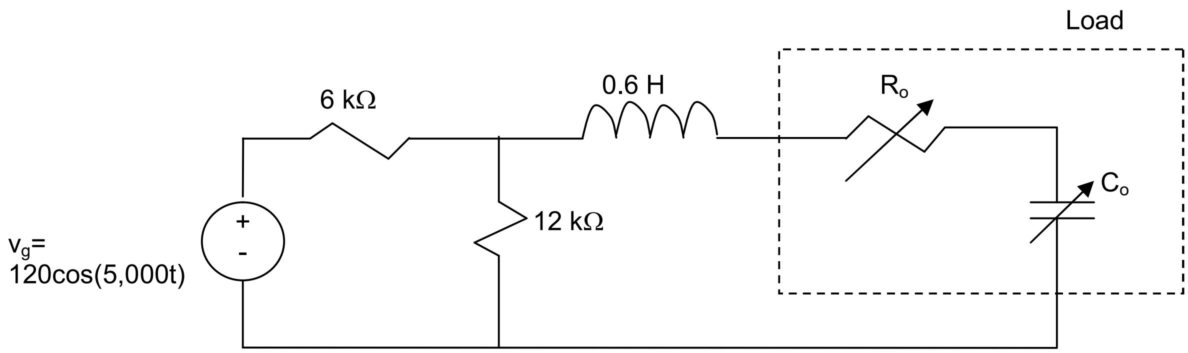
8U. The peak amplitude of the sinusoidal voltage source in the circuit shown in following circuit is 2 V, and its frequency is 20,000π rad/sec. The load resistor can be varied from 1 to 40 KΩ, and the load capacitor can be varied from 0.2 to 2.0 uF.



a) Calculate the average power delivered to the load when R_o = 20 kΩ and C_o = 1.1 uF.
b) Determine the settings of R_o and C_o that will result in the most average power being transferred to R_o.
c) What is the most average power in (b)? Is it greater then power in (a)?
d) If there are no constraints on R_o and C_o, what is the maximum average power that can be delivered to a load?
e) What are the values of Ro and Co for the condition of (d)?
f) Is the average power calculated in (d) larger than that calculated in (c)?

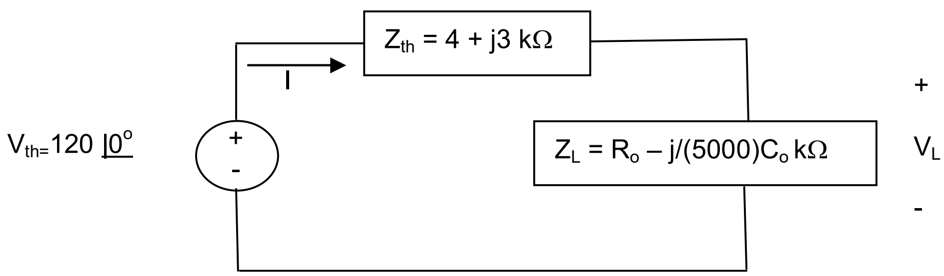
Solution:

9S. Assuming that R_o can be varied between 0 and 10 kΩ, determine the settings of R_o and C_o that will result in the most average power being transferred to load. Also, calculate the Maxium Average Power delivered to load.



Solution:

We can leverage last problem since the Thevenin equivalent will be the same only the load is different



The condition for maximum power transfer is $Z_{th} = Z_L^*$ therefore
4 + j3 = R_o + j/5C_o → Ro = 4 kΩ & 1/(5000)C_o = 3 kΩ → Co = 66.67 nf

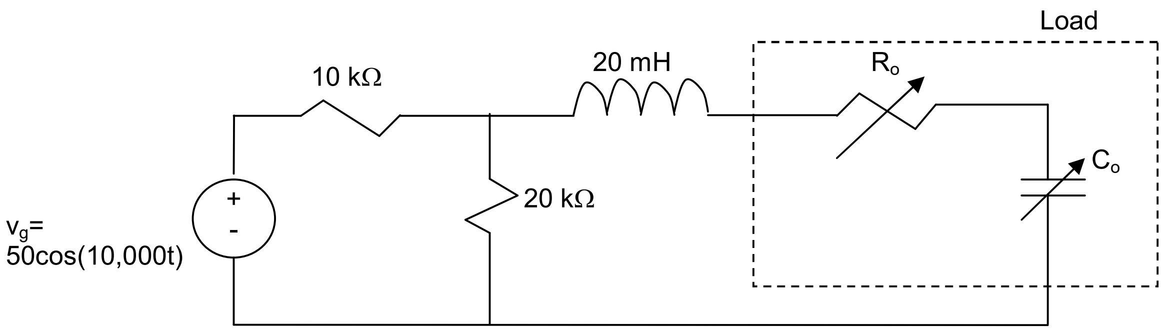
But we have a limitation that the lowest value of cap→ Co = 0.1 uf → -j2 kΩ
which means we can not do anymore to reach the maximum transfer condition $X_L = - X_{th}$ But we may have opportunity to work on $dP/dR_L = 0$ condition →

$$R_L = \sqrt{R^2{}_{th} + (X_{th} + X_L)^2} = \sqrt{4^2 + (-2+3)^2} = 4.12\ \Omega$$ → Z_L = 4.12 – j2 kΩ

$$I = (120)\left(\frac{1}{4 + j3 + 4.12 - j2}\right) = \frac{120}{8.12 + 1j} = \frac{120}{\sqrt{8.12^2 + 1}\ \underline{|\tan^{-1} 1/8.12}} = 14.66\ \underline{|-7.02^o}\ mA$$

P = average power = $|I_{eff}|^2 R_L$ = $(1/2)|I_m|^2 R_L$ = $(1/2)(14.66)^2(4.12)$ = 442.7 mW

9U. Assuming that R_o can only be varied between 0 and 30 kΩ, determine the settings of R_o and C_o that will result in the most average power being transferred to R_o. Also, calculate the Maxium Average Power delivered to load.



Solution: