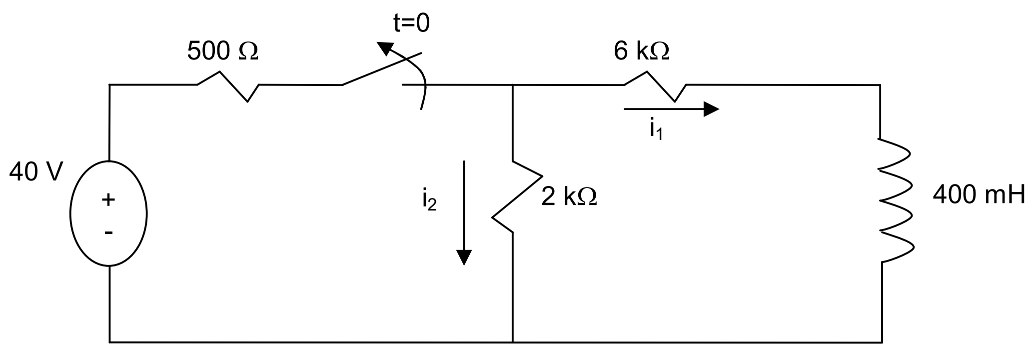
Fundamentals of Electrical Circuits - Chapter 7

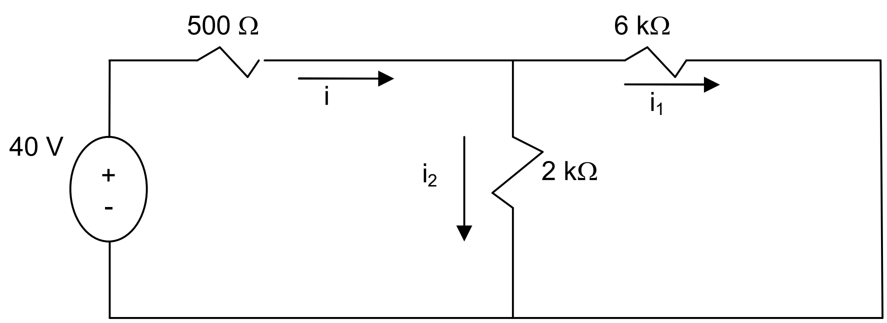
1S. In the following circuit, the switch is opened at t=0, after the switch being closed for a long time.



a) Find $i_1(0^-)$ and $i_2(0^-)$.
b) Find $i_1(0^+)$ and $i_2(0^+)$.
c) Find $i_1(t)$ for $t \geq 0$.
d) Find $i_2(t)$ for $t \geq 0$.
e) Explain why $i_2(0^-) \neq i_2(0^+)$.

Solution:

a) Find $i_1(0-)$ and $i_2(0-)$.
At t=0-, the switch has been closed for a long time. Therefore the inductor appears as a short..



Apply KVL →

$-40 + 500i + 2000(i - i_1) = 0 \rightarrow 2500i - 2000i_1 = 40$

$6000i_1 + 2000(i_1 - i) = 0 \rightarrow -2000i + 8000i_1 = 0$

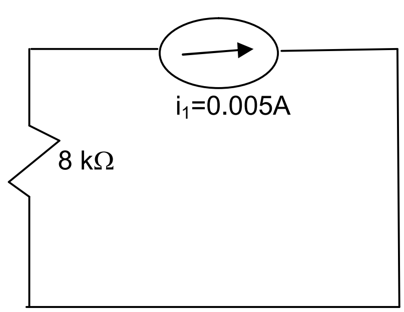
Solve and we know $i = i_1 + i_2$

$i(0^-) = 0.020$ A

$i_1(0^-) = 0.005$ A

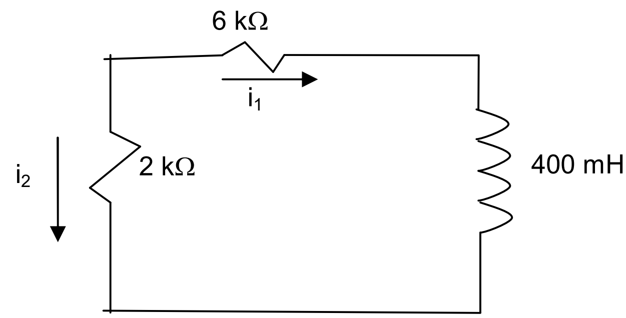
$i_2(0^-) = 0.015$ A

b) Find $i_1(0+)$ and $i_2(0+)$.
Immediately after switch is Opened at $t=0^+$, the inductor supplies $i_1(0^+) = i_1(0^-) = 0.005$ A current.



$i_2(0^+) = -\ i_1(0^+) = -\ 0.005$ A

c) Find $i_1(t)$ for $t \geq 0$.



Apply the Natural Response relationships → $i(t) = i(0)e^{-(R/L)t}\ for \quad t \geq 0$

$$i_1(t) = i_1(0^+)e^{-(R/L)t} = 0.005e^{-(8000/0.4)t}\ for \quad t \geq 0$$

$$i_1(t) = 0.005e^{-20000t} A \quad for \quad t \geq 0$$

d) Find $i_2(t)$ for $t \geq 0+$.

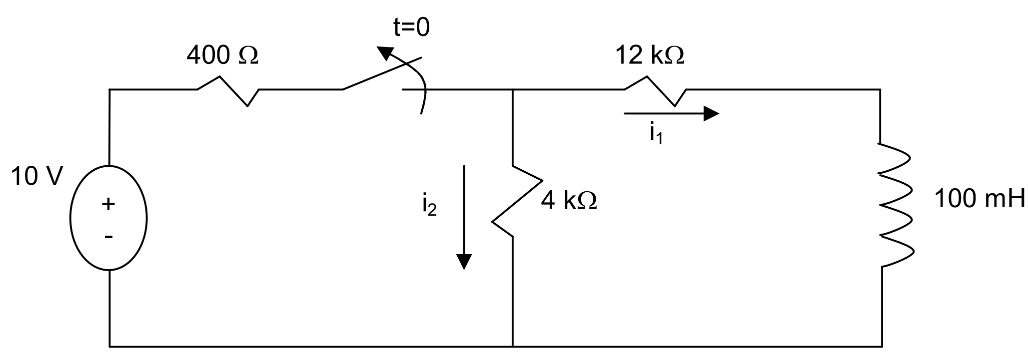
$$i_2(t) = -i_1(t) = -0.005e^{-20000t}\ for \quad t \geq 0^+$$

The only difference with part c is that t cannot be equal to 0.

e) Explain why $i_2(0-) \neq i_2(0+)$.

The current in resistor changes instantly. While the switching operation forces $i_2(0^-)$ to be 0.015 A and $i_2(0^+)$ to be -0.005 A

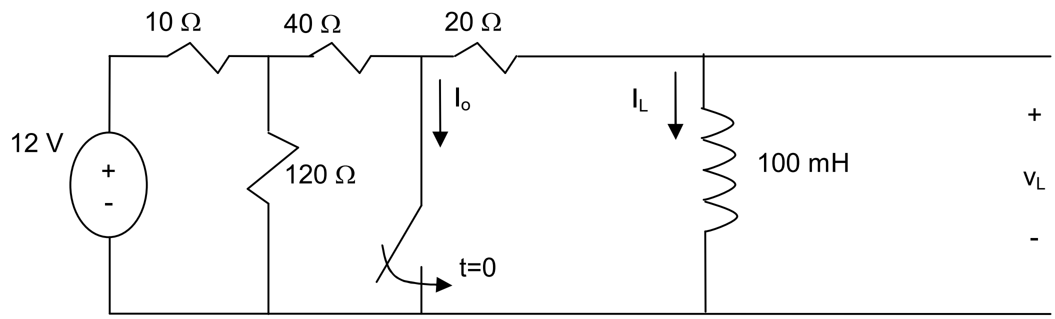
1U. In the following circuit, the switch is opened at t=0, after the switch being closed for a long time.



a) Find $i_1(0^-)$ and $i_2(0^-)$.
b) Find $i_1(0^+)$ and $i_2(0^+)$.
c) Find $i_1(t)$ for $t \geq 0$.
d) Find $i_2(t)$ for $t \geq 0$.
e) Explain why $i_2(0^-) \neq i_2(0^+)$.

Solution:

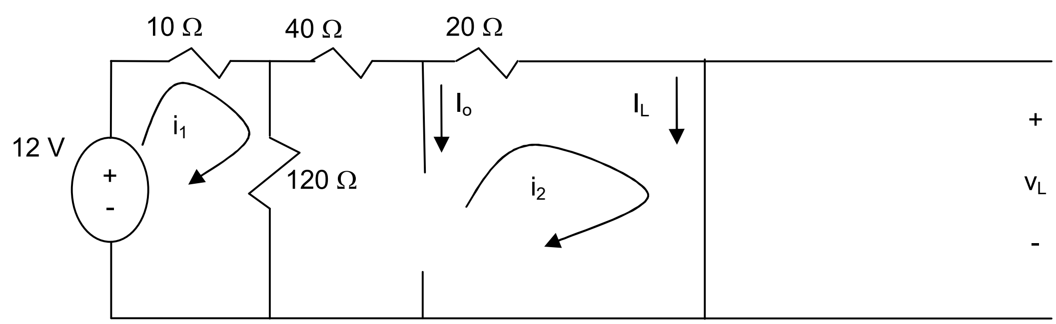
2S. The switch shown in the following figure has been open a long time before closing at t=0.



a) Find $i_o(0^-)$.
b) Find $i_L(0^-)$.
c) Find $i_o(0^+)$.
d) Find $i_L(0^+)$.
e) Find $i_o(\infty)$.
f) Find $i_L(\infty)$.
g) Write the expression for $i_L(t)$ for $t \geq 0$.
h) Find $V_L(0^-)$.
i) Find $V_L(0^+)$.
j) Find $V_L(\infty)$.
k) Write the expression for $V_L(t)$.for $t \geq 0^+$.
l) Write the expression for $i_o(t)$.for $t \geq 0^+$.

Solution:

a) Find $i_o(0^-)$.
at $t=0^-$ → circuit before the switch is closed and the inductor appear as a short.

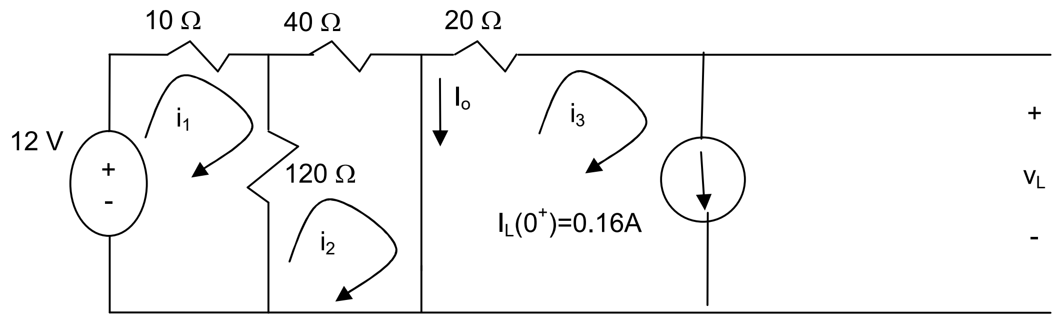


$i_o(0^-)=0$ since the switch is open.

b) Find $i_L(0^-)$.

KVL → $-12 + 10i_1 + 120(i_1 - i_2) = 0$ → $130i_1 - 120i_2 = 12$
KVL → $20i_2 + 40i_2 + 120(i_2 - i_1) = 0$ → $-120i_1 + 180i_2 = 0$
Solve
$i_1=0.24$ A ; $i_2=0.16$ A ;
→ ; $i_L(0^-) = i_2 = 0.16$ A

c) Find $i_o(0^+)$.
at $t=0^-+$ → circuit after the switch is closed and the inductor appear as a current source.

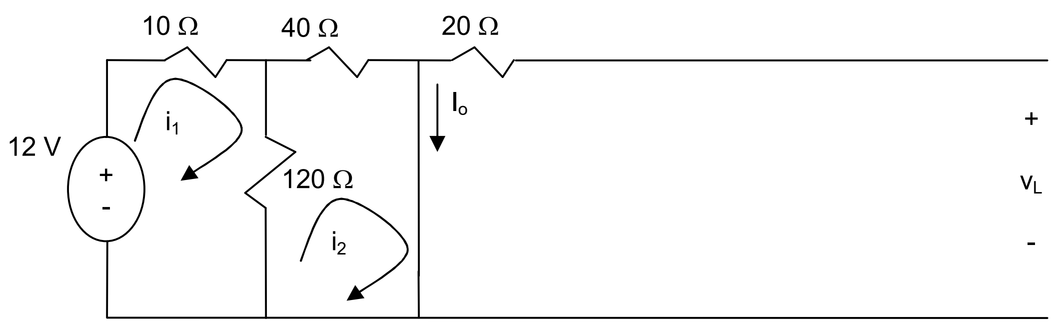


KVL @ i1 → $-12 + 10\, i_1 + 120\,(i_1 - i_2) = 0$ → $130 i_1 - 120 i_2 = 12$ → $520 i_2/3 - 120 i_2 = 12$ → $i_2 = 0.225$ A
KVL @ i2 → $120(i2 - i1) + 40\, i2 = 0$ → $-120 i_1 + 160 i_2 = 0$ → $i_1 = 4 i_2/3$
KVL @ i3 → i3 = 0.16

$Io = i_o(0^+) = i_2 - i_3 = 0.065$ A

d) Find $i_L(0^+)$.
$i_L(0^+) .= i_L(0^-) .= 0.16$ A

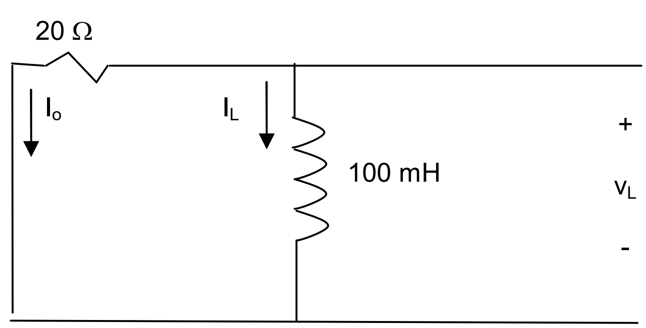
e) Find $i_o(\infty)$.
Inductor will have current of 0 (open) after long period of switch being closed



Same as part d → $i_o(\infty) .= i_2 = 0.225$ A

f) Find $i_L(\infty)$.
$i_L(\infty) .= 0$;

g) Write the expression for $i_L(t)$ for $t \ge 0$.



Apply the Natural Response relationships → $i(t) = i(0)e^{-(R/L)t}$ *for* $t \ge 0$

$$i_L(t) = 0.16e^{-(20/.1)t} \; for \quad t \ge 0$$

$$i_L(t) = 0.16e^{-200t}$$

h) Find $V_L(0^-)$.
$V_L(0^-) = 0$

i) Find $V_L(0^+)$.
refer to circuit in part c and write KVL around the most right loop →
$20 * (0.16) + V_L(0^+) = 0 \rightarrow V_L(0^+) = -3.2$ V

j) Find $V_L(\infty)$.
$V_L(\infty) = 0$

k) Write the expression for $V_L(t)$ for $t \geq 0^+$.

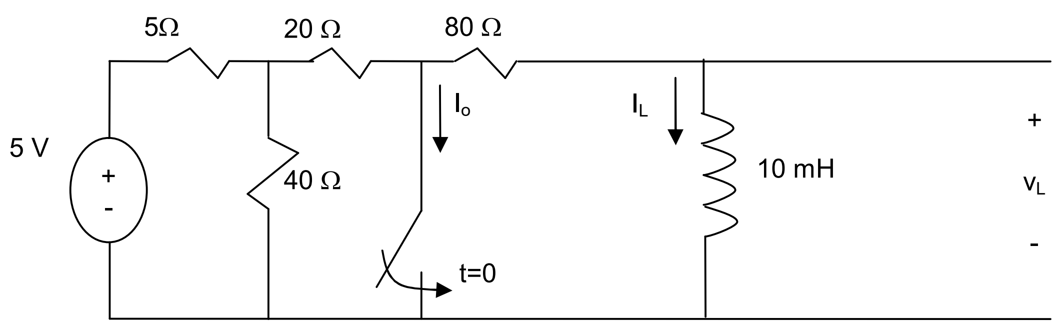
$$v_L(t) = L\frac{di_L}{dt} = (0.1)\frac{di_L}{dt} = 0.1 * 0.16 * (-200)e^{-(20/.1)t} \; for \quad t \geq 0$$

$$v_L(t) = -3.2e^{-200t}$$

l) Write the expression for $i_o(t)$ for $t \geq 0^+$.

$$i_0(t) = i_2(t) - i_L(t) = 0.225 - 0.16e^{-200t} \, A$$

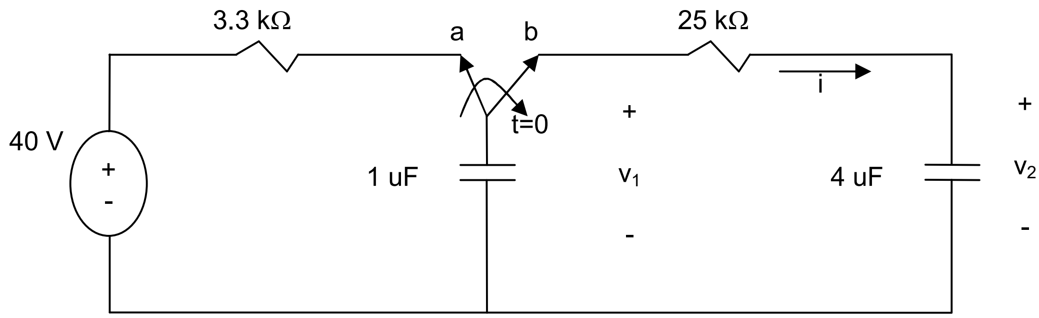
2U. The switch shown in the following figure has been open a long time before closing at t=0.



a) Find $i_o(0^-)$.
b) Find $i_L(0^-)$.
c) Find $i_o(0^+)$.
d) Find $i_L(0^+)$.
e) Find $i_o(\infty)$.
f) Find $i_L(\infty)$.
g) Write the expression for $i_L(t)$ for $t \geq 0$.
h) Find $V_L(0^-)$.
i) Find $V_L(0^+)$.
j) Find $V_L(\infty)$.
k) Write the expression for $V_L(t)$ for $t \geq 0^+$.
l) Write the expression for $i_o(t)$ for $t \geq 0^+$.

Solution:

3S. The switch in the following circuit has been in position for a long time. At t=0, the switch is thrown to position b.



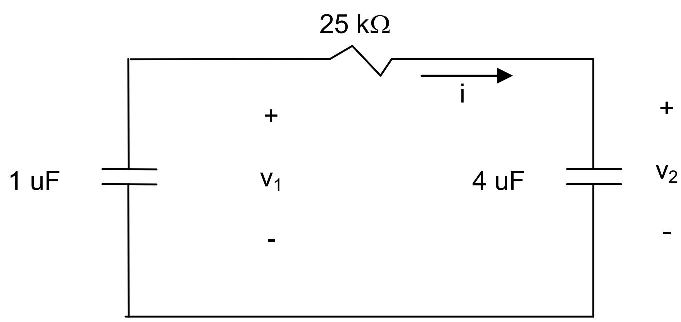
Calculate:

a) I, v_1 and v_2 for $t \geq 0^+$.
b) energy stored in the capacitors at t = 0.
c) energy trapped in the circuit and the total energy dissipated in the 25 kΩ resistor if the switch remains in position b indefinitely.

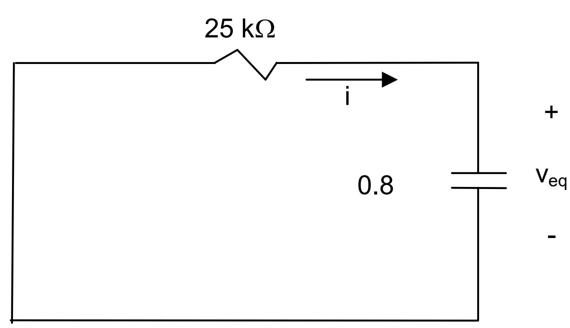
Solution:

a) i, v_1 and v_2 for $t \geq 0^+$.

After a long time with switch in position a results in capacitor appear as opening
→ $v_1(0^-)=v_1(0^+)=40$ V; $v_0(0^-)=0$ V;
When switch is put to position b, voltage is applied to 4uF which would appear open at $t=0^+$
→ $v_1(0^+)=40$ V
→ $i(0^+) = 0$
→ $v_2(0^+) = 40$ V
After Switch is changes from a to b



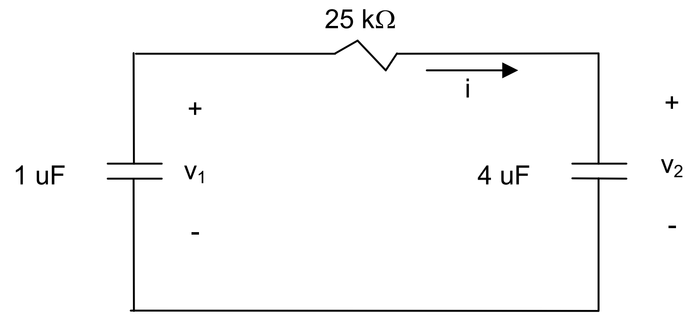
$C_{eq} = 1/(1/1 + 1/4) = 0.8$ uF



Apply the Natural Response relationships $v(t) = v(0)e^{-t/RC}$ *for* $t \geq 0$

$$i = C\frac{dv}{dt} = -C\frac{v(0)}{RC}e^{-t/RC} = -\frac{v(0)}{R}e^{-t/RC} \qquad for \quad t \geq 0$$

$$i(t) = -\frac{40}{25000}e^{-t/(25*10^3*0.8*10^{-6}} = -1.6e^{-50t}mA \qquad for \quad t \geq 0$$



$$v_1 = \frac{1}{C}\int_0^t i(\tau)d\tau + v(0^-) = \frac{1}{1*10^{-6}}\int_0^t 1.6*10^{-3}e^{-50\tau}d\tau + 40 = -32e^{-50t} + 72\,V \quad for \quad t \geq 0$$

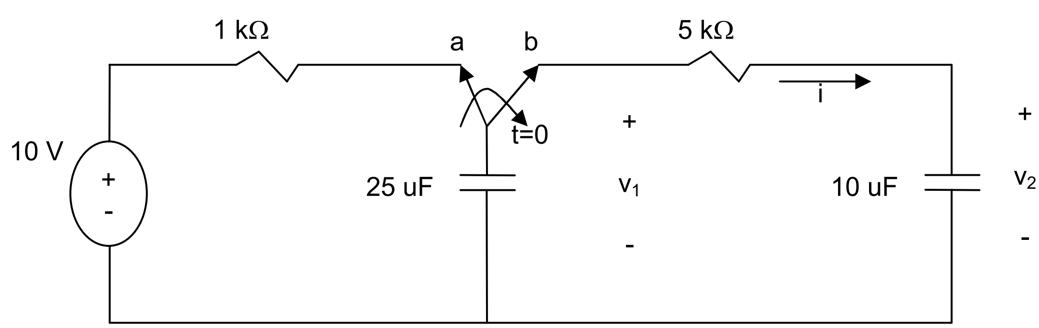
$$v_2 = \frac{1}{C}\int_0^t i(\tau)d\tau + v(0^-) = \frac{1}{4*10^{-6}}\int_0^t -1.6*10^{-3}e^{-50\tau}d\tau + 0 = 8e^{-50t} - 8\,V \quad for \quad t \geq 0$$

b) Energy stored in the capacitor at t = 0.
The only capacitor with energy is 1 uF capacitor
$w(0) = 1/2Cv(0)^2 = 0.5*10^{-6}*(40)^2 = 800$ uJ

c) Energy trapped in the circuit and the total energy dissipated in the 25 kΩ resistor if the switch remains in position b indefinitely.
$w_{trapped}(\infty) = 1/2C_1v_1^2 + 1/2C_2v_2^2 = 0.5*10^{-6}*(72)^2 + 0.5*4*10^{-6}*(8)^2 = 2624$ uJ

The energy that R will dissipate is equal to the amount of energy that was in the equivalent C at t=0+ since after a long-time the stored energy in the combined capacitor will be zero:
$w_{Ceq}(0) = 1/2Cv(0)^2 = 0.5*0.8*10^{-6}*(40)^2 = 640$ uJ

3U. The switch in the following circuit has been in position for a long time. At t=0, the switch is thrown to position b.



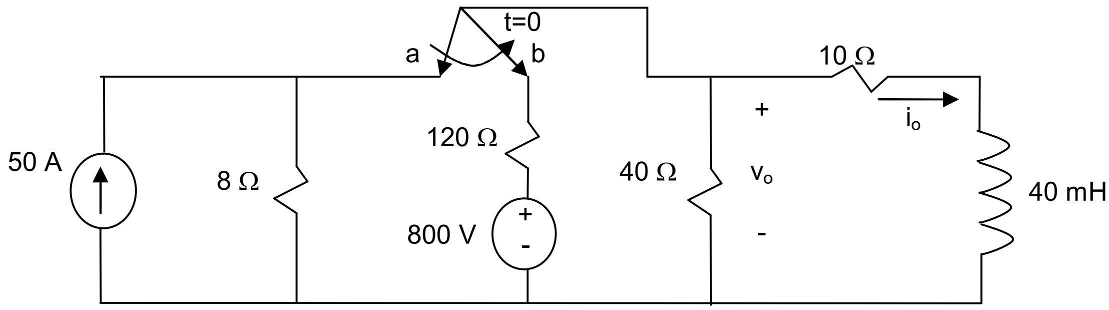
Calculate
a) I, v_1 and v_2 for $t \geq 0^+$.
b) energy stored in the capacitor at t = 0.
c) energy trapped in the circuit and the total energy dissipated in the 5 kΩ resistor if the switch remains in position b indefinitely.

Solution:

4S. The switch in the following circuit has been in position a for a long time. At t=0, the switch moves instantaneously to position b.

a) Find the numerical expression for $i_o(t)$ where $t \geq 0$.

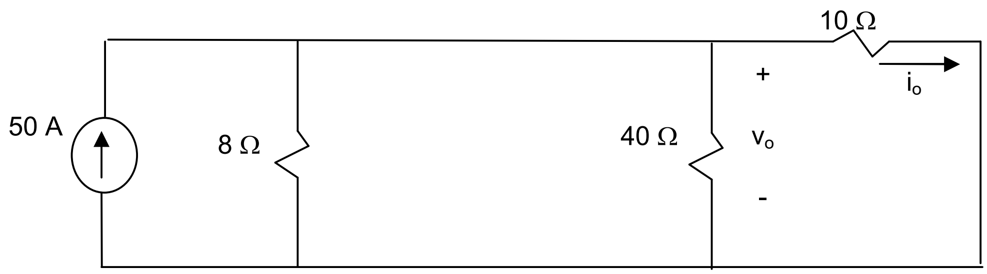
b) Find the numerical expression for $v_o(t)$ where $t \geq 0^+$.

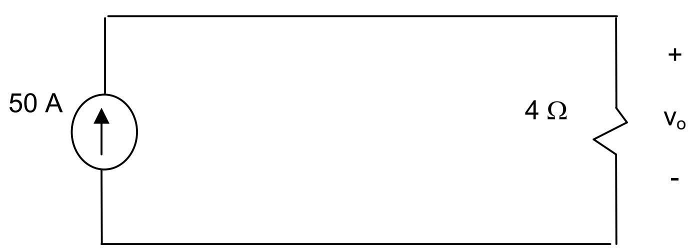


Solution:

a) Find the numerical expression for $i_o(t)$ where $t \geq 0$.

When the switch has been in position a for a long-time → Inductor will appear as a short

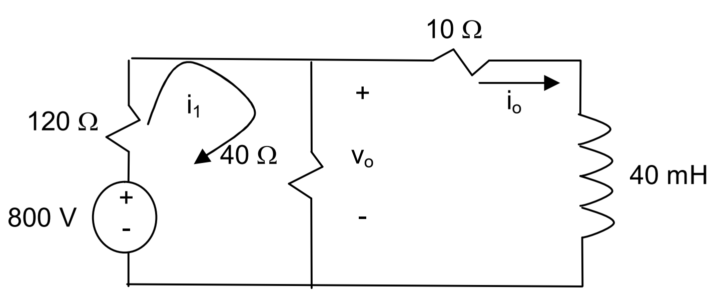




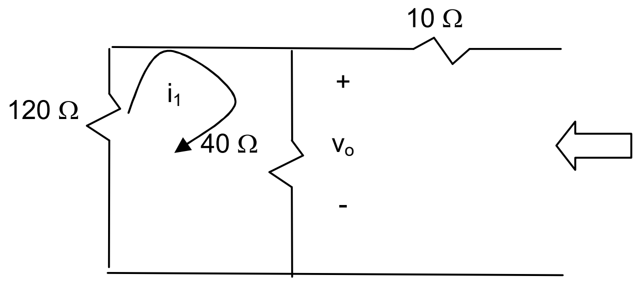
Simplified circuit → $V(o^-) = 50*4 = 200v$

Original circuit → $i_0(o^-) = 200 / 10 = 20$ A

After the switch is change to position b , circuit is redrawn and find the Thevenin equivalent with respect to Inductor terminals:

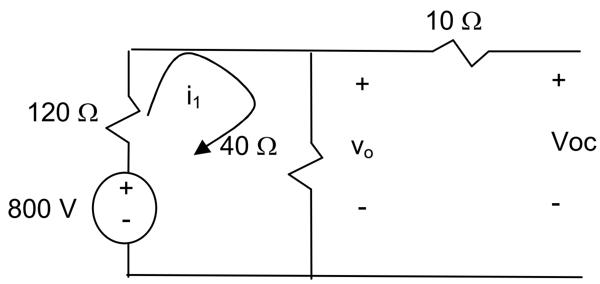


To Find Rth disable Voltage source (short)



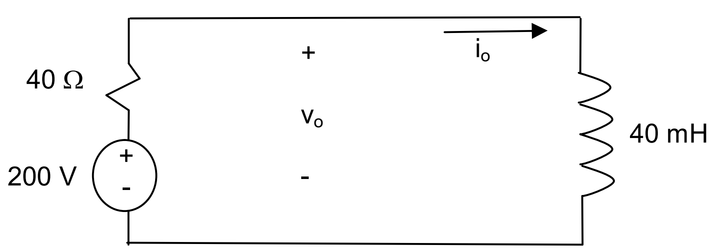
Rth = (120 || 40)+10 = 40 Ω

To Find Vth or Voc disable Voltage source (short)



KVL → -800 + 120 i_1 + 40 i_1=0 → 160 i_1 = 800 → i_1 = 5 A → Voc = 40 * 5 = 200 V

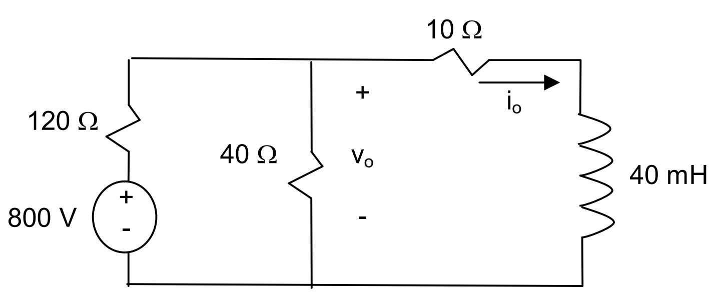
Place the Thevenin Equivalent back into the circuit:



Apply the step response relationships $i(t)=\dfrac{Vs}{R}+\left(I_0-\dfrac{V_s}{R}\right)e^{-(R/L)t}\ for \quad t\geq 0$

→ $i_0(t)=\dfrac{200}{40}+\left(20-\dfrac{200}{40}\right)e^{-(40/.004)t}=5+15e^{-1000t}A \quad for \quad t\geq 0$

b) Find the numerical expression for $v_o(t)$ where $t \geq 0^+$.

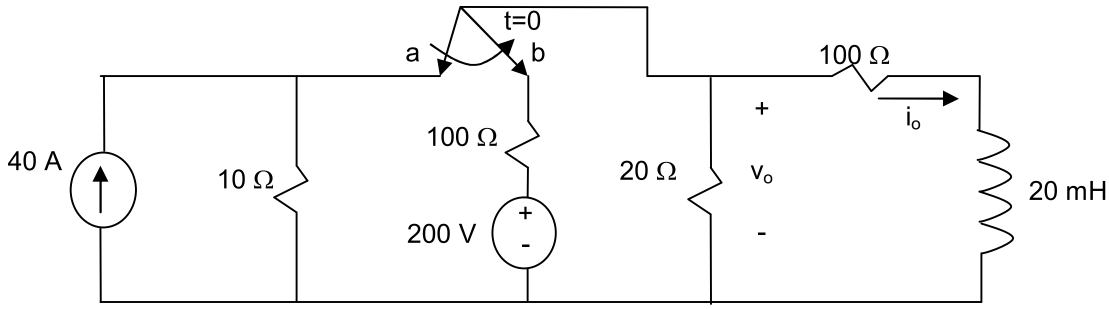


$$v_0(t)=10i_0+L\frac{di}{dt}=10(5+15e^{-1000t})+0.04(-1000)(15e^{-1000t})=50-450e^{-1000t} \quad for \quad t\geq 0$$

4U. The switch in the following circuit has been in position a for a long time. At t=0, the switch moves instantaneously to position b.

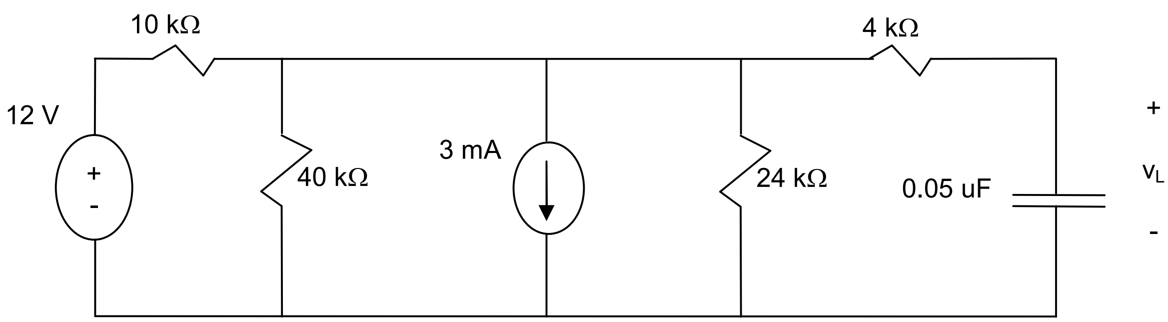
a) Find the numerical expression for $i_o(t)$ where $t \geq 0$.

b) Find the numerical expression for $v_o(t)$ where $t \geq 0^+$.



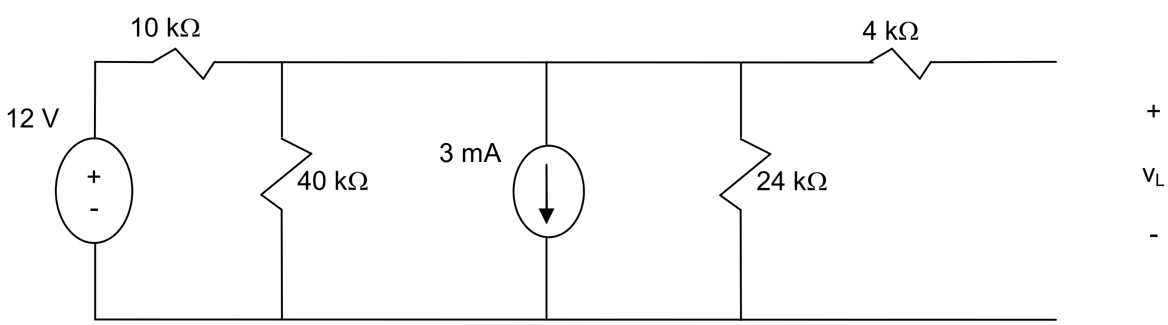
Solution:

5S. The following circuit has been in operation for a long time. At t=0, the voltage source reverses polarity and the current source drops from 3 mA to 2 mA. Find $v_L(t)$ for $t \geq 0$.



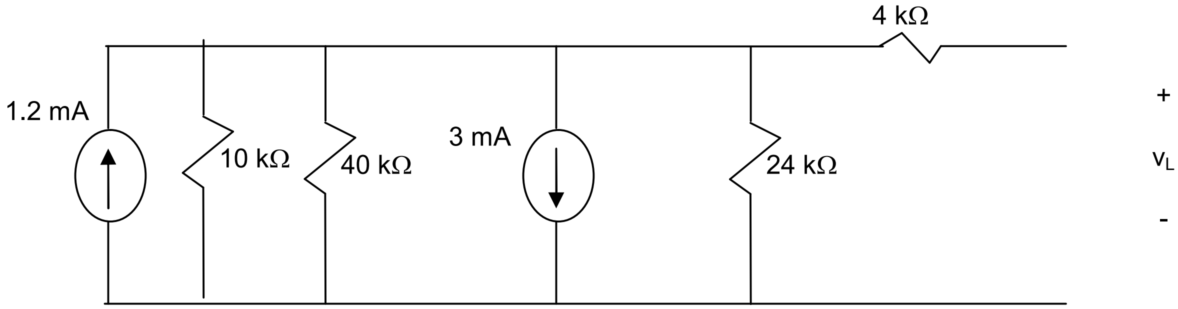
Solution:

After circuit has been in current state for a long-time ($t=0^-$), Capacitor will appear as an open

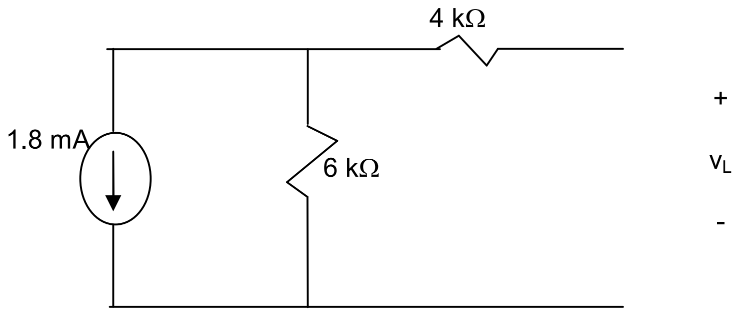


Use source transformation to simplify the circuit:

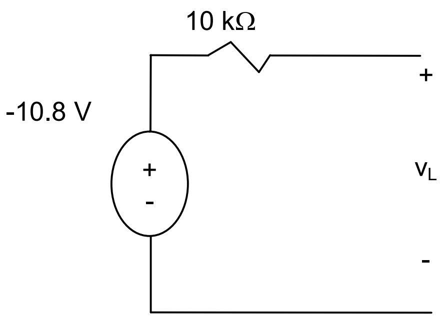
12v & series 10 kΩ → 1.2 mA and parallel 10 kΩ



R = (10 || 40 || 24)= 6 kΩ

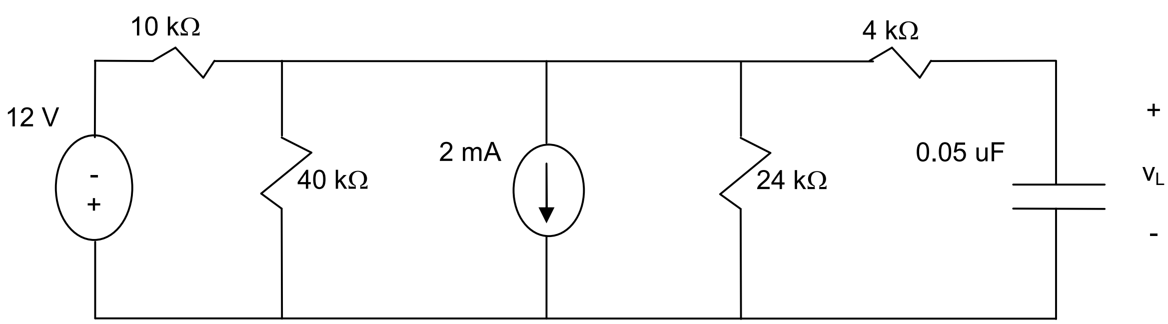


1.8 mA & parallel 6 kΩ → 10.8 V and series 6 kΩ



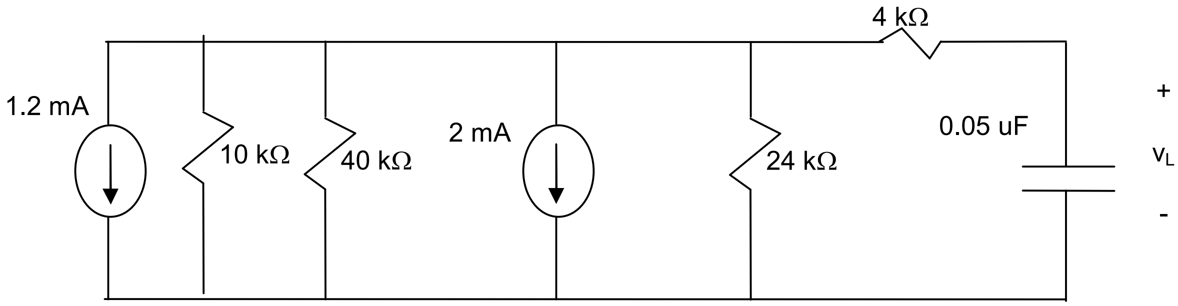
$v_{L(0^-)}$ =1.8 mA * 6 KΩ = -10.8 V

at t=0 (t > 0):circuit changes to

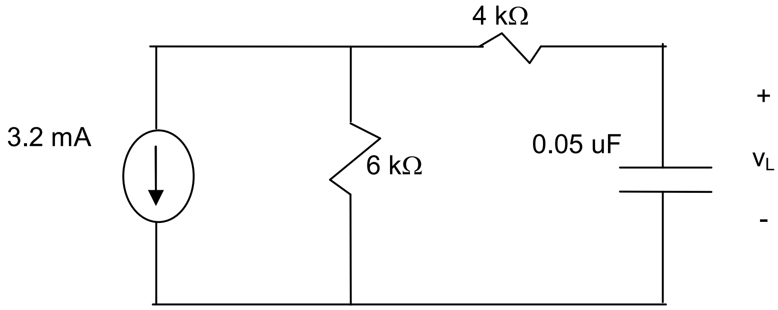


Use similar source transformation technique as used earlier will simplify the circuit into Norton equivalent.

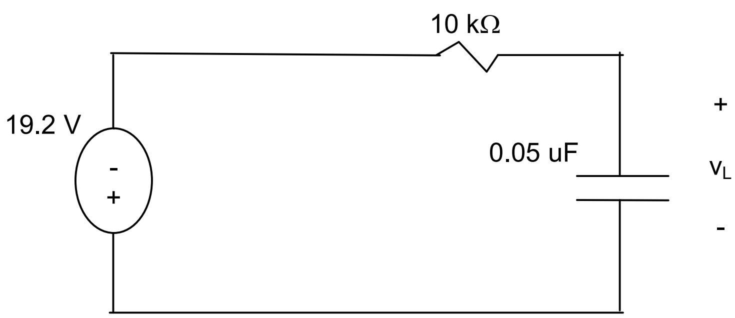
12v & series 10 kΩ → 1.2 mA and parallel 10 kΩ



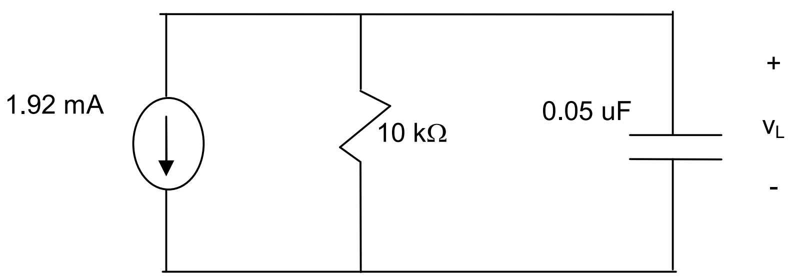
R = (10 || 40 || 24)= 6 kΩ



3.2 mA & parallel 6 kΩ → 19.2 V and series 6 kΩ
Another source transformation reduces the circuit to:



Another transformation get us back to the standard form:
19.2v & series 10 kΩ → 1.92 mA and parallel 10 kΩ



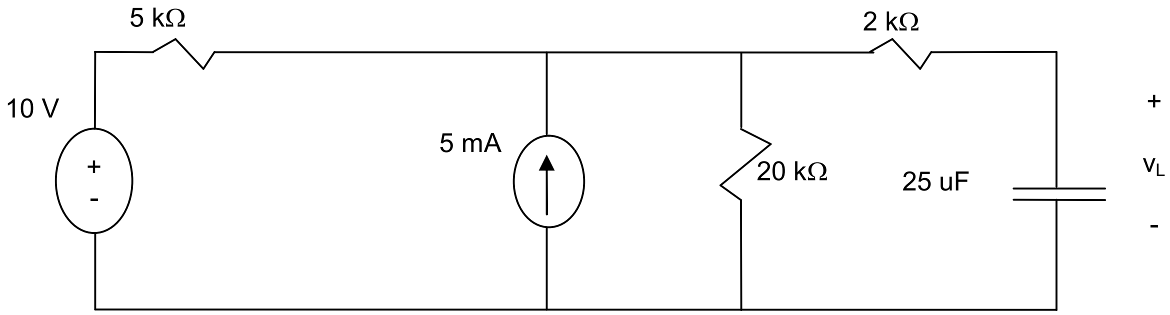
Apply the step response for RC circuit $v(t) = I_s R + (V_0 - I_s R)e^{-t/RC}\ for \quad t \geq 0$

$$v_L(t) = -(0.00192) * (10{,}000) - (-10.8 - (-0.00192 * 10{,}000))e^{-t/0.0005}\ for \quad t \geq 0$$

$$v_L(t) = -19.2 + (+10.8 - 19.2)e^{-2000t}\ for \quad t \geq 0$$

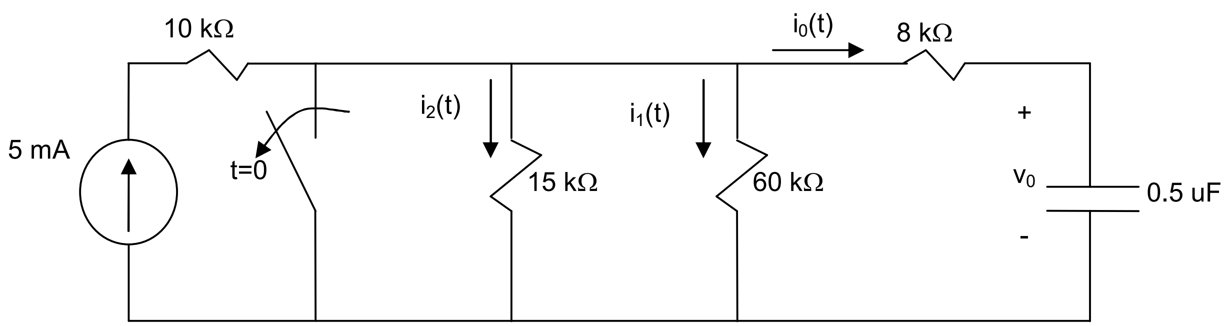
$$v_L(t) = -19.2 - 8.4e^{-2000t}\ for \quad t \geq 0$$

5U. The following circuit has been in operation for a long time. At t=0, the voltage source reverses polarity and the current source drops from 5 mA to 3 mA. Find $v_L(t)$ for $t \geq 0$.



Solution:

6S. The switch in the following circuit has been closed a long time before opening at t = 0.



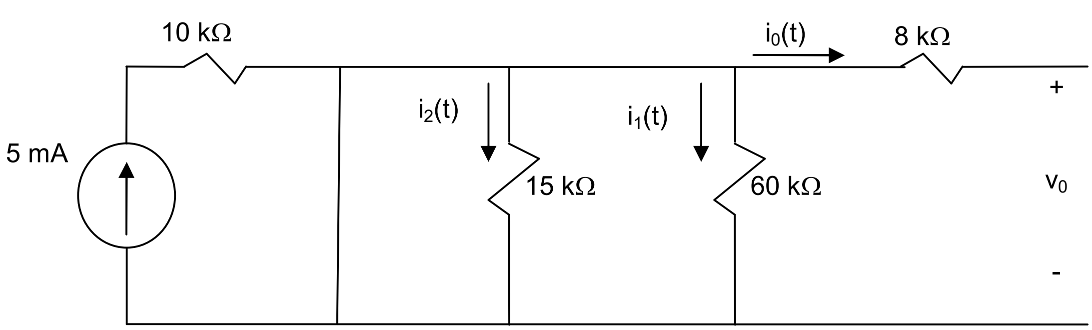
For $t \geq 0+$, find:

a) $v_0(t)$. b) $i_0(t)$. c) $i_1(t)$. d) $i_2(t)$. e) $i_1(0^+)$.

Solution:

a) $v_0(t)$

After a long time at $t=0^-$, capacitor appears as open…

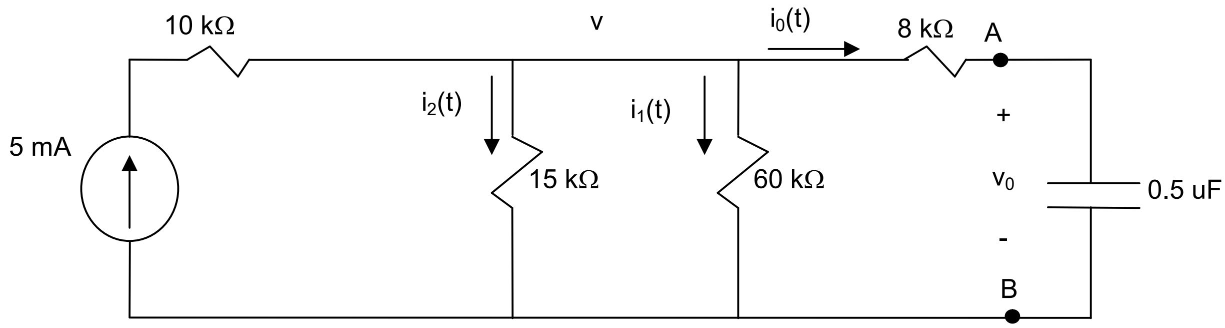


Short across the source then no power is deliver to the rest of the circuit →

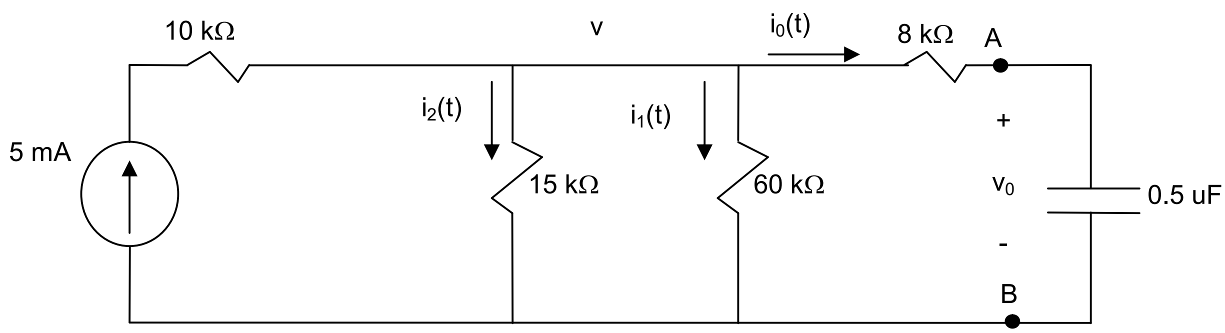
$v_0(0^-) = 0$ V

$i_0(0^-) = i_1(0^-) = i_2(0^-) = 0$ A

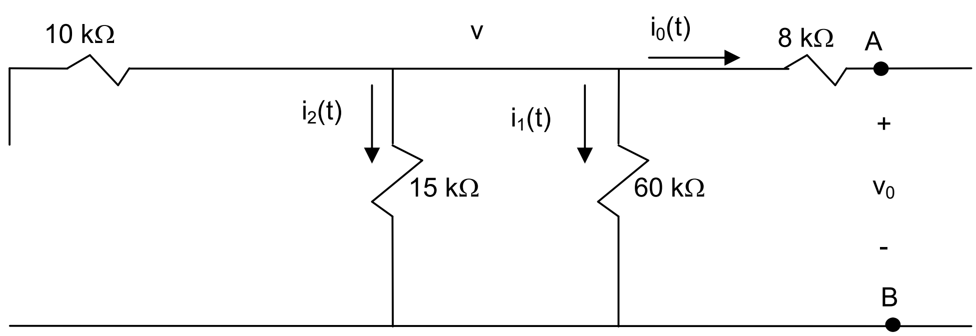
After t=0 (t>0), circuit is redrawn as:



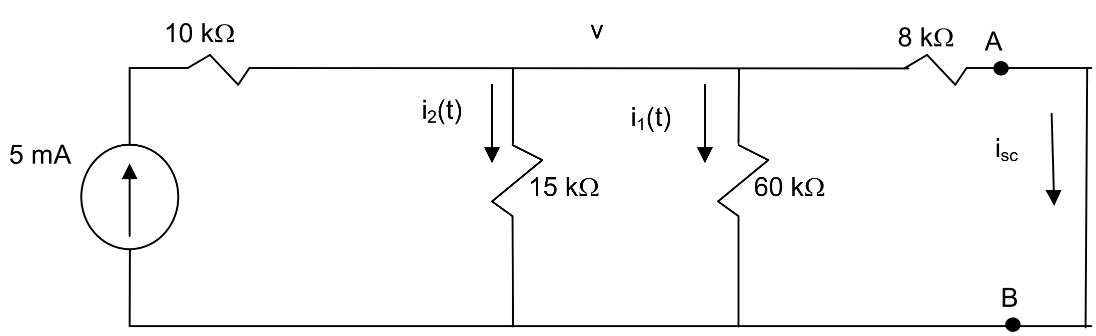
At $t=0^+$, Capacitor appears as a short → $V_o(0^+)=0$ V.
KCL → $-5 + v/15 + v/60 + v/8 = 0$ → $25v = 5*120$ → $v(0^+) = 24$ V → $i_0(0^+) = 3$ mA



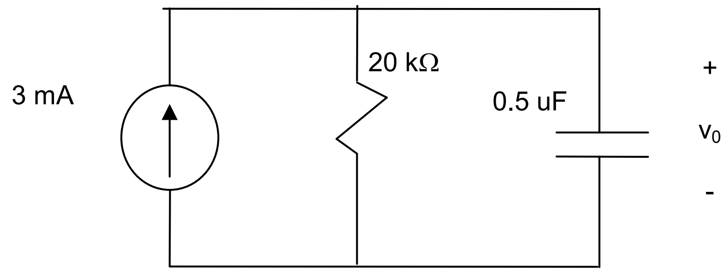
Find the Norton Equivalent at terminal AB→
Disable current source (open) to find Rth:



Rth = (15 || 60) + 8 = 20 kΩ
Find Isc:



KCL → $-5 + v/15 + v/60 + v/8 = 0$ → $25v = 5*120$ → $v(0^+) = 24$ V → $i_{sc} = 3$ mA



Apply the step response for RC circuit → $v(t) = I_s R + (V_0 - I_s R)e^{-t/RC}\ for \quad t \geq 0$

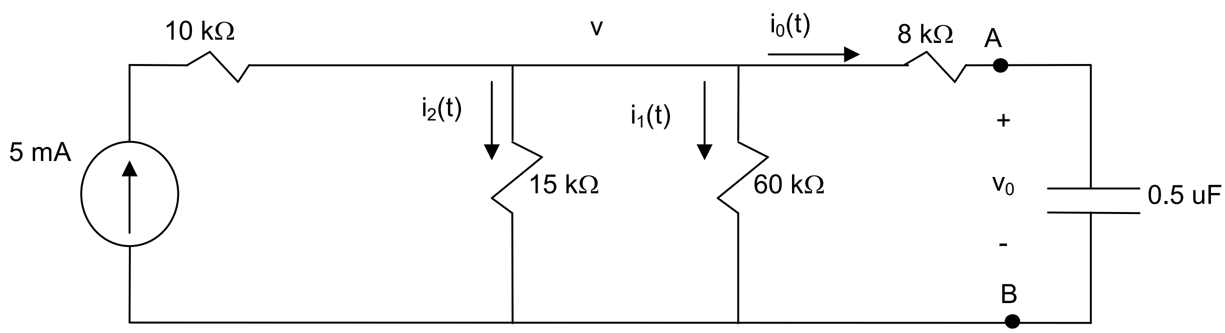
$$v_0(t) = 0.003 * 20{,}000 + (0 - 0.003 * 20{,}000)e^{-t/(20{,}000*0.5*10^{-6})}\ for \quad t \geq 0$$

$$v_0(t) = 60 - 60e^{-100t} V \quad for \quad t \geq 0$$

b) $i_0(t)$

$$i_0(t) = C\frac{dv}{dt} = 0.5*10^{-6}*(-60)*(-100)e^{-100t} = 3e^{-100t} mA \quad for \quad t \geq 0$$

c) $i_1(t)$



$$v(t) = 8{,}000 * i_0(t) + v_0(t) = 8*3e^{-100t} + 60 - 60e^{-100t} = 60 - 36e^{-100t} \quad for \quad t \geq 0$$

$$i_1(t) = \frac{v(t)}{60{,}000} = 1 + 0.6e^{-100t} mA \quad for \quad t \geq 0$$

d) $i_2(t)$

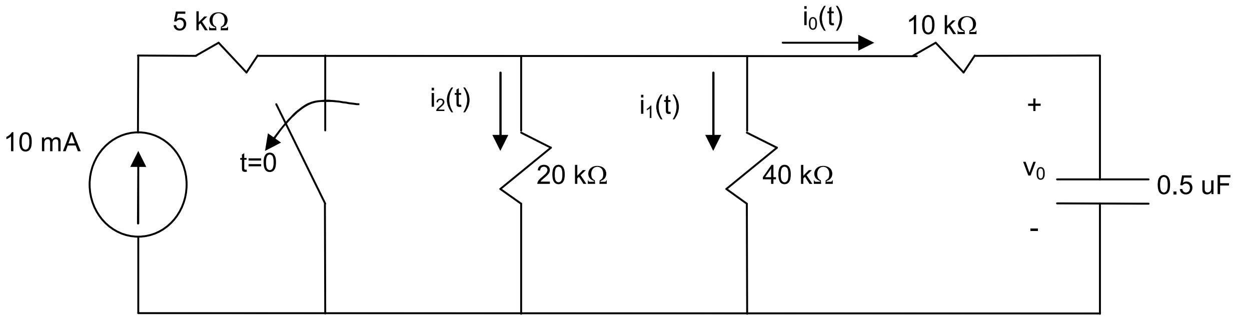
$$i_2(t) = \frac{v(t)}{15{,}000} = 4 + 2.4e^{-100t} mA \quad for \quad t \geq 0$$

e) $i_1(0^+)$

$v(0^+)$ = 24 V   from earlier part.

$i_1(0^+)$.= $v(0^+)$/60,000 = 0.4 mA

6U. The switch in the following circuit has been closed a long time before opening at t = 0.

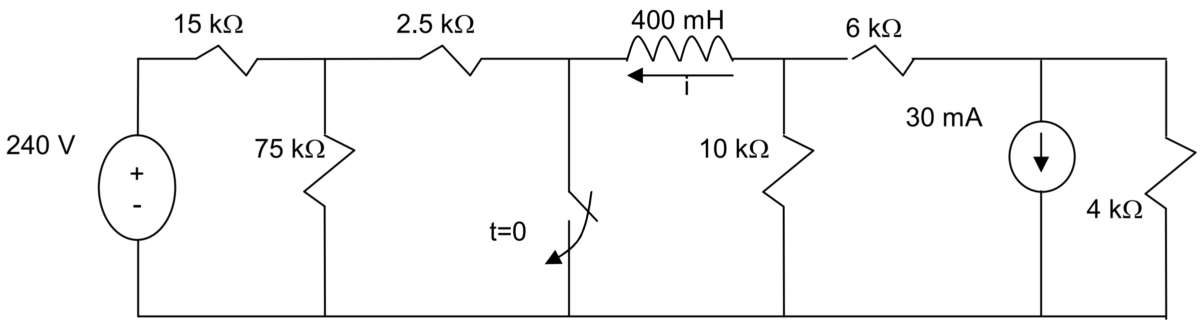


For t ≥ 0+, find:

a) $v_0(t)$. b) $i_0(t)$. c) $i_1(t)$. d) $i_2(t)$. e) $i_1(0^+)$.

Solution:

7S. At t=0 the switch is closed in the following circuit after the switch being open for a long time.



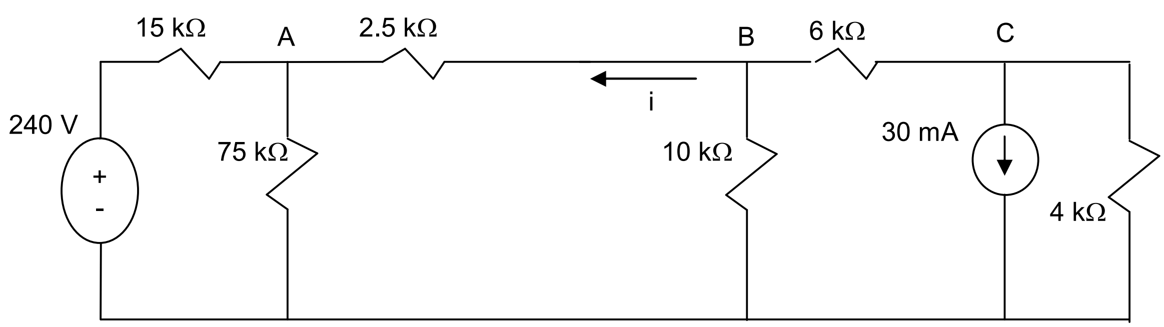
Calculate:

a) the initial value of i
b) the final value of i
c) the time constant for t ≥ 0
d) the numerical expression for i(t) when t ≥ 0.

Solution:

a) the initial value of i

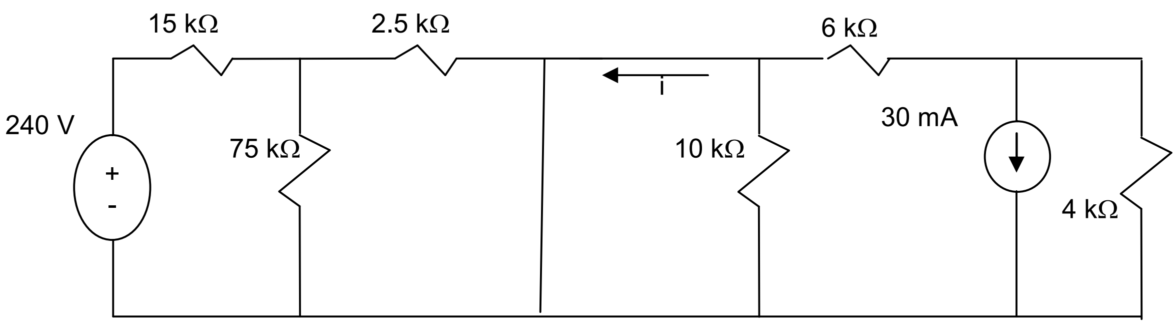
At $t=0^-$, the inductor appears as a short since the circuit has been stabilized for a long time. The circuit can be redrawn as:



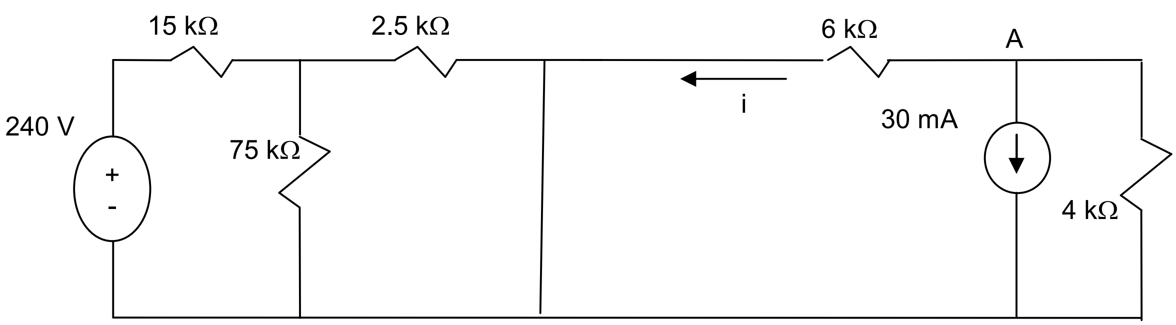
KCL Node A → $(V_A - 240)/15 + V_A/75 + (V_A - V_B)/2.5 = 0$ → $36V_A - 30V_B = 1200$
KCL Node B → $(V_B - V_A)/2.5 + V_B/10 + (V_B - V_C)/6 = 0$ → $-12V_A + 20V_B - 5V_C = 0$
KCL Node C → $(V_C - V_B)/6 + 30 + V_C/4 = 0$ → $-2V_B + 5V_C = -360$
Solve → $V_A = 37.5V$; $V_B = 5V$; $V_C = -70V$
$i(0^-) = (V_B - V_A)/2.5 = (5 - 37.5)/2.5 = -13$ mA Initial value of i

b) Final value of i

At t=∞, the inductor appears as a short and the circuit can be redrawn as:



Or can be redrawn as:

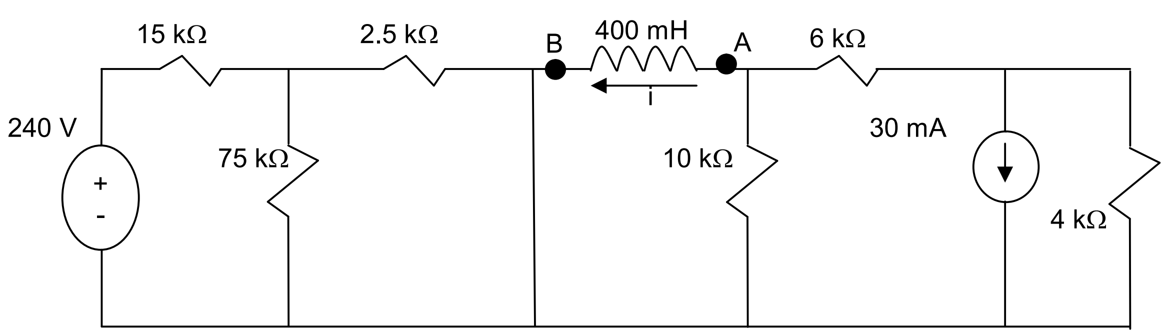


KCL → $V_A / 6 + 30 + V_A/4 = 0$ → $V_A = -72$

$i(\infty) = -72 / 6 = -12$ mA

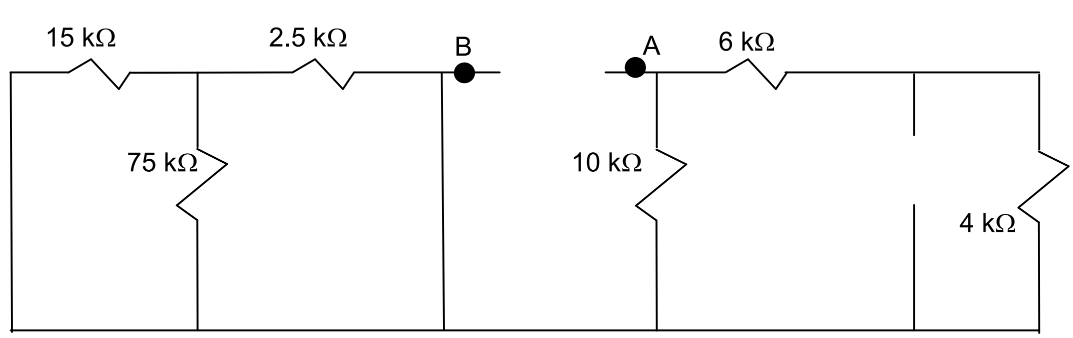
c) Time constant for $t \geq 0$

After t=0, the circuit may be redrawn as:

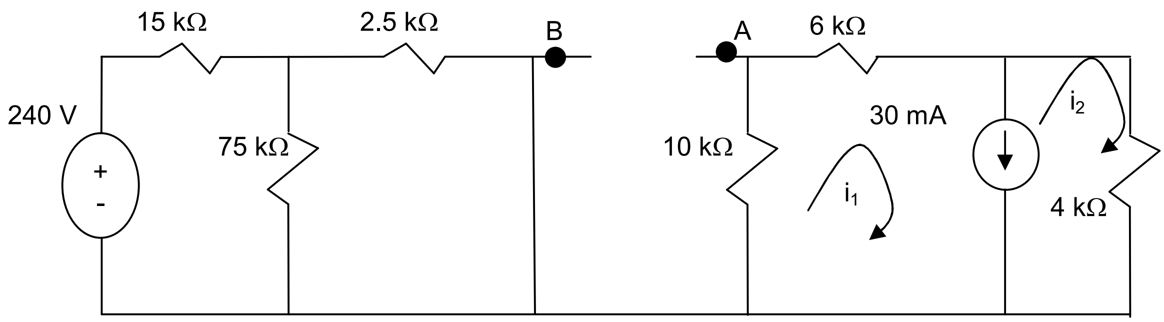


Find the Thevenin equivalent…

Calculate R_{th} by disable the sources (short for voltage source and open for current source):

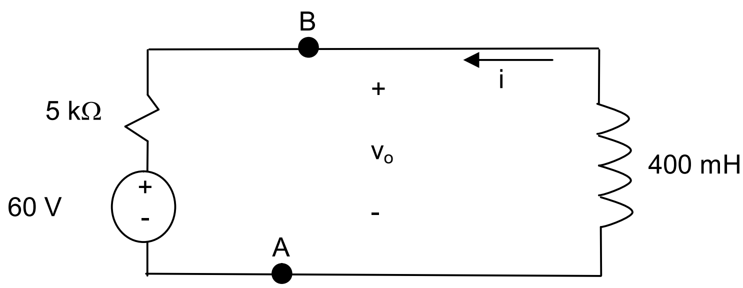


Rth = (10 || (6+4)) = 5 kΩ



Choose B as the reference (v=0) and write the supper mesh current equation:
$10i_1 + 6i_1 + 4i_2 = 0$
$i_1 - i_2 = 30$ mA
Solve → $i_1 = 6$ mA
Vth = Voc = V_{BA} = - (10,000)(0.006) = -60 V

Place the Thevenin Equivalent back into the circuit:



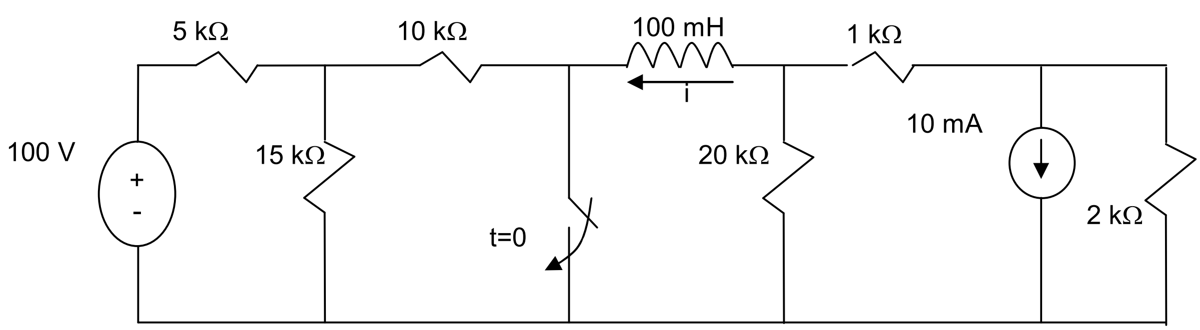
Time Constant = L/R = 0.4/5000/0.4 = 80 uSec

d) The numerical expression for i(t) when t ≥ 0.

Apply Step response for RL Circuit → $i(t) = \frac{Vs}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-(R/L)t} \; for \quad t \geq 0$

$$i(t) = -\frac{60}{5000} + \left(-0.013 - \frac{-60}{5000}\right)e^{-12,500t}) = -.012 - .001e^{-12,500t} A \quad for \quad t \geq 0$$

7U. At t=0 the switch is closed in the following circuit after the switch being open for a long time.

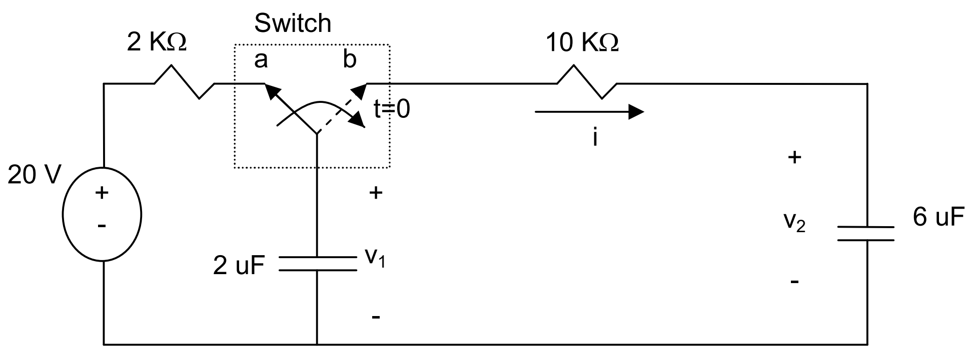


Calculate:
a) the initial value of i
b) the final value of i
c) the time constant for t ≥ 0

d) the numerical expression for i(t) when $t \geq 0$.

Solution:

8S. In the following circuit, switch has been in the "a" position for a long time. At t=0, the switch is moved to position "b":



For the above circuit:

a) Find the values of $i(0^+)$, $v_1(0^+)$ and $V_2(0^+)$.

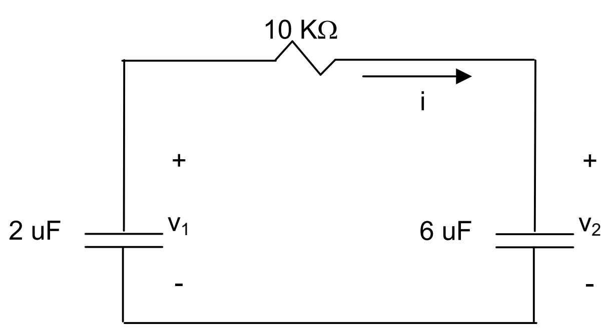
b) Calculate i(t) for t>0

Solution:

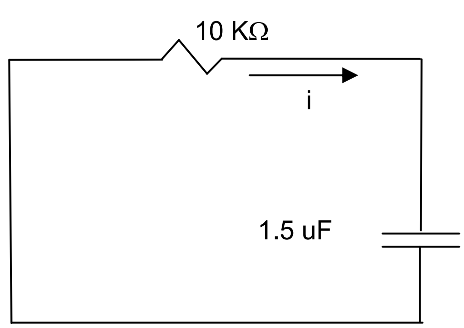
a) $i(0^+) = 0$

$v_1(0^+) = v_2(0^+) = 20$ V

b) Redraw Circuit with the new switch position



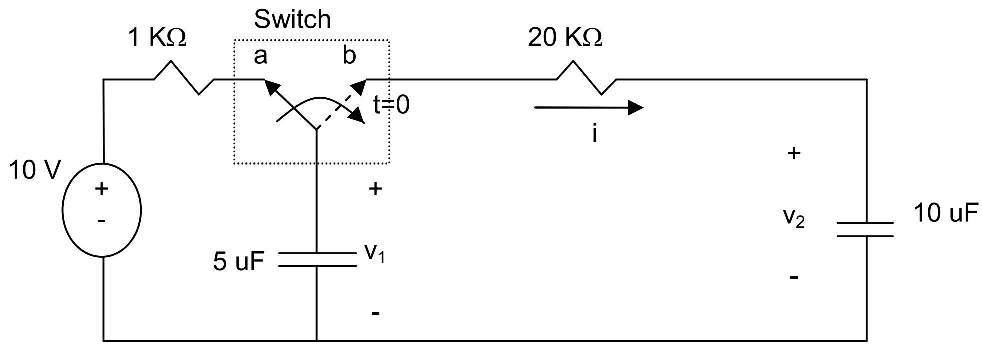
Ceq = 1/(1/2 + 1/6) = 1.5 uF



Natural Response → $v(t) = v(0)e^{-t/RC} = 20e^{-66.67t} \quad for \quad t \geq 0$

$$i(t) = C\frac{dv}{dt} = (1.5*10^{-6})(1333.4)e^{-66.67t} \quad for \quad t \geq 0$$

8U. In the following circuit, switch has been in the “a” position for a long time. At t=0, the switch is moved to position “b”:



For the above circuit:

a) Find the values of $i(0^+)$, $v_1(0^+)$ and $V_2(0^+)$.

b) Calculate i(t) for t>0

Solution: