Fundamentals of Electrical Circuits - Chapter 9

1S. Consider the Sinusoidal Voltage → $v(t) = 40 \cos(100\pi t + 60^o)$ V
a) What is the Maximum Amplitude of the Voltage?
b) What is the frequency in Hertz?
c) What is the frequency in Radians per second?
d) What is the phase angle in radians?
e) What is the phase angle in degrees?
f) What is the period in milliseconds?
g) What is the first time after t=0 that v=-40 V?
h) The sinusoidal function is shifted 10/3 ms to the right along the time axis. What is the expression for v(t)?
i) What is the minimum number of milliseconds that the function must be shifted to the right if the expression for v(t) is $40 \sin(100\pi t)$
j) What is the minimum number of milliseconds that the function must be shifted to the left if the expression for v(t) is $40 \cos(100\pi t)$ V.

Solution:

a) Vmax = 40 V

b) $w = 2\pi f = 100\pi$ → $f=(100\pi)/2\pi = 50$ Hz

c) $w = 100\pi$ rad./Sec.

d) $\Phi = (60^o)(\pi \text{ rad}/180^o) = \pi/3$

e) $\Phi = (60^o)$

f) T=1/f = 1/50 = 20 ms

g) t=? where $v(t) = -40 = 40\cos(100\pi t + 60^o)$
$\cos(100\pi t + 60^o) = -1 = \cos((2k+1)\pi)$ where k = 0, 1, 2, 3, …
$100\pi t + \pi/3 = (2k+1)\pi$ → $t = (2k + 1 - 1/3)/100 = (6k + 2)/300$
first time after t=0, v(t) =40 at t = 2/300 (K=0)

h) Δt = 10/3 ms "shift to the right on time axis"
$\Delta\Phi = \Delta t * w = (1/300 \text{ s})(100\pi \text{ rad/s}) = \pi/3$
$v_{shifted}(t) = 40\cos(100\pi t + \pi/3 - \pi/3) = 40\cos(100\pi t)$

i) We need $\Delta\Phi = -\pi/2 - \pi/3 = -5\pi/6$ (to convert sine to cosine and then shift to zero)
Now convert to time from radian
$\Delta t = \Delta\Phi / w = (-5\pi/6) / (100\pi) = -5/600 = -8.33$ ms (negative indicates shift to the right)

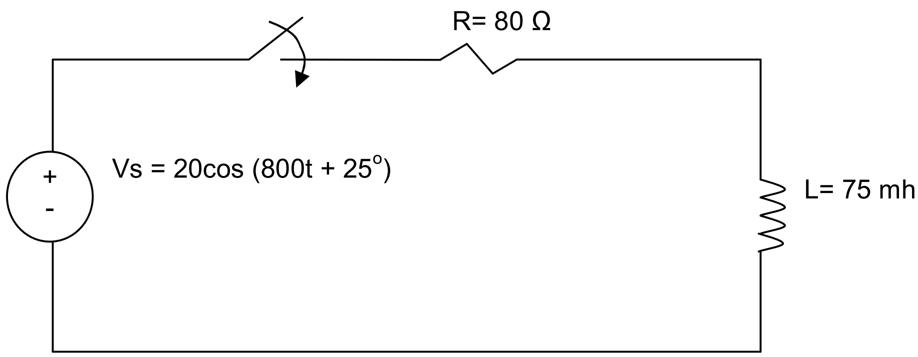
j) We need go forward by 2π to find the left shift in other word add 2π to the $\Delta\Phi$ in section i.
$\Delta\Phi = 2\pi - 5\pi/6 = 7\pi/6$ (to convert cosine to sine and then shift to zero)
Now convert to time from radian
$\Delta t = \Delta\Phi / w = (7\pi/6) / (100\pi) = 7/600 = 11.67$ ms (positive indicates shift to the left)

1U. Consider the Sinusoidal Voltage → $v(t) = 15 \cos(250\pi t + 75^o)$ V
a) What is the Maximum Amplitude of the Voltage?
b) What is the frequency in Hertz?
c) What is the frequency in Radians per second?
d) What is the phase angle in radians?
e) What is the phase angle in degrees?
f) What is the period in milliseconds?
g) What is the first time after t=0 that v=-10 V?

h) The sinusoidal function is shifted 10/3 ms to the right along the time axis. What is the expression for v(t)?
i) What is the minimum number of milliseconds that the function must be shifted to the right if the expression for v(t) is 15 sin(250πt)
j) What is the minimum number of milliseconds that the function must be shifted to the left if the expression for v(t) is15cos(250πt) V.

Solution:

2S. The Voltage applied to the circuit shown below at t=0 is 20cos (800t + 25°). The circuit resistance is 80 Ω, and the initial current in the 75 mh inductor is zero.
a) Find i(t) for t ≥ 0.
b) Write the expression for the transient and steady state component of i(t).
c) Find the numerical value of I after the switch has been closed for 1.875 ms.
d) What are the maximum amplitude, frequency (rad./s), and phase angle of the steady state current?
e) By how many degrees are the voltage and steady state current out of phase?



Solution:

a) Since Here we are asked to find the total response i(t), we can use the following equation where the first term is the transient component and second term ins the steady state component.

$$i(t) = \frac{-Vm}{\sqrt{R^2 + w^2L^2}}\cos(\phi - \theta)e^{-(R/L)t} + \frac{Vm}{\sqrt{R^2 + w^2L^2}}\cos(wt + \phi - \theta) \quad where \quad \theta = \tan^{-1}(wL/R)$$

$$w = 800, \quad \phi = 25^o, \quad Vm = 20 \quad V$$

$$\theta = \tan^{-1}(800 * 0.075/80) = 36.87^o$$

$$i(t) = \frac{-20}{\sqrt{80^2 + 800^2(.075)^2}}\cos(25^o - 36.87^o)e^{-(80/0.075)t} + \frac{20}{\sqrt{80^2 + 800^2(.075)^2}}\cos(800t + 25^o - 36.87^o)$$

$$i(t) = -0.2\cos(-11.87)e^{-1066.67t} + 0.2\cos(800t - 11.87)$$

b)

Transient part of i(t) = $i(t) = -0.2\cos(-11.87^o)e^{-1066.67t}$

Steady State part of i(t) = $+0.2\cos(800t - 11.87^o)$

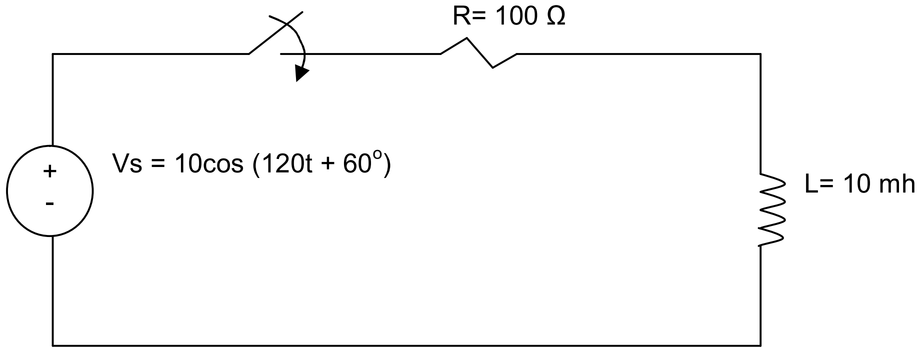
c) Numerial value of I(t=1.875 ms)
i(1.875 ms) = -0.2cos(-11.87°)e$^{-(1066.67*1.875/1000)}$ + 0.2cos(800*1.87/1000 – 11.87°*π/180)

i(1.875 ms) = 173.46 ma

d) For steady state part:
Maximum Amplitude = Imax = 0.2
Angular Frequency (Rad/s) = w = 800 rad/sec.
Phase Shift = Φ = -11.87^{o}

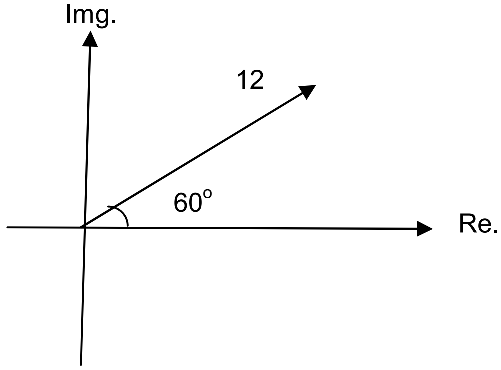
e) Phase difference = 25 – (-11.87) = 36.87^{o}

2U. The Voltage applied to the circuit shown below at t=0 is 10cos (120t + 60^{o}). The circuit resistance is 100 Ω, and the initial current in the 10 mh inductor is zero.
a) Find i(t) for t ≥ 0.
b) Write the expression for the transient and steady state component of i(t).
c) Find the numerical value of I after the switch has been closed for 2 ms.
d) What are the maximum amplitude, frequency (rad./s), and phase angle of the steady state current?
e) By how many degrees are the voltage and steady state current out of phase?



Solution:

3S. Write the time domain equation and phasor Polar, Rectangular and Angular forms for the following Signals (use w=3000 rad/sec. if not provided)
a)



b) X = 20 $e^{j\pi/3}$
c) x(t) = 25 Cos(300πt + π/3)

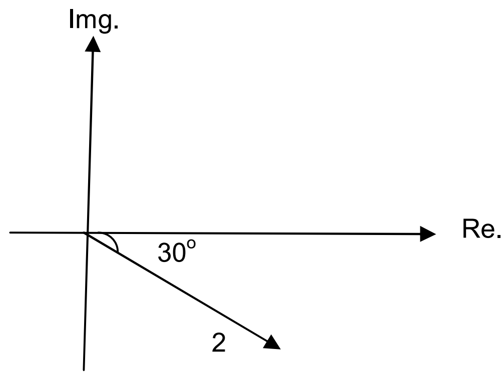
Solution:

Part	Time Domain	Angular	Polar	Rectangular

a	12cos(3000t +π/3)	$12 \angle \pi/3$	$12e^{j\pi/3}$	6 + j10.4
b	20cos(3000t +π/3)	$20 \angle \pi/3$	$20e^{j\pi/3}$	6 + j17.32
c	25 Cos(300πt + π/3)	$25\angle \pi/3$	$25e^{j\pi/3}$	12.5 + j21.65

3U. Write the time domain equation and phasor Polar, Rectangular and Angular forms for the following Signals (use w=3000 rad/sec. if not provided)

a)



b) $X = 25\ e^{j\pi/4}$

c) $X = 4\ e^{-\pi/3}$

d) x(t) = 25 Cos(300πt + π/3)

e) x(t) = 10 Sin(500πt + π/2)

f) $X = 3\ \angle 2\pi/3$

g) X = 25 + j32

h) x(t) = 20 Cos (30πt + 6π/3)

Solution:

4S. A 1000 Hz Sinusoidal voltage with a maximum amplitude of 200 V at t=0 is applied across the terminals of an inductor. The maximum amplitude of the steady-state current in the inductor is 25 A.

a) What is the frequency of the inductor current?

b) What is the phase angle of the voltage?

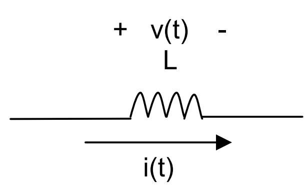
c) What is the phase angle of the current?

d) What is the inductive reactance of the inductor?

e) What is the inductance of the inductor in milihenrys?

f) What is the impedance of the inductor?

Solution:



a) Frequency = f = 1000 Hz (same as voltage)

b) Since the maximum amplitude of voltage occurs at t=0 then phase of voltage is 0
$\Phi_v = 0$

c) We know the Inductor shift current by -π/2 rad compared to voltage →
Current Phase Shift = $\Phi_i = 0 - \pi/2 = -\pi/2$

d) Inductive reactance = wL =?
Z=V/I → wl = Vmax/Imax = 200/25 = 8 Ω (reactance)

e) w = 2πf = 2000π
wL = (2000π) L = 8 → L = 1.27 mH

f) Impendance of inductor = Z = jwL = j8 Ω

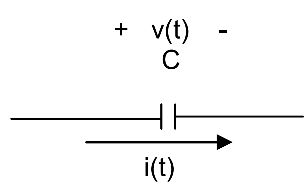
4U. A 2500 Hz Sinusoidal voltage with a maximum amplitude of 80 V at t=0 is applied across the terminals of an inductor. The maximum amplitude of the steady-state current in the inductor is 10 A.
a) What is the frequency of the inductor current?
b) What is the phase angle of the voltage?
c) What is the phase angle of the current?
d) What is the inductive reactance of the inductor?
e) What is the inductance of the inductor in milihenrys?
f) What is the impedance of the inductor?

Solution:

5S. A 50 kHz sinusoidal voltage has zero phase angle and a maximum amplitude of 10 mV. When this voltage is applied across the terminals of a capacitor, the resulting steady-state current has a maximum amplitude of 628.32 uA.

a) What is the frequency of the current in radians per second?
b) What is the phase angle of the current?
c) What is the capacitive reactance of the capacitor?
d) What is the capacitance of the capacitor in microfarads?
e) What is the impedance of the capacitor?

Solution:



a) Angular Frequency (rad/s) = w = 2πf = 2π(50,000) = 100,000π rad/sec

b) Phase Angle = Φ_i = π/2 Since Current is +π/2 shift from voltage (current leads the voltage by π/2)

c) Capacitive reactance = 1/wC = Vmax/Imax
1/wC = (10*1000)/628.32 = 15.92 Ω

d) -1/wC =-15.92 → C = 1/(15.92 * 100,000π) = 0.2 uF

e) Impedance of the capacitor = Z = J(-1/wC) = -j15.92 Ω

5U. A 15 kHz sinusoidal voltage has 45^o phase angle and a maximum amplitude of 50 mV. When this voltage is applied across the terminals of a capacitor, the resulting steady-state current has a maximum amplitude of 250 uA.

a) What is the frequency of the current in radians per second?
b) What is the phase angle of the current?
c) What is the capacitive reactance of the capacitor?

d) What is the capacitance of the capacitor in microfarads?
e) What is the impedance of the capacitor?

Solution:

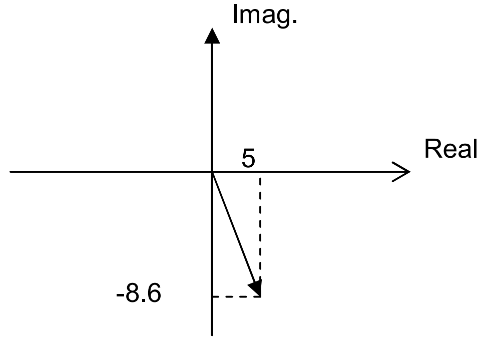
6S. Convert Steady State current i(t) = 10 cos (150t - π/3) to Phasor Domain and write out the equivalent current in Angular, Polar and Rectangular forms. Also draw the Phasor representation of this signal.

Solution

i(t) = 10 cos(150t - π/3) = 10 cos(150t - π/3) → w=150 rad/sec; Phase = - π/3; V_{max} = 10v

Angular Form → 10 $\underline{|-\pi/3}$
Polar Form → 10 $e^{-j\pi/3}$
Rectangular Form → 10cos(-π/3) + j10sin(-π/3) = 5 – j8.6

Phasor Diagram →



6U. Convert Steady State current i(t) = 30 cos (200t + π/6) to Phasor Domain and write out the equivalent current in Angular, Polar and Rectangular forms. Also draw the Phasor representation of this signal.

Solution

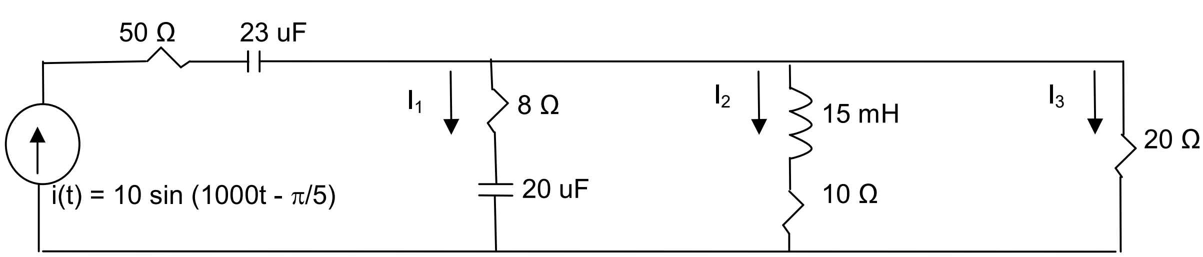
6Sb. Convert Steady State voltage v(t) = 25 sin(2000t + π/8) to Phasor domain and write out the equivalent voltage in Angular, Polar and Rectangular forms.

Solution:

V(t) = 25 sin(2,000t + π/8) Need to convert to cosine by (-π/2 shift)
V(t) = 25 cos(2,000t + π/8 - π/2) = 25 cos(2,000t - 3π/8) → w=2,000 rad/sec; Phase = - 3π/8; V_{max} = 25v

Angular Form → 25 $\underline{|-3\pi/8}$
Polar Form → 25 $e^{-3\pi/8}$
Rectangular Form → 25cos(-3π/8) + j25sin(-3π/8) = 9.6 – j23.1

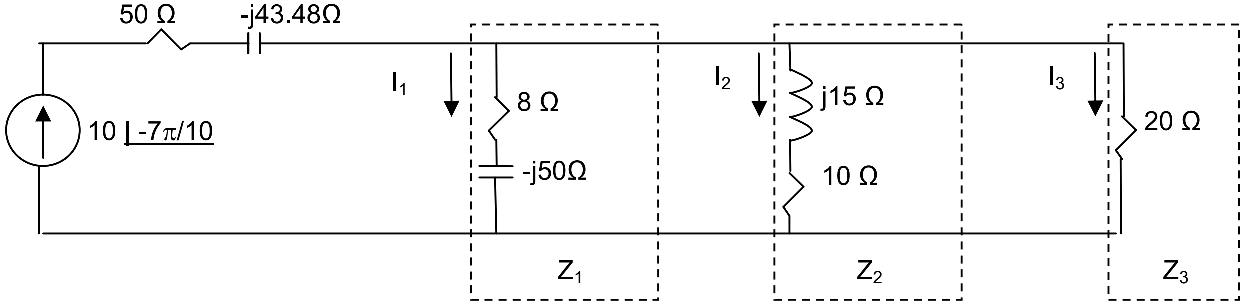
7S. Draw the phasor-domain equivalent circuit of the circuit shown below. Also identify the currents with the largest and the smallest magnitude among the three parallel branch currents (I_1, I_2, I_3)
Hint: Finding the exact value of I_1, I_2 and I_3 is not required.



Solution

Step 1. Covert to Phasor Domain (C→1/jwc; L→jwL; R→R)

w= 1000



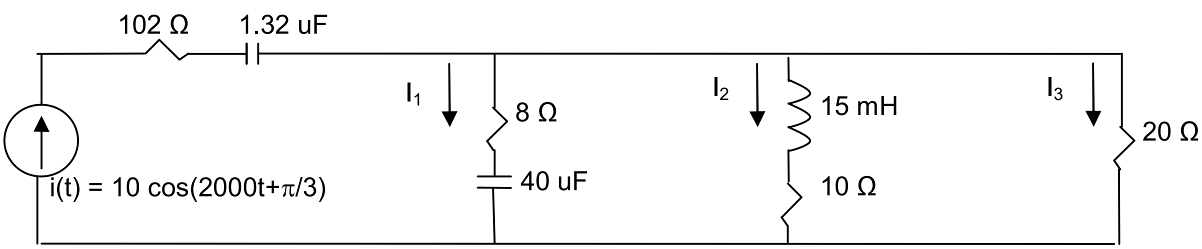
This is a current driver when In is inversely proportional to Zn

$|Z_1| = \sqrt{8^2 + 50^2} = 50.7\Omega$ Largest Impedance therefore Smallest current |I1|

$|Z_3| = 20\Omega$ Middle value Impedance therefore middle value current |I3|

$|Z_2| = \sqrt{10^2 + 15^2} = 18.03\Omega$ Smallest Impedance therefore Largest current |I2|

7U. Draw the Phasor domain equivalent of the circuit shown below. Also identify the currents with the largest and the smallest magnitude among the three parallel branch currents (I_1, I_2, I_3)



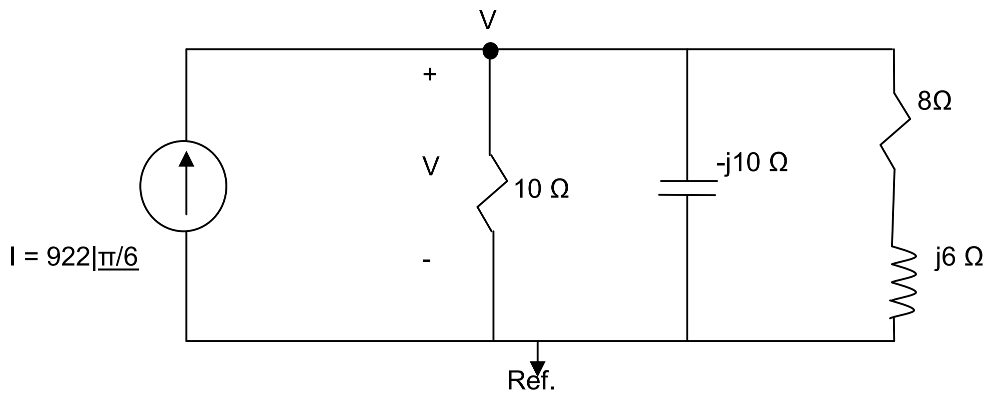
Solution:

8S. A 10 Ω resistor and a 5 uF Capacitor are connected in parallel. This parallel combination is also in parallel with the series combination of an 8 Ω resistor and an 300 uH inductor. These three parallel branches are driven by a sinusoidal current source whose current is 922cos(20,000t + 30°) A.

a) Draw the Frequency-domain equivalent circuit.
b) Reference the voltage across the current source as a rise in the direction of the source current, and find the phasor voltage.
c) Find the steady-state expression for v(t).

Solution:

a) Note (R→R, L → jwL , C→ 1/JwC = -J/wC where w=20,000



b) To find V, user Node-Voltage method and write the KCL equation for node V

First write the I in Rectangular form → I = 922cos(π/6) + j922sin(π/6) = 798.48 +j461

Now the KCL for V

-(798.48 + J461) + V/10 + V/(-j10) + V/(8 + j6) = 0

V= (798.48 + J461) / (1/10 +J/10 + 1/(8 + j6) = 4769.6 + j1501.2 (rectangular- Phasor form)

Amplitude = Vmax = $\sqrt{4769.6^2 + 1501.2^2} = 5000.2 \quad V$

Phase = Φ = $\tan^{-1}(\frac{1501}{4769.6}) = 0.3 \quad Rad.$ or 17.5°

V = 5000 |17.5°

c) Steady State v(t) = 5000cos(20,000t + 17.5°)

8U. A 20 Ω resistor and a 50 uF Capacitor are connected in parallel. This parallel combination is also in parallel with the series combination of an 50 Ω resistor and an 200 uH inductor. These three parallel branches are driven by a sinusoidal current source whose current is 120cos(10,000t + 45°) A.

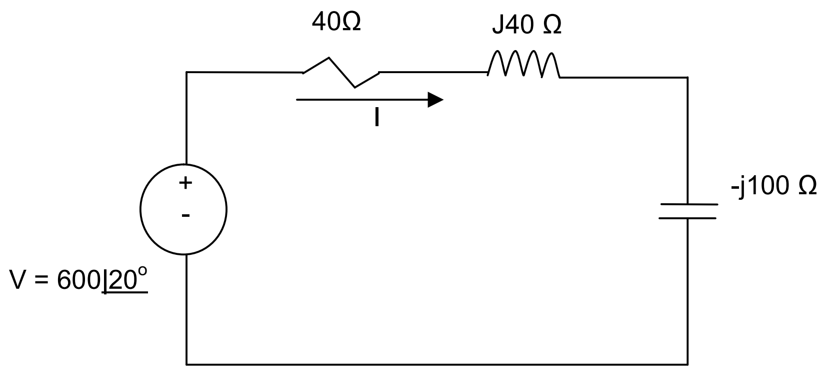
a) Draw the Frequency-domain equivalent circuit.
b) Reference the voltage across the current source as a rise in the direction of the source current, and find the phasor voltage.
c) Find the steady-state expression for v(t).

Solution:

9S. A 40 Ω resistor, a 5 mH inductor and a 1.25 uF capacitor are connected in series. The series-connected elements are energized by a sinusoidal voltage source whose voltage is 600 cos(8000t + 20°) V.
a) Draw the frequency-domain equivalent circuit.
b) Reference the current in the direction of the voltage rise across the source, and find the phasor current.
c) Find the steady state expression for i(t).

Solution:

a) Note: R→R, L → jwL , C→ 1/JwC = -J/wC where w=8,000



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b) I = V/Z

If we write Z also in in angular form then we can divide magnitudes and subtract phase to get the angular form of I

Z= (40 + j40 – j100) = (40 – j60) = $\sqrt{40^2+60^2}\ \angle \tan^{-1}(-\frac{60}{40}) = 72.11\angle -56.31^o$

I = (600$\angle 20^o$) / (72.11$\angle -56.31^o$) = (600/72.11)$\angle (20 - (-56.31))^o$

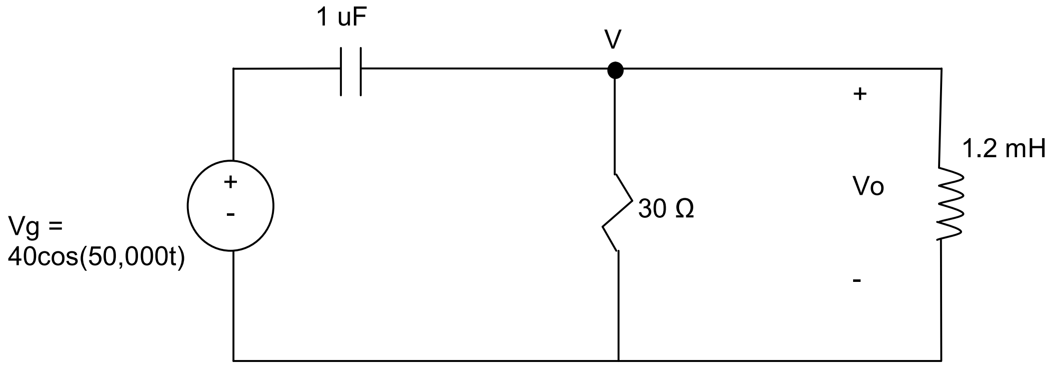
I = 8.32$\angle 76.31^o$ A

c) Steady State i(t) = 8.32 cos (8,000t + 76.31°) A

9U. A 40 Ω resistor, a 5 mH inductor and a 1.25 uF capacitor are connected in series. The series-connected elements are energized by a sinusoidal voltage source whose voltage is 600 cos(8000t + 20°) V.
a) Draw the frequency-domain equivalent circuit.
b) Reference the current in the direction of the voltage rise across the source, and find the phasor current.
c) Find the steady state expression for i(t).

Solution:

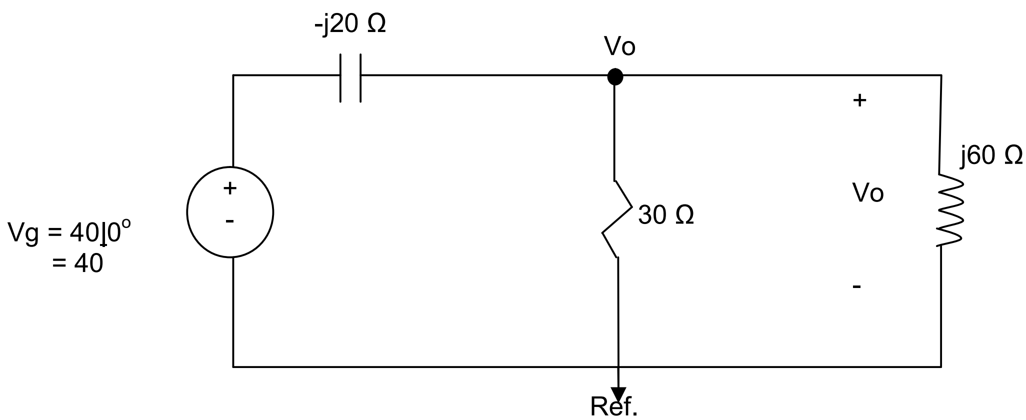
10S. The Circuit in the following figure is operating in the sinusoidal steady state. Find the steady-state expression for $v_o(t)$ if $v_g(t) = 40 \cos(50{,}000t)$ V.



Solution:

Step 1) Convert the circuit to Frequency domain using phasor transformation

Note (R→R, L → jwL , C→ 1/JwC = -J/wC where w=50,000



Step 2) Decide on Analysis method, Node Voltage is the simplest approach with 1 KCL equation

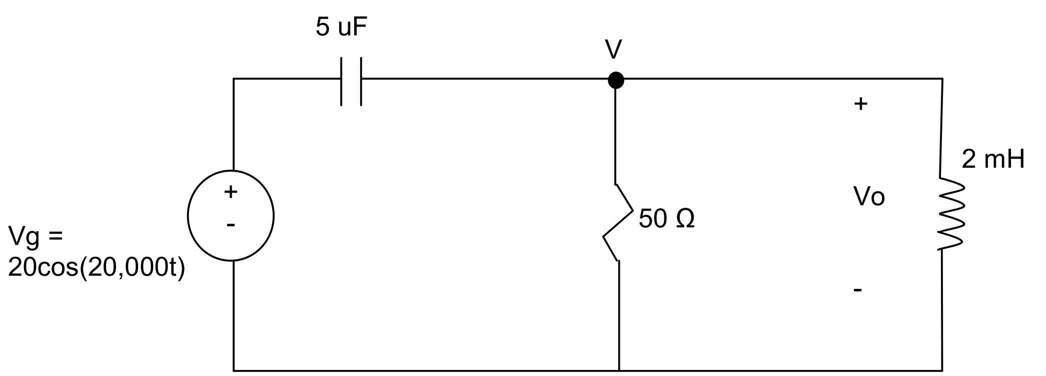
So write the KCL for Node V_o

$(V_o - 40) / -j20 + V_o /30 + V_o / j60 = 0$

$V_o = (-40/j20)/ (1/-j20 + 1/30 + 1/j60) = 30 + j30 = 42.43 \angle 45^{o}$

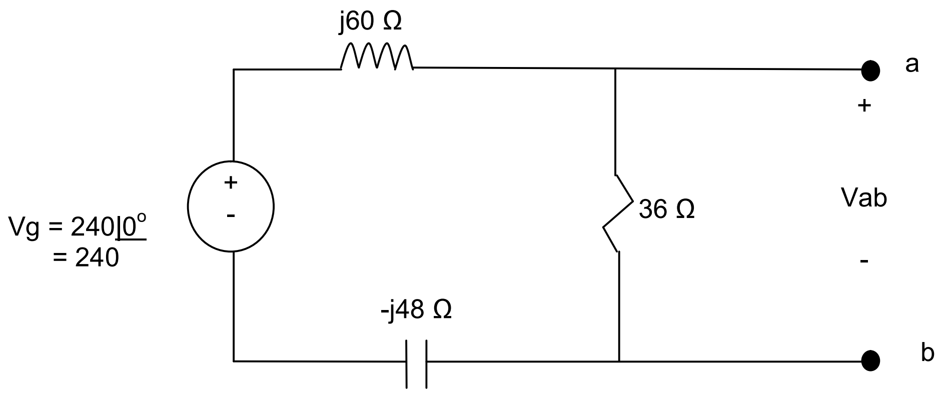
Steady State expression → $v(t) = 42.43\cos(50{,}000t + 45^{o})$

10U. The Circuit in the following figure is operating in the sinusoidal steady state. Find the steady-state expression for $v_o(t)$ if $v_g(t) = 20 \cos(20{,}000t)$ V.



Solution:

11S. Find the Thevenin equivalent circuit with respect to the terminals a, b for the circuit shown in the following figure.



Solution:

Step 1) Find the $V_{oc} = V_{th}$

You could look at this circuit as voltage divider where

Vth=Voc = Vab = (240/ (j60 + 36 – j48))*36 = 8640/ (36 + j12)

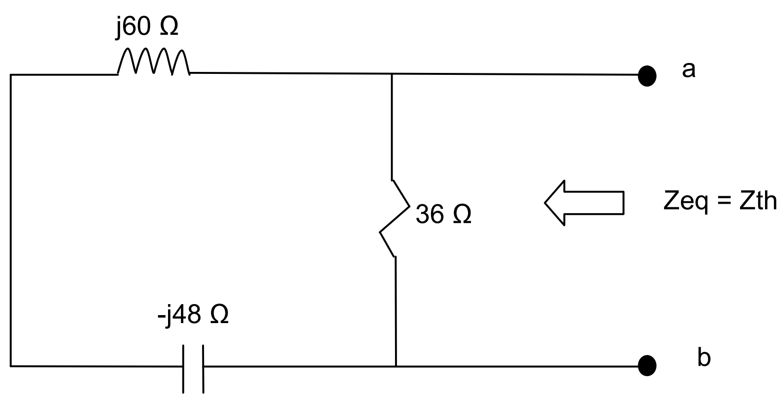
If we rewrite the bottom as a angular form phase then we can divide amplitudes and subtract phases

36 + j12 = $\sqrt{36^2+12^2} \,\Big|\, \underline{tan^{-1}(\frac{12}{36})} = 37.95\underline{|18.43^o}$

Vth = (8640 $\underline{|0^o}$) / (37.95 $\underline{|18.43^o}$) = (8640/37.95)$\underline{|(0-18.43)^o}$

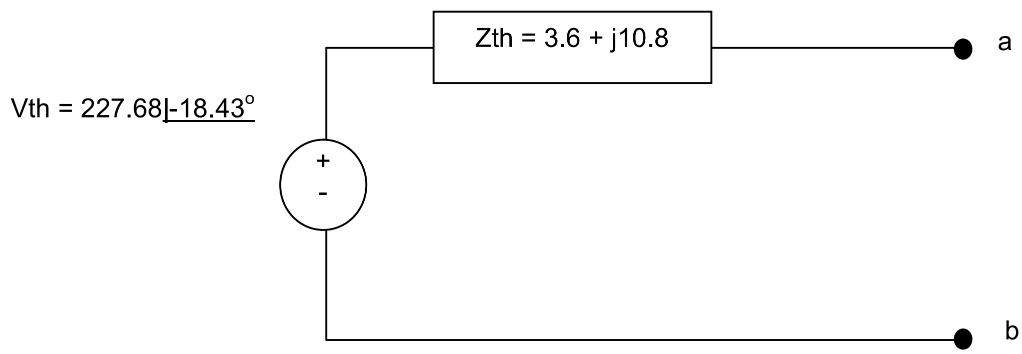
Vth = 227.668$\underline{|-18.43^o}$

Step 2) Find the Rth = Req when all independent sources are deactivated

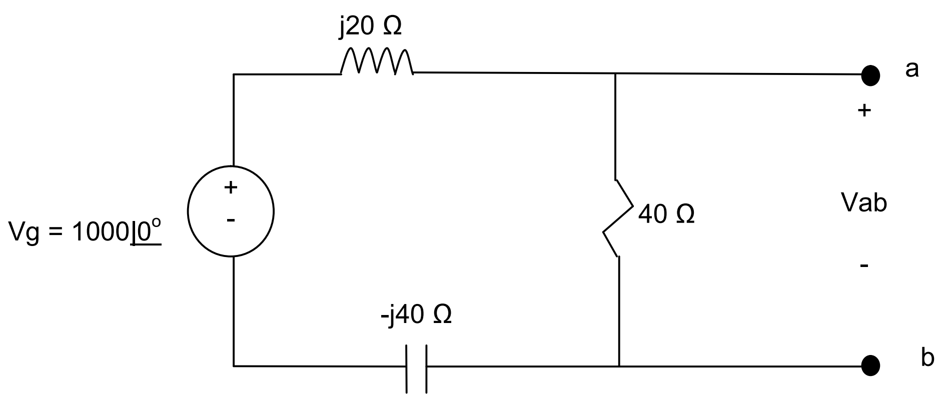


Zth = 1/(1/(j60 –j48) + 1/36) = 3.6 + j10.8

Step 3) Draw the Thevenin Equivalent Circuit



11U. Find the Thevenin equivalent circuit with respect to the terminals a, b for the circuit shown in the following figure.

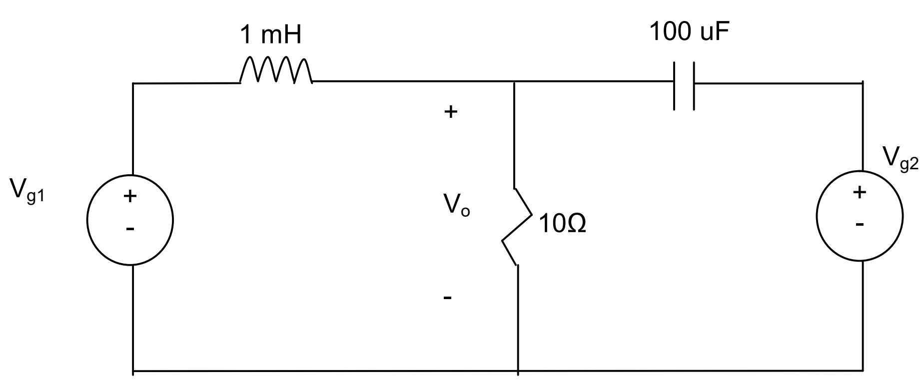


Solution:

12S. Use the mesh-current method to find the steady-state expression for the voltage $v_o(t)$ for the circuit shown below when:

v_{g1} = 20 cos(2000t – 36.87°) V

v_{g2} = 50 sin(2000t – 16.26°) V

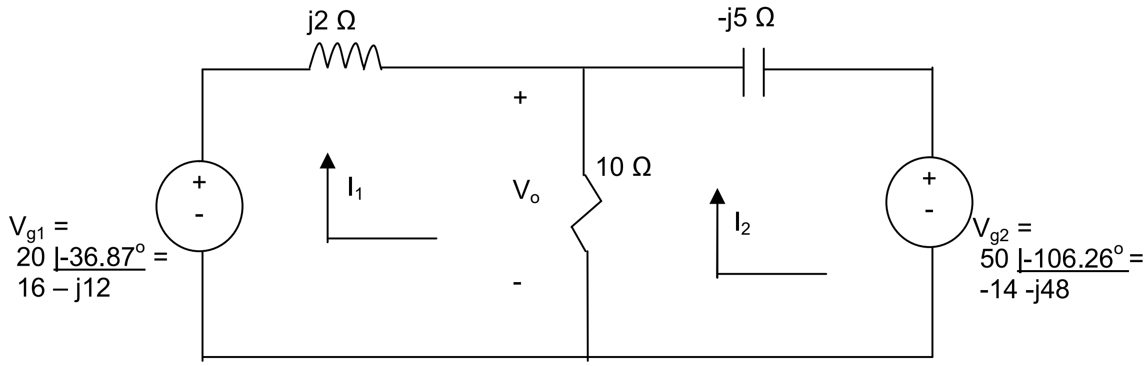


Solution:

Step 1) Convert v_{g2} to Cosine standard form by subtraction 90° from the phase.:

v_{g2} = 50 sin(2000t – 16.26°) = 50 cos(2000t – 106.26°)

Step 2) Convert the circuit to Frequency domain using phasor transformation
Note (R→R, L → jwL , C→ 1/JwC = -J/wC where w=2000

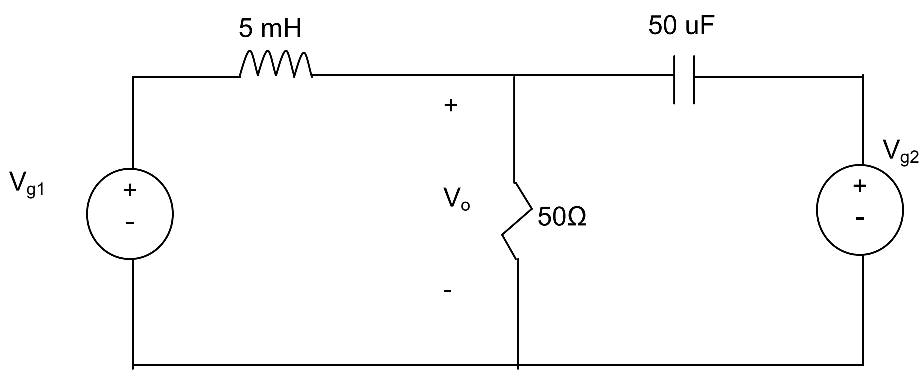


Step 3) Write the KVL equations for the two meshes:
Mesh 1 → - (16 - j12) + j2I_1 + 10 (I_1 - I_2) = 0
Mesh 2 → (-14 – j48) – j5I_2 + 10 (I_2 – I_1) = 0

From Mesh 1 equation find I_2 in term of I_1, plug the expression in Mesh 2 equation for I_2
I_1 = -6 + j10
I_2 = -9.6 + j10

Step 4) Find the V_o
Vo = (I1 – I2)*10 = (3.6*10) = 36|0°
v(t) = 36cos(2000t)

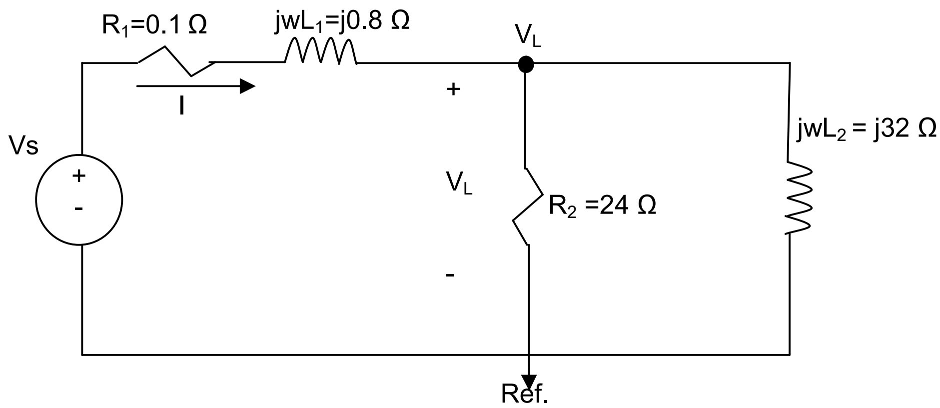
12U. Use the mesh-current method to find the steady-state expression for the voltage $v_o(t)$ for the circuit shown below when:
v_{g1} = 10 cos(1000t – 30°) V
v_{g2} = 20 sin(1000t – 45°) V



Solution:

13S. The parameters in the circuit shown below are R_1 = 0.1 Ω, wL_1 = 0.8 Ω, R_2 = 24 Ω, wL_2 = 32 Ω and V_L = 240 + j0.
a) Calculate the phasor voltage Vs.
b) Connect a capacitor in parallel with the inductor L2, hold V_L constant, and adjust the capacitor until the magnitude of I is a minimum. What is the capacitive reactance? What is the Value of Vs?
c) Find the value of the capacitive reactance that keeps the magnitude of I as small as possible while

$|V_s| = |V_L| = 240$ V.



Solution:

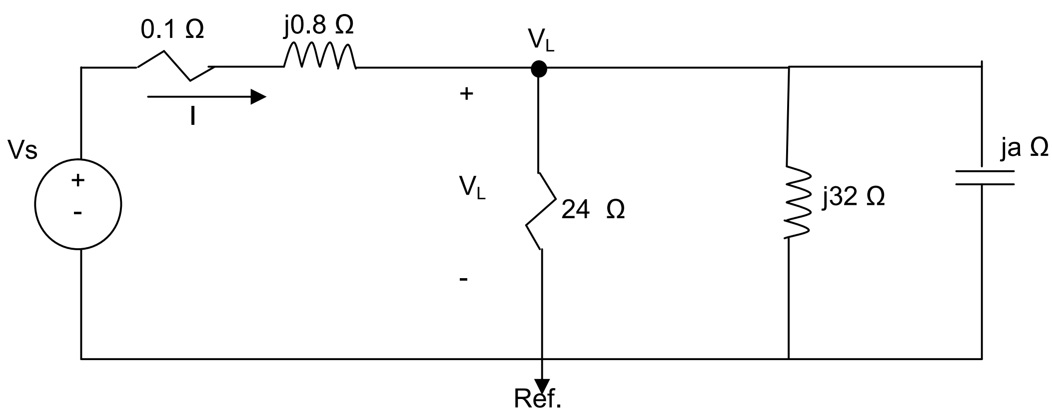
a) Vs = ?

Write KCL at node V_L → $(V_L – V_s)/(R_1 + jwL_1) + V_L/R_2 + V_L/jwL_2 = 0$

$(240 - V_s)/(0.1 + j0.8) + 240/24 + 240/j32 = 0$

$V_s = (0.1 + j0.8)(240/(0.1 + J0.8) + 10 - j7.5) = 247.35\angle 1.85^\circ$

b) Find capacitance reactance = a = -1/wc when VL = 240 V and magnitude of I is minimum.



I = 240/24 + 240/j32 + 240/ja =10 – j(7.5 + 240/a)

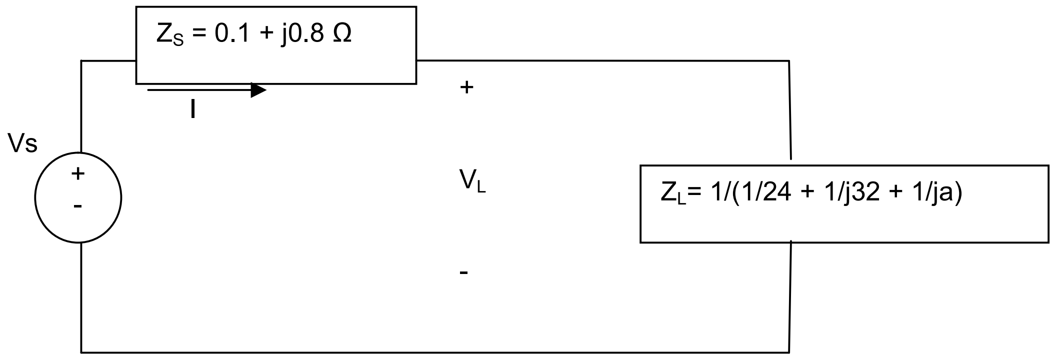
magnitude of I = $|I| = \sqrt{10^2 + (7.5 + 240/a)^2}$

Min of |I| only when Cap. Cancels out the Inductor so → $(7.5 + 240/a)^2 = 0$ → a = -32

Which means I = 10 A

$Vs = (0.1 + j0.8)*10 + 240 = 241 + j8 = 241.13\angle 1.9^\circ$

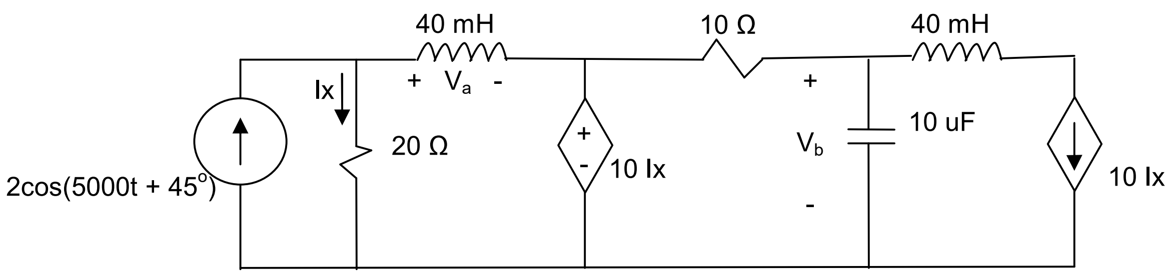
c) Find capacitance reactance = a = -1/wc when $|V_L| = |V_S|$ and magnitude of I is minimum



|VL| = |Vs| (Z_L/ (Z_L + Z_S))
to Meet the condition → (Z_L/ (Z_L + Z_S)) =1 → since Z_S is not 0 then Z_L must be infinite

or the denominator |Z_L| of is 0 → |j4+3-j96a| = $\sqrt{3^2 + (4-96a)^2} = 0$ →
$(4-96a)^2 = -9$ Not Possible

13U. Given the following Circuit:



Find the relationship between $V_a(t)$ and $V_b(t)$ when the circuit is in steady state.

Solution: