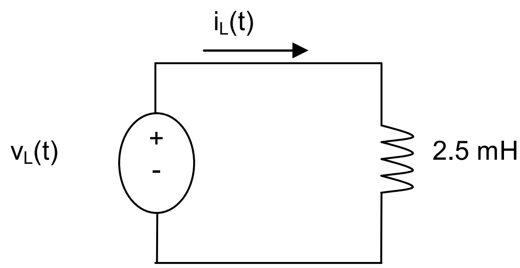
Fundamentals of Electrical Circuits - Chapter 6

1S. The current in the 2.5 mH Inductor in the following figure is known to be 0A for t<0. The inductor voltage for t≥0 is given by the expression:

$$v_L(t) = \begin{cases} 3\,e^{-4t}\ \text{mV} & 0 \le t \le 2\ \text{s} \\ -3\,e^{-4(t-2)}\ \text{mV} & 2\ \text{s} \le t \le \infty \end{cases}$$



Sketch $v_L(t)$ and $i_L(t)$ for $0 \le t \le \infty$

Solution:

$$i_L(t) = \frac{1}{L}\int_{t_0}^{t} V dt + i(t_0)$$

$0 \le t \le 2$

$$i_L(t) = \frac{1}{2.5*10^{-3}}\int_0^t 3*10^{-3} e^{-4\tau} d\tau + i_L(0)$$

$$i_L(t) = \frac{3}{2.5}\int_0^t e^{-4\tau} d\tau + 0 = \frac{3}{2.5} * \frac{1}{-4} e^{-4\tau}\Big|_0^t$$

$$i_L(t) = -.3e^{-4t} + .3 \quad \{eq1\}$$

$2 \le t \le \infty$

$$i_L(t) = \frac{1}{2.5*10^{-3}}\int_2^t -3*10^{-3} e^{-4(\tau-2)} d\tau + i_L(2^-)$$

$$i_L(t) = \frac{-3}{2.5}\int_2^t e^{-4(\tau-2)} d\tau + i_L(2^-)$$

$$i_L(t) = \frac{-3}{2.5} * \frac{1}{-4} e^{-4(\tau-2)}\Big|_2^t + i_L(2^-) \quad \{eq\ 2\}$$

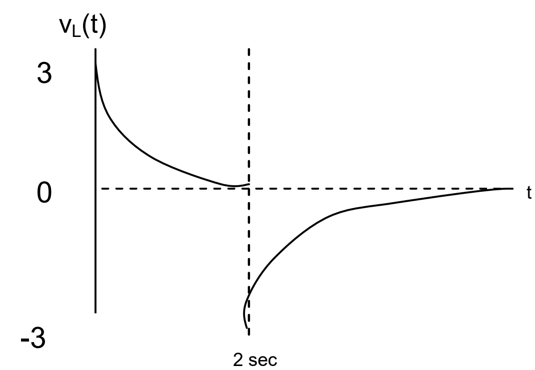
since initial current cannot change instantly then we can use $i_L(2^-)$ from equation {eq 1}:

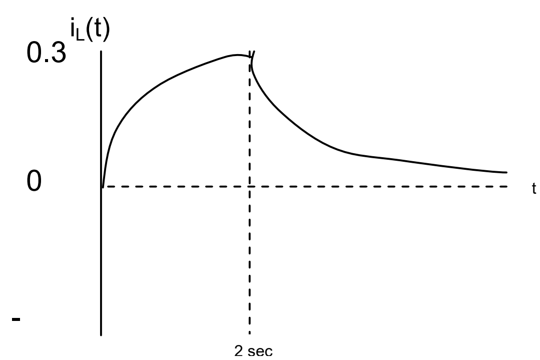
$$i_L(2^-) = -.3e^{-4(2)} + .3$$

Plug value in $i_L(2^-)$ *in {eq 2}*

$$i_L(t) = .3e^{-4(t-2)} - .3 - .3e^{-8} + .3$$

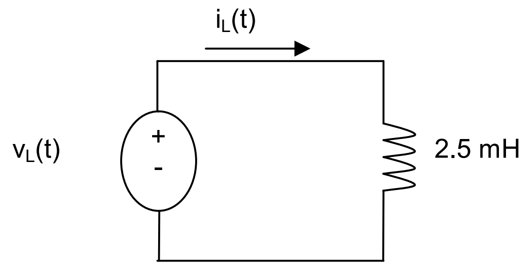
$$i_L(t) = .3e^{-4(t-2)} - .3e^{-8}$$





1U. The current in the 2.5 mH Inductor in the following figure is known to be 0A for t<0. The inductor voltage for $t \geq 0$ is given by the expression:

$$v_L(t) = \begin{cases} 3\,e^{-9t}\ \text{mV} & 0 \leq t \leq 6\ \text{s} \\ -3\,e^{-8(t-2)}\ \text{mV} & 6\ \text{s} \leq t \leq \infty \end{cases}$$



Sketch $v_L(t)$ and $i_L(t)$ for $0 \leq t \leq \infty$

Solution:

2S. A 20 uF capacitor is subjected to a voltage pulse having a duration of one seconds. The pulse is described by the following equations:

$$v_C(t) = \begin{cases} 30\,t^2\ \text{V} & 0 \leq t \leq 0.5\ \text{s} \\ 30(t-1)^2\ \text{V} & 0.5\ \text{s} \leq t \leq 1.0\ \text{s} \\ 0 & \text{elsewhere} \end{cases}$$

Sketch the current pulse that exists in the capacitor during the 0 to 1 second interval.

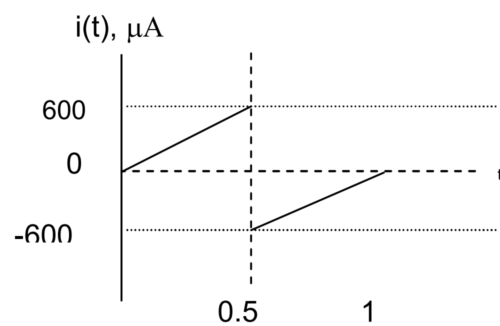
Solution:

i(0)=0 , $i = \dfrac{cdv}{dt}$

$0 \leq t \leq .5 \ = 1200t\ \mu A$

$.5 \leq t \leq 1 \ = \ 1200(t-1)\ \mu A$

elsewhere = 0 A



2U. A 10 uF capacitor is subjected to a voltage pulse having a duration of two seconds. The pulse is described by the following equations:

$$v_C(t) = \begin{cases} 10\,t^2\ \text{V} & 0 \leq t \leq 1.5\ \text{s} \\ 20(t-2)^2\ \text{V} & 1.5\ \text{s} \leq t \leq 2.0\ \text{s} \\ 0 & \text{elsewhere} \end{cases}$$

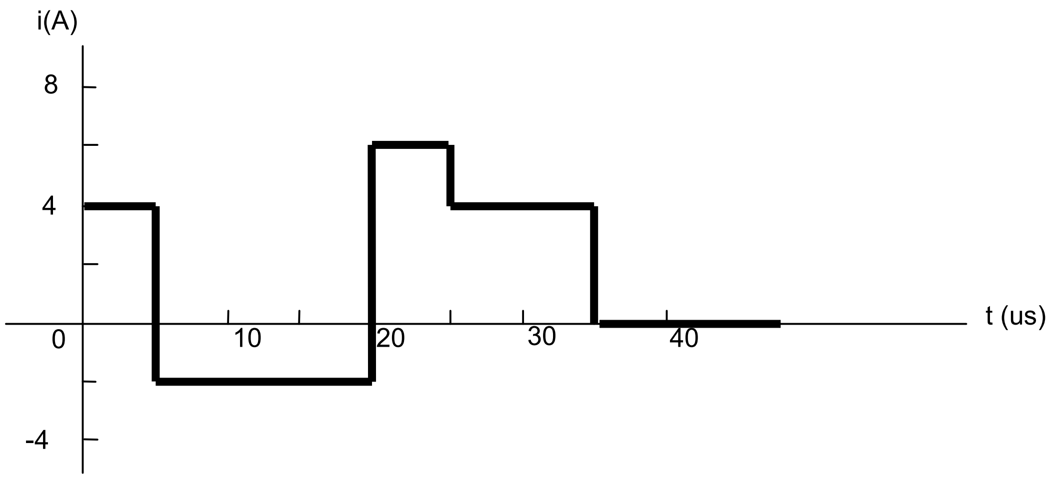
Sketch the current pulse that exists in the capacitor from t=0 to 2 seconds.

Solution:

3S. The rectangular-shaped current pulse shown in the following figure is applied to a 5 uF capacitor. The initial voltage on the capacitor is 12 V drop in the reference direction of the current. Assume the passive sign convention. Derive the expression for the capacitor voltage for the time intervals in (a)-(e).

a) $0 \leq t \leq 5$ us
b) 5 us $\leq t \leq$ 20 us
c) 20 us $\leq t \leq$ 25 us
d) 25 us $\leq t \leq$ 35 us

e) 35 us ≤ t ≤ ∞
f) Sketch v(t) over the interval a) -50 us ≤ t ≤ 300 us.



Solution:
c=5 μF
V(0)=12V

$$V = \frac{1}{c}\int_{t_0}^{t} i(t)dt + V(0)$$

a) 0 ≤ t ≤ 5 μSec

$$V(t) = 200,000\int_{0}^{t} 4dt + 12V$$ =200,000(4t)+12V = $8*10^5$t + 12V

@ t=$5*10^{-6}$ v(t)=16V

b) 5μSec ≤ t ≤ 20 μSec

$$V(t) = 2*10^5 \int_{5*10^{-6}}^{t} -2dt + V(5*10^{-6})$$ = $-4*10^5[t-5*10^{-6}]+16$ = $-4*10^5$ t + 18

@ t=$20*10^{-6}$ v(t)=10V

c) 20 μSec ≤ t ≤ 25 μSec

$$V(t) = 2*10^5 \int_{20*10^{-6}}^{t} 6dt + V(20*10^{-6})$$ =$12*10^5(t-2*10^{-5})+10$ = $12*10^5$t - 14

@ t=$25*10^{-6}$, V($25*10^{-6}$)=16V

d) 25 μSec ≤ t ≤ 35 μSec

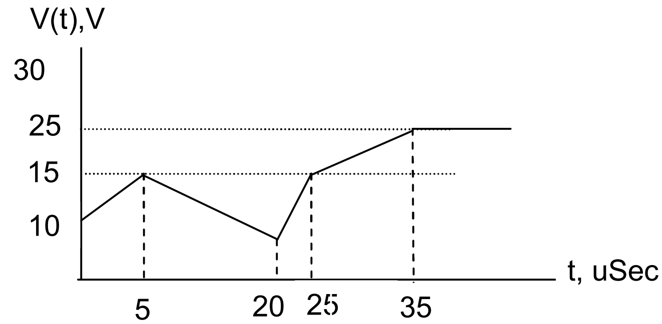
$$V(t) = 2*10^5 \int_{25*10^{-6}}^{t} 4dt + V(25*10^{-6})$$ = $8*10^5(t-25*10^{-6})+16$ = $8*10^5$t – 4

@t=$35*10^{-6}$, V($35*10^{-6}$) = 24V

e) 35 μSec ≤ t ≤ ∞

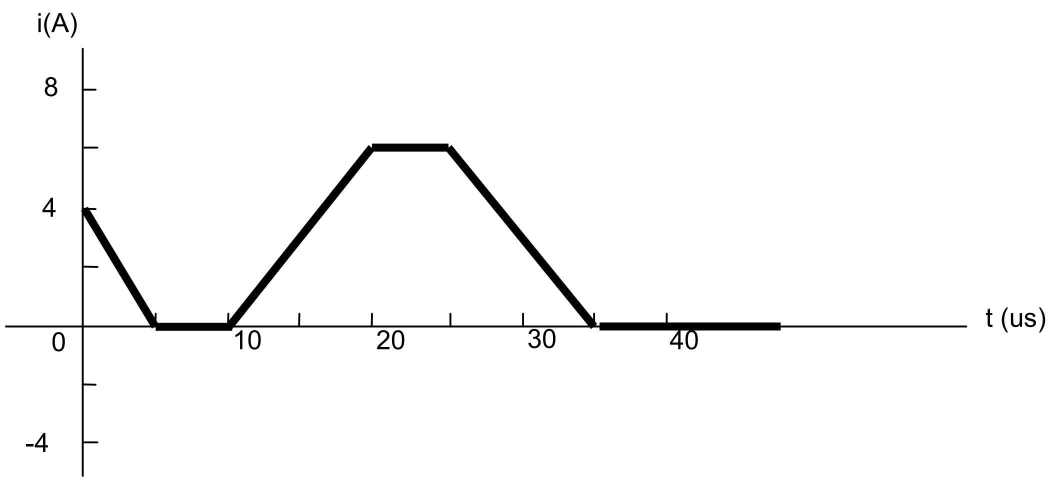
$$V(t) = 2*10^5 \int_{35*10^{-6}}^{t} 0dt + V(35*10^{-6}) = 24V$$

f)



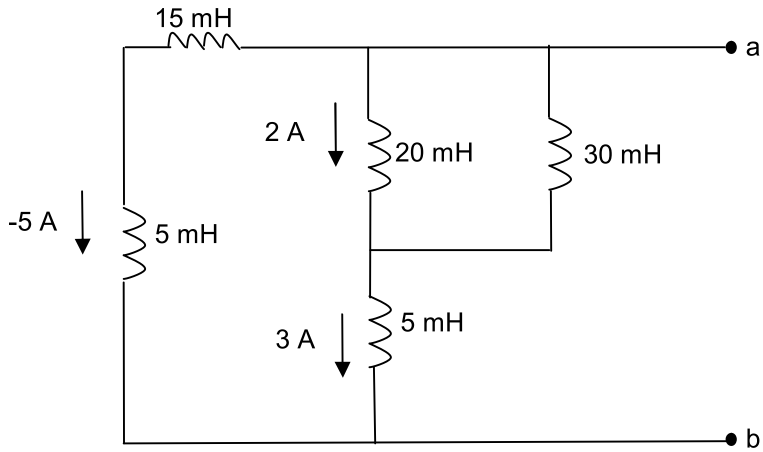
3U. The current signal is shown in the following figure is applied to a 5 uF capacitor. The initial voltage on the capacitor is 12 V drop in the reference direction of the current. Assume the passive sign convention. Derive the expression for the capacitor voltage for the time intervals in (a)-(e).

a) $0 \le t \le 5$ us
b) 5 us $\le t \le 20$ us
c) 20 us $\le t \le 25$ us
d) 25 us $\le t \le 35$ us
e) 35 us $\le t \le \infty$
f) Sketch v(t) over the interval a) -50 us $\le t \le 300$ us.



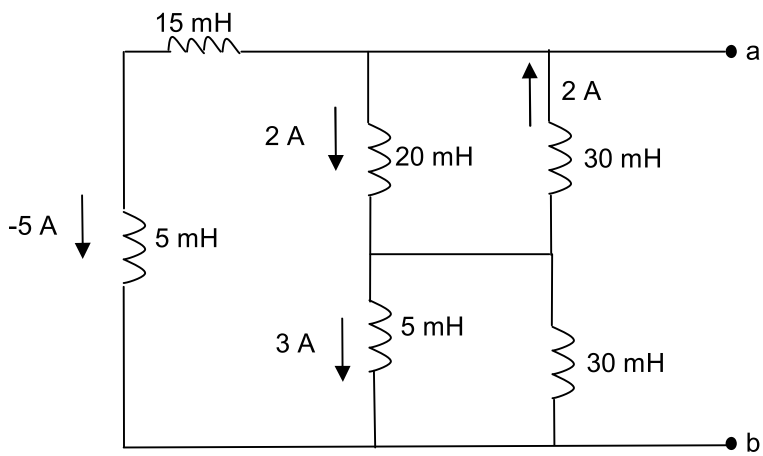
Solution:

4S. For the following circuit, find the equivalent Inductor (fully define L_{eq}) with respect to terminals a and b:



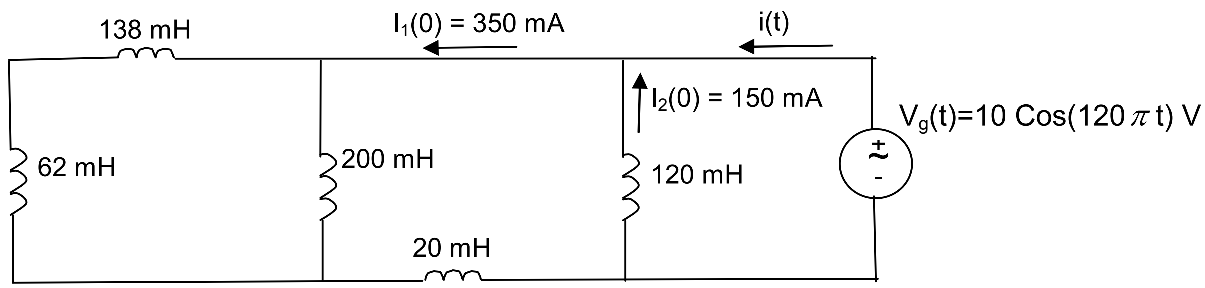
Solution
I = -5 + 3 = -2 A "Initial Condition"
L_{eq} = (5 + 15) || (20 || 30 + 5) = 9.19 mH

4U. For the following circuit, find the equivalent Inductor (fully define L_{eq}) with respect to terminals a and b:



Solution:

5S. In the following circuit, Write i(t) equation for t ≥ 0 sec.



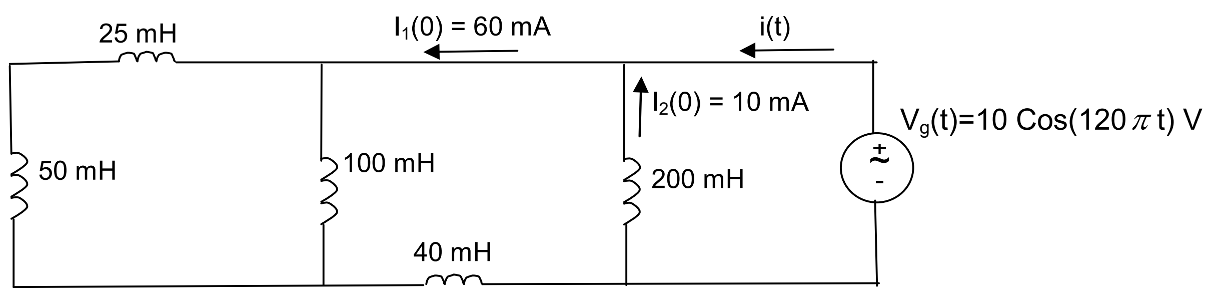
Solution:
L_{eq} = ((138+62) || 200) +20) || (120) = 60 mH

$i_{eq}(2) = 350 – 150 = 200$ mA

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0) = \frac{1}{0.06}\int_0^t 10Cos(120\pi\tau)d\tau + i(0) = +\frac{10}{0.06x120\pi}\sin(120\pi\tau\Big|_0^t + 0.2$$

$$i(t) = 0.44\sin(120\pi t) + 0.2 \quad A$$

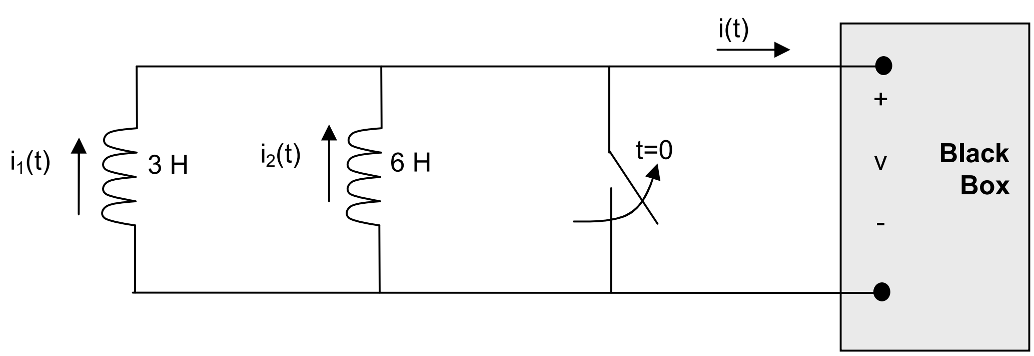
5U. In the following circuit, Write i(t) equation for t ≥ 0 sec.



Solution:

6S. The two parallel inductors in the following figure are connected across the terminals of a black box at t=0. The resulting voltage v for t ≥ 0 is known to be $12e^{-t}$ V. It is also known that $i_1(0) = 2$ A and $i_2(0) = 4$ A.

a) Replace the original inductors with an equivalent inductor and find i(t) for t ≥ 0.
b) Find $i_1(t)$ for t ≥ 0.
c) Find $i_2(t)$ for t ≥ 0.
d) How much energy is delivered to the black box in the time interval $0 \le t \le \infty$?
e) How much energy was initially stored in the parallel inductors?
f) How much energy is trapped in the ideal inductors?
g) Do your solutions for i_1 and i_2 agree with the answer obtained in (f)?



Solution:
$V(t)=12e^{-t}$ V for t≥0
$i_1(0)=2$A
$i_2(0)=4$A

a) Leq=(3||6)= 1/[(1/3)+(1/6)] =2H
i(0) = $i_1(0) + i_2(0)$ = 2+4=6 [NOTE: this is entering the negative terminal of Inductors]

$$i(t) = \frac{-1}{L}\int_0^t Vdt + i(0) = \frac{-1}{2}\int_0^t 12e^{-t}dt + 6 = 6e^{-t}\Big|_0^t + 6 = 6e^{-t} \quad A$$

b) **Refer to passive sign convention. Here i1(t) is entering Negative Voltage Point

$$i_1(t) = -\frac{1}{L_1}\int_0^t Vdt + i_1(0) = -\frac{1}{3}\int_0^t 12e^{-t} + 2$$

$$= (+4)[e^{-t}\Big|_0^t + 2 = 4e^{-t} - 2A$$

c) $i_2(t) = -\frac{1}{L_2}\int_0^t V dt + i_2(0) = -\frac{1}{6}\int_0^t 12e^{-t} + 4 = +2[e^{-t}\Big|_0^t + 4$ = $2e^{-t}+2$ A

d) How much Energy is delivered to Black Box?
$W=(1/2)L_{eq}i^2 = (1/2)(2)(6e^{-t})^2 = 36e^{-2t}$ Joules for $t \geq 0$

$t = 0$ ➔ $W_0 = 36$ Joules
$t = \infty$ ➔ $W_\infty = 0$ Joules → $W_{\text{Black Box } (0 \leq t \leq \infty)} = 36$ joules

e) Initial Energy (t=0)
L_1 → $W_0= (1/2)L_1 i_1^2 = (1/2)(3)(4e^{-t}-2)^2= (1/2)(3)(4-2)^2$
$W_{0L1}=6$ Joules

L_2 → $W_0 = (1/2)L_2 i_2^2= (1/2)(6)(2e^{-t}+2)^2 = ½ (6) (2+2)^2$
$W_{0L2}=48$ Joules

$W_{0\text{total stored energy}} = W_{0L1} + W_{0L2} = 6 + 48 = 54$ Joules

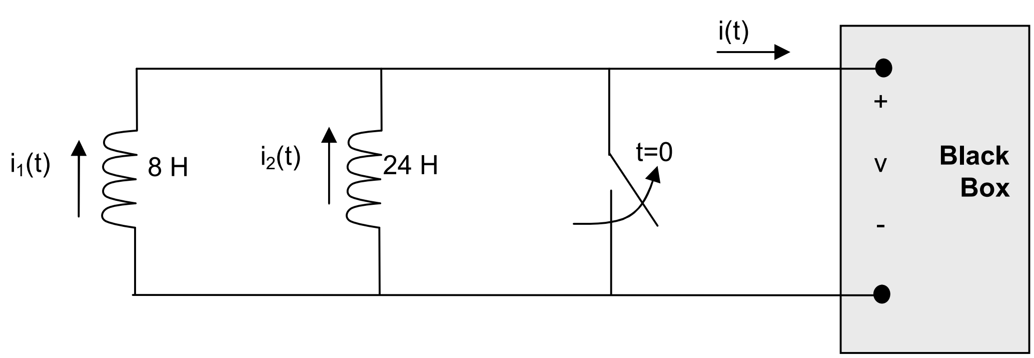
f) Trapped energy in Inductors? ($t = \infty$)
L_1 → $W_{L1}=(1/2) L_1 i_1^2 = (1/2)(3)(4e^{-t}-2)^2 = (1/2)(3)(4e^{-\infty}- 2)^2 = 6$ Joules

L_2 → $W_{L2}=(1/2)L_2 i_2^2 = (1/2)(6)(2e^{-t}+2)^2 = (1/2)(6) (4e^{-\infty}- 2)^2 = 12$ Joules

$W_{\text{total trapped}} = 6 + 12 = 18$ Joules

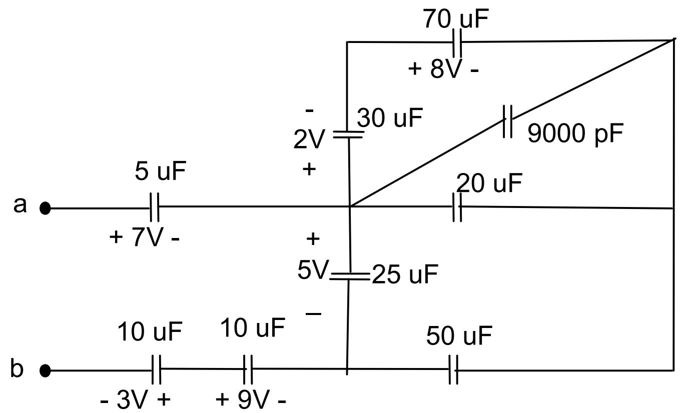
g) Energy delivered to Black Box = Inductor Initial Energy – Inductor Trapped Energy
36 = 54 – 18
36 = 36
Yes, they are equal!

6U. The two parallel inductors in the following figure are connected across the terminals of a black box at t=0. The resulting voltage v for $t \geq 0$ is known to be $8e^{-2t}$ V. It is also known that $i_1(0) = 5$ A and $i_2(0) = 8$ A.
a) Replace the original inductors with an equivalent inductor and find i(t) for $t \geq 0$.
b) Find $i_1(t)$ for $t \geq 0$.
c) Find $i_2(t)$ for $t \geq 0$.
d) How much energy is delivered to the black box in the time interval $0 \leq t \leq \infty$?
e) How much energy was initially stored in the parallel inductors?
f) How much energy is trapped in the ideal inductors?
g) Do your solutions for i_1 and i_2 agree with the answer obtained in (f)?

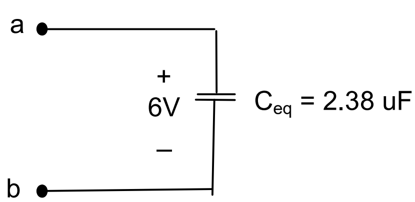


Solution:

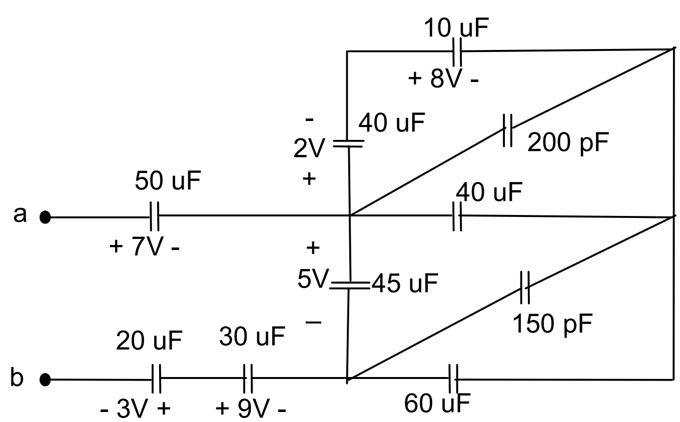
7S. Calculate the equivalent capacitance with respect to terminals a and b of the following circuit:



Solution:
Series: 30 & 70 uF → 1/(1/30 + 1/70) = 21 uF with Initial cond. 2+8 = 10 V
Parallel: 21, 20, .009 uf → 21+20+0.009= 41 uF with Initial cond. 10 V
Series: 50, 41 uf → 1/(1/50 + 1/41) = 22.5 uF
Parallel: 22.5, 25 uF → 25+22.5 = 47.5 uF with Initial cond. 5 V
Series: 5, 47.5, 10, 10 uF → 1/(1/5 + 1/47.5+1/10+1/10) = 2.38 uF
with initial Cond. +7 + 5 – 9 + 3 = 6 V

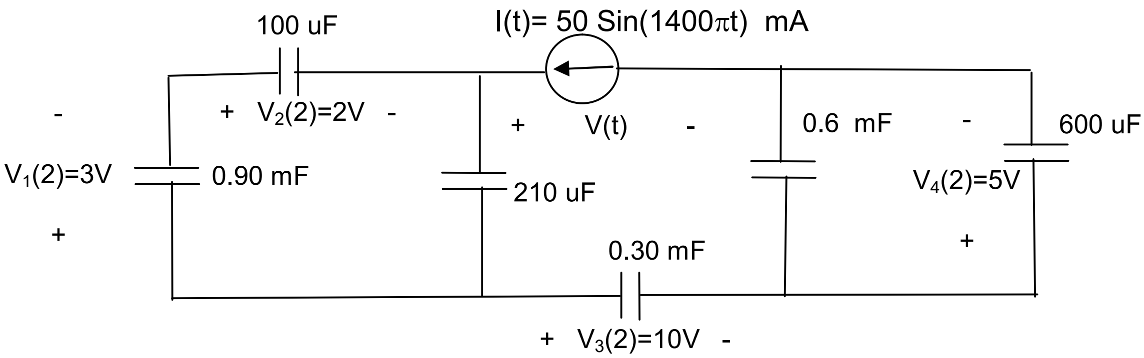


7U. Calculate the equivalent capacitance with respect to terminals a and b of the following circuit:



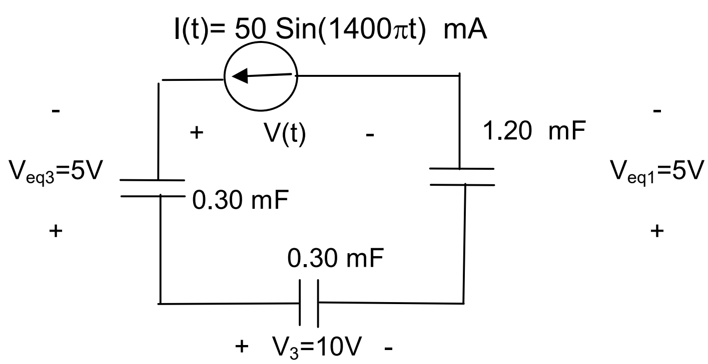
Solution:

8S. In the following circuit, find the equation for voltage across the current source, v(t) for t ≥ 2 sec.

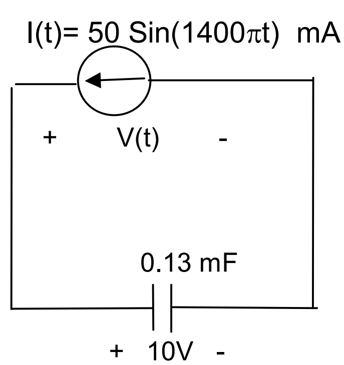


Solution:

0.6 mF in Parlallel with 600 uF → Ceq1 = 1.20 mF with V = 5 V
0.9 mF in Series with 0.1 mF → Ceq2 = 0.09 mF with V = 5V
0.09 mF in Parallel with 0.21 mF → Ceq3 = 0.30 mF with V=5 V



0.30 mF, 0.30 mF and 1.20 mF are in series → Ceq4 = 0.13 mF with V = 10-5 +5 = 10 V

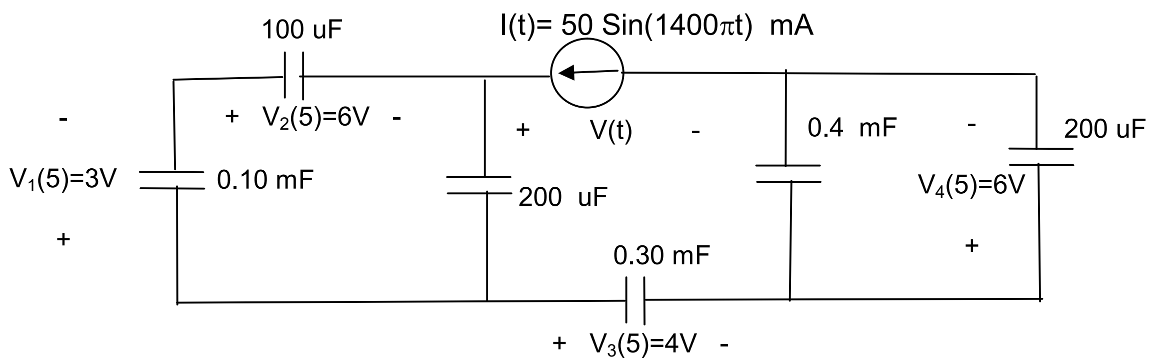


$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0) = \frac{1}{0.133*10^{-3}}\int_{2}^{t} 50*10^{-3}*\sin(1400\pi\tau)d\tau + v(2)$$

$$v(t) = -\frac{50*10^{-3}}{.133*10^{-3}*1400\pi}\cos(1400\pi t)\Big|_2^t + 10$$

$$v(t) = -0.085\cos(1400\pi t) - 10.085\ V$$

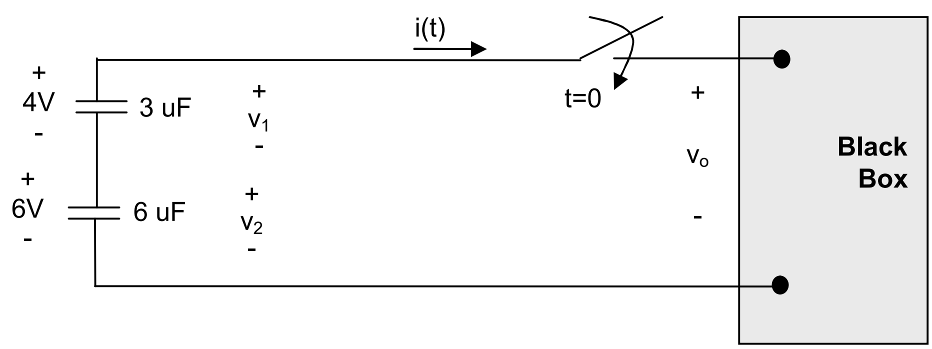
8U. In the following circuit, find the equation for voltage across the current source, v(t) for t ≥ 5 sec.



Solution:

8Sb. The two series-connected capacitors are connected to the terminals of a black box at t=0 as shown below. The resulting current i(t) for t ≥ 0 is known to be $20e^{-t}$ uA.

a) Replace the original capacitors with an equivalent capacitor and find $v_o(t)$ for t ≥ 0.
b) Find $v_1(t)$ for t ≥ 0.
c) Find $v_2(t)$ for t ≥ 0.
d) How much energy is delivered to the black box in the time interval $0 \le t \le \infty$?
e) How much energy was initially stored in the series capacitors?
f) How much energy is trapped in the ideal capacitors?
g) Do the solutions for v_1 and v_2 agree with the answer obtained in (f)?



Solution:

a) $i(t) = 20e^{-t}$ uA $\quad t \ge 0$
find C_{eq} and $V_0(t)$
$(1/C_{eq}) = (1/C_1) + (1/C_2)$ → $C_{eq} = 2$ uF

Note again that the current is flowing from negative terminal to positive which means need to have "-" between I and V relation

$$V_0 = \frac{-1}{Ceq}\int_0^t idt + V_0(0)$$

$$V_0 = \frac{-10^6}{2}\int_0^t 20e^{-t} * 10^{-6} dt + 10$$

$$V_0 = 10e^{-t}\big|_0^t + 10 = 10e^{-t}$$

b)

$$V_1(t) = -\frac{1}{3*10^{-6}}\int_0^t 20e^{-t} * 10^{-6} dt + 4 = -\frac{20}{3}\left[-e^{-t}\right]_0^t + 4 = 6.67e^{-t} - 2.67\ V$$

c)

$$V_2(t) = -\frac{1}{6*10^{-6}}\int_0^t 20e^{-t} * 10^{-6} dt + 6 = -\frac{20}{6}\left[-e^{-t}\right] + 6 = 3.33e^{-t} + 2.67\ V$$

d) We know from step a that

$$V_0 = \frac{-1}{Ceq}\int_0^t idt + V_0(0)$$

$$V_0 = \frac{-10^6}{2}\int_0^t 20e^{-t} * 10^{-6} dt + 10$$

$$V_0 = 10e^{-t}\big|_0^t + 10 = 10e^{-t}$$

$$W = (1/2)\int_0^\infty 10e^{-t} * 20e^{-t} * 10^{-6} = 10^{-6}(100)\int_0^\infty e^{-2t} = 100 * 10^{-6}$$

W = 100 μJoules

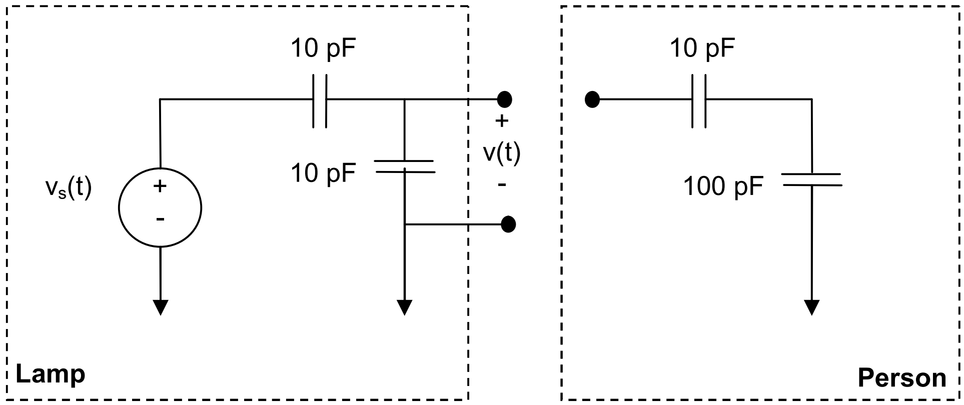
e) @ t=0
$W_{1□ =}$ (½)$cv_1(t)^2$ = (1/2)(3*10^{-6})$(4)^2$ = 24 μJoules
$W_{2 =}$ (½)$cv_2(t)^2$ = ^{2} = (1/2)(6*10^{-6})$(6)^2$ = 108 μJoules
24 μJoules + 108 μJoules = 132 μJoules

f) @ t=∞
W_1= (1/2)$CV_1(t=\infty)^2$ = (1/2)(3*10^{-6})$(-2.67)^2$ = 10.69 μJoules
W_2= (1/2)$CV_1(t=\infty)^2$ = (1/2)(6*10^{-6})$(2.67)^2$ = 21.38 μJoules
10.69 μJoules + 21.31 μJoules = 32.0 μJoules

g) 100 μJoules + 32 μJoules = 132 μJoules (Verified)

8Sc. Some lamps are made to turn on or off when the base is touched. These use a one-terminal variation of the capacitive switch design. Calculate the change in voltage v(t) when a person touches the lamp. Assume that all capacitors are initially discharged and a person can be modeled with a series of 10 pF and

100 pF capacitors.



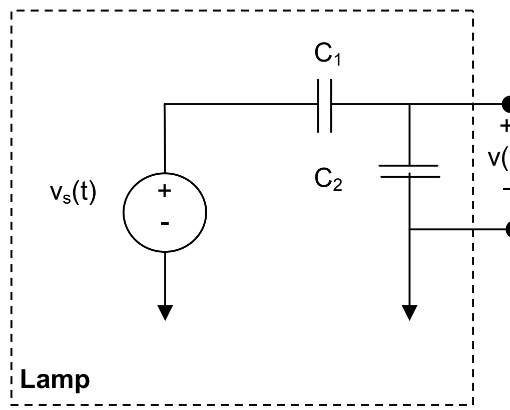
Solution:
Note that capacitors divide voltage as shown below:

$$V_s(t) = \frac{1}{C_1}\int i(t)dt + \frac{1}{C_2}\int i(t)dt$$

$$V_s(t) = (\frac{1}{C_1} + \frac{1}{C_2})\int i(t)dt$$

$$\int i(t)dt = \frac{C_1 C_2}{C_1 + C_2} V_s(t)$$

$$V(t) = \frac{1}{C_2}\int i(t)dt = \frac{C_1}{C_1 + C_2} V_s(t)$$



Using the above analysis result fot the circuit when no person touching, only 10 pF goes to ground. Therefore

v(t) = (10/20) v_s(t) =0.5 v_s(t)

With figure touching, then the capacitors representing person must be considers. There we have to calculate the equivalent capacitor to ground:

$$Ceq = \frac{1}{\frac{1}{100} + \frac{1}{10}} + 10 = 19.091\ pF$$

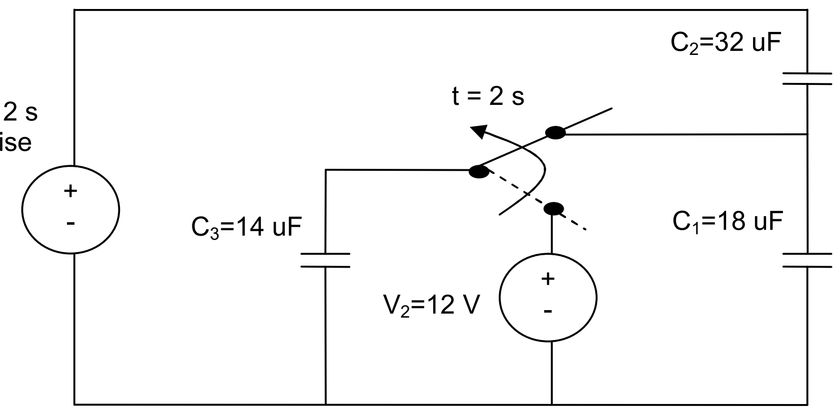
$$v(t) = \frac{10}{19.091 + 10} v_s(t) = 0.344 v_s(t)$$

$$Therefore$$

$$\Delta v(t) = 0.156 v_s(t)$$

9S. For the following circuit find the current through C_2 for t ≥ 2 s, and the total energy delivered to C_2.

$v_1(t) = 5e^{-4,000t}$ V for $t \geq 2$ s
0 Otherwise



Solution:

$V_{C3}(2) = 12$ V, $V_{C2}(2) = 0$ V, $V_{C1}(2) = 0$ V

$t \geq 2$ s

Ceq = (18 + 14) || 32 = 16 uF

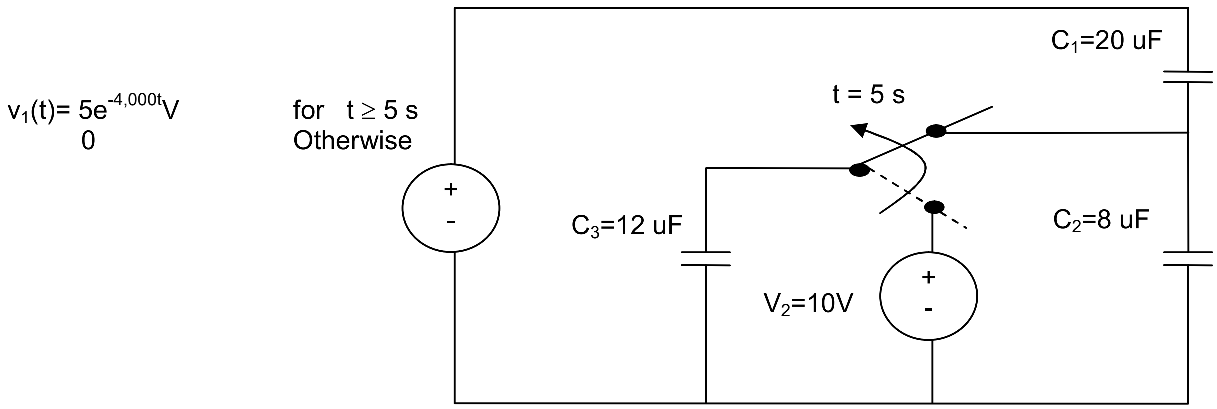
$i(t) = C dv/dt = 16\,(-20,000\, e^{-4,000t}) = -0.32\, e^{-4,000t}$ A

$$v_{c2} = \frac{1}{C_2}\int_2^t i\,dt = \frac{1}{32x10^{-6}}\int_2^t \{-0.32e^{-4000t}\}dt + v_{c2}(2) = 2.5\{e^{-4000t} - e^{-8000}\} + 0$$

$$v_{c2} = 2.5e^{-4000t} - 2.5e^{-8000} V$$

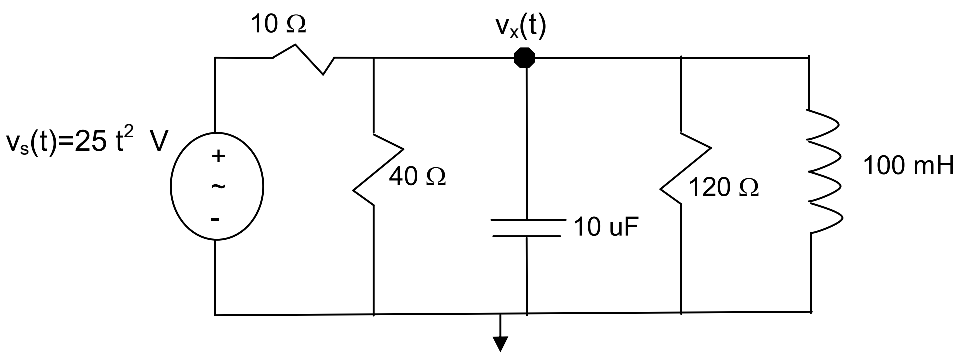
$$E_{c2,t\to\infty} = \frac{1}{2}CV^2 = \frac{32x10^{-6}}{2}\left[2.5e^{-4000t} - 2.5e^{-8000}\right]^2 \text{ Jouls}$$

9U. For the following circuit find the current through C_1 for t ≥ 5 s, and the total energy delivered to C_3.



Solution:

10S. Write an differential equation in term of $v_x(t)$ for the following circuit:



Solution:

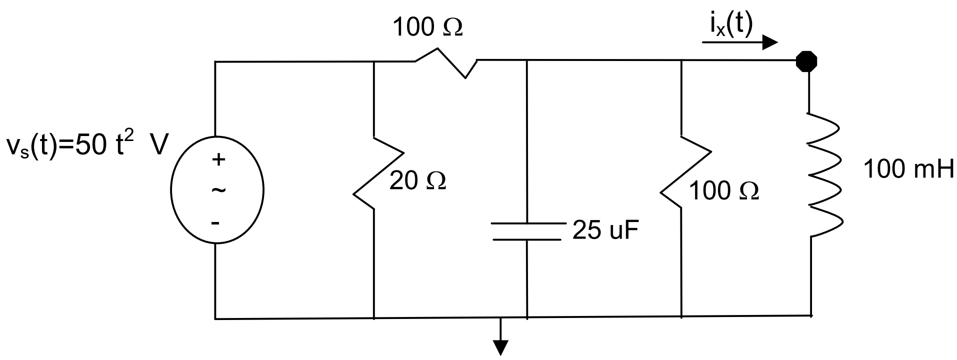
Use KCL at Node $v_x(t)$ → $\frac{v_x - 25t^2}{10} + \frac{v_x}{40} + 10^{-5}\frac{dv_x}{dt} + \frac{v_x}{120} + \frac{1}{0.1}\int_{t_0}^{t} v_x(\tau)d\tau + i_L(t_0) = 0$

Take a derivate of both side →

$$0.1\frac{dv_x}{dt} - 5t + 0.025\frac{dv_x}{dt} + 10^{-5}\frac{d^2v_x}{dt^2} + \frac{1}{120}\frac{dv_x}{dt} + \frac{v_x}{0.1} = 0$$

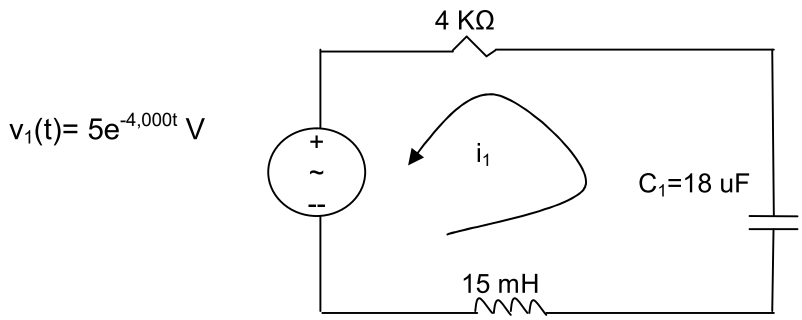
$$\frac{d^2v_x}{dt^2} + 13{,}333.33\frac{dv_x}{dt} + 10^6 v_x - 5x10^5 t = 0$$

10U. Write an differential equation in term of $i_x(t)$ for the following circuit:



Solution:

10Sb. Use KVL to write a differential equation in terms of i_1 for the following circuit:



Solution

$$4000i_1 + 5e^{-4000t} + 15*10^{-3}\frac{di_1}{dt} + \frac{10^6}{18}\int i_1 dt = 0$$

derivative:

$$15*10^{-3}\frac{d^2 i_1}{dt^2} + 4000\frac{di_1}{dt} + \frac{10^6}{18}i_1 = 20{,}000e^{-4000t}$$