## Fundamentals of Electrical Circuits - Chapter 5

1S. The Op Amp in the circuit shown below is ideal.
a) Calculate Vo if $\mathrm{Va}=4 \mathrm{~V}$ and $\mathrm{Vb}=0 \mathrm{~V}$.
b) Calculate Vo if $\mathrm{Va}=2 \mathrm{~V}$ and $\mathrm{Vb}=0 \mathrm{~V}$.
c) Calculate Vo if $\mathrm{Va}=2 \mathrm{~V}$ and $\mathrm{Vb}=1 \mathrm{~V}$.
d) Calculate Vo if $\mathrm{Va}=1 \mathrm{~V}$ and $\mathrm{Vb}=2 \mathrm{~V}$.
e) If $\mathrm{Vb}=1.6 \mathrm{~V}$, specify the range of Va such that the amplifier does not saturate.


## Solution:

a, b, c and d)
KCL at Node $V n \rightarrow \frac{V_{n}-V_{a}}{20}+\frac{V_{n}-V_{o}}{100}=0$
Ideal Op Amp $\rightarrow$ Ip $=\mathrm{In}=0 \rightarrow V_{n}=V_{p}=V_{b}$
Apply to KCL equation $\rightarrow \frac{V_{n}-V_{a}}{20}+\frac{V_{n}-V_{o}}{100}=0 \rightarrow \frac{V_{b}-V_{a}}{20}+\frac{V_{b}-V_{o}}{100}=0$
$\rightarrow V_{o}=6 V_{b}-5 V_{a}$
Use the above relationship to answer the problem parts $a, b, c$ and $d$. Note that calculated value of Vo must be limited by the $\mathrm{V}_{\mathrm{CC}}$ limits ( -15 and +15 )

| Part | $\mathrm{Va}(\mathrm{V})$ | $\mathrm{Vb}(\mathrm{V})$ | $\mathrm{Vo}(\mathrm{V})$ | Comment |
| :---: | :---: | :---: | :---: | :--- |
| a | 4 | 0 | -20 | Vo will be limited to -15 V due to saturation |
| b | 2 | 0 | -10 | Linear Region |
| c | 2 | 1 | -4 | Linear Region |
| d | 1 | 2 | 7 | Linear Region |

e)

In order for the Op Amp not to saturate its output must satisfy $\quad-15 \leq$ Vo $\leq 15$
given $V_{b}=1.6 \mathrm{~V}$
$\rightarrow-15 \leq 6 \mathrm{~V}_{\mathrm{b}}-5 \mathrm{~V}_{\mathrm{a}} \leq 15 \rightarrow-1.08 \leq \mathrm{V}_{\mathrm{a}} \leq 4.92$

1U. The Op Amp in the circuit shown below is ideal.
a) Calculate Vo if $\mathrm{Va}=2 \mathrm{mV}$ and $\mathrm{Vb}=0 \mathrm{~V}$.
b) Calculate Vo if $\mathrm{Va}=100 \mathrm{mV}$ and $\mathrm{Vb}=20 \mathrm{mV}$.
c) Calculate Vo if $\mathrm{Va}=2 \mathrm{~V}$ and $\mathrm{Vb}=1 \mathrm{~V}$.
d) Calculate Vo if $\mathrm{Va}=2 \mathrm{~V}$ and $\mathrm{Vb}=2.1 \mathrm{~V}$.
e) If $\mathrm{Vb}=1.2 \mathrm{~V}$, specify the range of Va such that the amplifier does not saturate.


## Solution:

2S. Find $\mathrm{i}_{0}$ in the following circuit if the Op Amp is ideal


## Solution:


$K C L$ at Node $V n \rightarrow-0.5+\frac{V_{n}-V_{o}}{10}=0$
Ideal Op Amp $\rightarrow$ Ip $=I n=0 \rightarrow V_{n}=V_{p}=0$
Apply to $K C L$ equation $\rightarrow V_{o}=-5 \mathrm{~V}$
$\rightarrow I \sigma=\frac{V_{o}}{R}=\frac{-5}{5}=-1 \mathrm{~mA}$

2 U . Find Ix in the following circuit if the Op Amp is ideal


## Solution:

3S. The Op Amp in the following circuit is ideal
a) What Op Amp circuit configuration is this?
b) Calculate Vo


## Solution:

a) non-Inverting amplifier - Source is connected to positive input terminal
b) $\mathrm{Vo}=$ ?

KCL at Node $V n \rightarrow \frac{V_{n}}{40}+\frac{V_{n}-V_{o}}{80}=0$
Ideal Op Amp $\rightarrow$ Ip $=$ In $=0 \rightarrow V_{n}=V_{p}=3$
Apply to $K C L$ equation $\rightarrow \frac{3}{40}+\frac{3-V_{o}}{80}=0 \rightarrow V_{o}=9 \mathrm{~V}$

3U. The Op Amp in the following circuit is ideal
a) What Op Amp circuit configuration is this?
b) Calculate Vo


## Solution:

4S. The Op Amp in the following figure is ideal.
a) What circuit configuration is shown in the figure?
b) Find Vo if $\mathrm{Va}=1.2 \mathrm{~V}, \mathrm{Vb}=-1.5 \mathrm{~V}$ and $\mathrm{Vc}=4 \mathrm{~V}$.
c) The voltage Va and Vc remain at 1.2 V and 4 V , respectively. What are the limits on Vb if the Op Amp
operates within its linear region?


Solution:
a) Inverting summing amplifier - Multiple sources are connected to negative input terminal
b) Find Vo?
$K C L$ at Node $V n \rightarrow \frac{V_{n}-V_{a}}{33}+\frac{V_{n}-V_{b}}{22}+\frac{V_{n}-V_{c}}{80}+\frac{V_{n}-V_{o}}{220}=0$
Ideal OpAmp $\rightarrow$ Ip $=I n=0 \rightarrow V_{n}=V_{p}=0$
Apply to KCL equation $\rightarrow-\frac{V_{a}}{33}-\frac{V_{b}}{22}-\frac{V_{c}}{80}-\frac{V_{o}}{220}=0 \rightarrow V_{o}=-220\left(\frac{V_{a}}{33}+\frac{V_{b}}{22}+\frac{V_{c}}{80}\right)$
$V_{o}=-4 \mathrm{~V}$
c) Limits of Vb if Va and Vc remain unchanged

We can rewrite Vo and set it to $V_{c c}$ limit to ensure linearity
$-6 \leq \mathrm{Vo} \leq 6$
$-6 \leq-220(+1.2 / 33+\mathrm{Vb} / 22+4 / 80) \leq 6$
$-6 \leq-10 \mathrm{Vb}-19 \leq 6$
$-2.5 \leq \mathrm{Vb} \leq-1.3$
4 U . The Op Amp in the following figure is ideal.
a) What circuit configuration is shown in the figure?
b) Find Vo if $\mathrm{Va}=2 \mathrm{~V}, \mathrm{Vb}=-2 \mathrm{~V}$ and $\mathrm{Vc}=4 \mathrm{~V}$.
c) The voltage Va and Vc remain at 2 V and 4 V , respectively. What are the limits on Vb if the Op Amp operates within its linear region?


## Solution:

4Sb.
a) The Op Amp in the following circuit is ideal. Find Vo if $\mathrm{Va}=4 \mathrm{~V}, \mathrm{Vb}=9 \mathrm{~V}, \mathrm{Vc}=13 \mathrm{~V}$ and $\mathrm{Vd}=8 \mathrm{~V}$.
b) Assume $\mathrm{Vb}, \mathrm{Vc}, \mathrm{Vd}$ retain their values from part (a). Specify the range Va such that the Op Amp operates within its linear region.


## Solution:

a) Find Vo where $\mathrm{Va}=4 \mathrm{~V}, \mathrm{Vb}=9 \mathrm{~V}, \mathrm{Vc}=13 \mathrm{~V}, \mathrm{Vd}=8 \mathrm{~V}$
$K C L$ at Node $V n \rightarrow \frac{V_{n}-V_{a}}{40}+\frac{V_{n}-V_{b}}{22}+\frac{V_{n}-V_{c}}{100}+\frac{V_{n}}{352}+\frac{V_{n}-V_{o}}{220}=0$
Ideal Op Amp $\rightarrow$ Ip $=$ In $=0 \rightarrow V_{n}=V_{p}=8$
Apply to KCL equation $\rightarrow \frac{8-4}{40}+\frac{8-9}{22}+\frac{8-13}{100}+\frac{8}{352}+\frac{8-V_{o}}{220}=0$
$V_{o}=14 \mathrm{~V}$
c) Range of Va so Op Amp is in linear region if all else stay the same as (a)

We can rewrite Vo in term of Va and set it to $\mathrm{V}_{\mathrm{CC}}$ limit to ensure linearity
$\frac{8-V_{a}}{40}+\frac{8-9}{22}+\frac{8-13}{100}+\frac{8}{352}+\frac{8-V_{o}}{220}=0$
$V_{o}=36-5.5 V_{a}$
$-15 \leq V_{o}=36-5.5 V_{a} \leq 15$ for linearity
$3.8 \leq V_{a} \leq 9.3$

5S. The Op Amp in the following circuit is ideal.
a) What Op Amp circuit configuration is this?
b) Find Vo in term of Vs.
c) Find the range of values for Vs such that Vo does not saturate and the Op Amp remains in its linear region of operation.


## Solution:

a) Non-inverting, source is connected to positive input terminal
b) Find Vo in-terms of Vs

KCL at Node $V n \rightarrow \frac{V_{n}}{4}+\frac{V_{n}-V_{o}}{28}=0$
Ideal Op Amp $\rightarrow I p=I n=0 \rightarrow V_{n}=V_{p}$
Voltage - divider $\rightarrow V_{p}=\left(\frac{V_{s}}{30+10}\right) * 10=\frac{V_{s}}{4}=V n$
Apply to $K C L$ equation $\rightarrow \frac{V_{s}}{16}+\frac{\frac{V_{s}}{4}-V_{o}}{28}=0 \rightarrow V_{o}=2 V_{s}$
c) Range of Vs so Op Amp is in linear region (not saturated)

$$
\begin{aligned}
& -12 \leq V_{o}=2 V_{s} \leq 8 \quad \text { for linearity } \\
& -6 \leq V_{s} \leq 4
\end{aligned}
$$

5 U . The Op Amp in the circuit shown below is ideal. What value of Rf will give the equation $\{\mathrm{Vo}=5-4 \mathrm{Va}\}$ for this circuit.


## Solution:

6 S . A voltmeter with a full-scale reading of 20 V and $10 \mathrm{M} \Omega$ internal resistances is used to measure the output voltage in the following circuit. Assuming the Op Amp is ideal, what is the reading of the voltmeter (Vm)?


## Solution:

KCL at $\mathrm{Vn} \rightarrow-250 \times 10^{-6}+(\mathrm{Vn}-(-\mathrm{Vm}) / 4000+\mathrm{In}=0$
Ideal Op Amp $\rightarrow \mathrm{In}=\mathrm{Ip}=0 ; \mathrm{Vn}=\mathrm{V} p=5 \mathrm{~V}$.
Combine the above two equations
$-250 \times 10^{-6}+(5+\mathrm{Vm}) / 4000+0=0 \rightarrow \mathrm{Vm}=-4 \mathrm{~V}$
Voltmeter Reads -4 V.
6U. Suppose the strain gage resistors in the bridge shown in following figure have the value of $120 \Omega \pm 1 \%$ ( $\Delta \mathrm{R}=1.2 \Omega$ ). The power supplies to the Op Amp $\pm 10 \mathrm{~V}$, and the reference voltage, Vref, is taken from the positive power supply.
a) Calculate the value of Rf so that when the strain gage that is lengthening reaches its maximum length, the output voltage is 5 V .
b) Suppose that we can accurately measure 50 mV changes in the output voltage. What change in strain gage resistance can be detected in milliohms?


Solution:

6Sb. Strain gage (SG) resistance is 350 ohms and is layed out on flat surface about 18 mm long SG's resitance increases by $10 \%$ when stretched by 1 mm and conversely, its resistance decreases by $10 \%$ when shrunk by 1 mm . Two SGs are attached to left and right side of a column at point of deflection.
a) Design a circuit using resistors, 12 v battery, Op Amps (Ideal), LEDs and SGs to turn on a red LED when the column is bending to the right and turn the green LED when the column is bending to the left.
b) For the the designed circuit, calculate Vog (Voltage at the Green LED) and Vor (Voltage at the Red LED) when the column is deflect at point of SG attachement by 1 mm .

## Solution:

One Possible solution:


6Sc. Find $\mathrm{i}_{\mathrm{o}}$ in the following circuit. Assume an Ideal Op Amp. Current at $\mathrm{I}_{\mathrm{a}}=0.55 \mathrm{~mA}$, at $\mathrm{I}_{\mathrm{b}}=0.75 \mathrm{~mA}$, and at $\mathrm{I}_{\mathrm{c}}=$ 0.25 mA .
$10 \mathrm{k} \Omega$


## Solution:

$\mathrm{KCL} @ \mathrm{Vn} \quad-0.55+0.75-0.25+\left(\mathrm{V}_{\mathrm{n}}-\mathrm{V}_{\mathrm{o}}\right) / 10=0$
Ideal Op Amp states: $\mathrm{I}_{\mathrm{n}}=\mathrm{I}_{\mathrm{p}}=0 \quad \& \quad \mathrm{~V}_{\mathrm{n}}=\mathrm{V}_{\mathrm{p}}=0$

$$
\begin{aligned}
-0.05 & =V_{o} / 10 \\
V_{0} & =-0.5 \mathrm{~V}
\end{aligned}
$$

$$
\mathrm{I}_{0}=\mathrm{V}_{0} / R=-0.5 / 2.5=-1 / 5=-0.20 \mathrm{~mA}
$$

7S. For the following circuit and using ideal OpAmp Model:
a) What Op Amp configuration is this?
b) Find $\mathrm{i}_{0}$.


## Solution:

a) The circuit is an inverting amplifier, because the source is connected to the negative input terminal.
b) KCL at $\mathrm{Vn}: \quad-1+(\mathrm{Vn}-\mathrm{Vo}) / 15+\mathrm{In}=0$

Apply Ideal Op Amp: $\mathrm{Ip}=\mathrm{In}=0$ and $\mathrm{Vn}=\mathrm{Vp}=0$

So,
Now,

$$
\text { Vo/15 = } 1 \text {--> Vo = -15V }
$$

$$
\mathrm{I}_{\mathrm{o}}=\mathrm{V}_{\mathrm{o}} / 5=-15 / 5 \quad-->\underline{\mathrm{I}}_{0}=-3 \mathrm{~mA}
$$

7U. Find $V_{o}$ assuming an Ideal Op Amp.


Solution:

KCL at $\mathrm{Vn} \rightarrow\left(\mathrm{V}_{\mathrm{n}}-7\right) / 5+\left(\mathrm{V}_{\mathrm{n}}-6\right) / 2+5+\left(\mathrm{V}_{\mathrm{n}}-\mathrm{V}_{\mathrm{o}}\right) / 0.250=0$
KCL at $\mathrm{Vp} \rightarrow\left(\mathrm{V}_{\mathrm{p}}-6\right) / 0.5+\mathrm{Vp} / 2.5=0$
Ideal Op Amp $\rightarrow I_{n}=I_{p}=0 \& V p=V n$
To be completed

7Sc. Calculate the value of $\mathrm{I}_{0}$ for the difference-amplifier circuit shown below (Idea Op Amp).


## Solution:

$\mathrm{KCL} @ \mathrm{Vn} \rightarrow \frac{V n-2}{3}+\frac{V n-V o}{2.5}=0$
Ideal Op Amp $\rightarrow \mathrm{Ip}=\mathrm{In}=0$ \& $\mathrm{Vn}=\mathrm{Vp}=5$
Therefore: $\frac{5-2}{3}+\frac{5-V o}{2.5}=0 \rightarrow \mathrm{Vo}=7.5 \rightarrow \mathrm{lo}=\mathrm{Vo} / 24 \mathrm{~K}=0.3125 \mathrm{~mA}$
7Sd. Calculate the value of $\mathrm{V}_{0}$ for the difference-amplifier circuit shown below (Using Idea OpAmp Model):


## Solution

$\mathrm{KCL} @ \mathrm{Vn} \rightarrow \frac{V n-6}{4}+\frac{V n-V o}{6}=0$
Ideal OpAmp $\rightarrow \mathrm{Ip}=0, \mathrm{In}=0 \& \mathrm{~V} p=\mathrm{Vn} \rightarrow \mathrm{Vp}=-15^{*} 2=-30 \mathrm{~V}$
Therefore: $\frac{-30-6}{4}+\frac{-30-V o}{6}=0 \rightarrow \mathrm{Vo}=-84 \mathrm{~V}$
The OpAmp saturates at $-5 \mathrm{~V} \rightarrow \mathrm{Vo}=-5 \mathrm{~V}$
7Se. In the following circuit, using ideal Op Amp model:
a) Find $V o$ in terms of $V_{a}$ and $I_{b}$
b) Calculate Vo for $\mathrm{V}_{\mathrm{a}}=0.1 \mathrm{~V}$ and $\mathrm{I}_{\mathrm{b}}=2 \mathrm{~mA}$


## Solution:

a) KCL at $\mathrm{Vn} \rightarrow(\mathrm{Vn}-\mathrm{Va}) / 5000+(\mathrm{Vn}-\mathrm{Vo}) / 25000=0$ $V n=V p=10000 I_{b}$

$$
\left(10000 I_{b}-\mathrm{Va}\right) / 5000+\left(10000 \mathrm{I}_{\mathrm{b}}-\mathrm{Vo}\right) / 25000=0 \rightarrow \mathrm{Vo}=60000 \mathrm{I}_{\mathrm{b}}-5 \mathrm{Va}
$$

b) $\mathrm{Vo}=5 \mathrm{~V}$ (limited by the rail - saturated)

7Sf. For the following Op Amp circuit, Find Vo using an Ideal op Amp model.


## Solution:


$K C L$ at Node $V n \rightarrow \frac{V_{n}+12}{10}+\frac{V_{n}-8}{5}+\frac{V_{n}}{40}+\frac{V_{n}-V_{o}}{40}=0$
Ideal $O p A m p \rightarrow I p=I n=0 \rightarrow V_{n}=V_{p}=4$
Apply to $K C L$ equation $\rightarrow \frac{4+12}{10}+\frac{4-8}{5}+\frac{4}{40}+\frac{4-V_{o}}{40}=0$
$V_{o}=40 \mathrm{~V}$
The output within the rail voltages...
8S. In the difference amplifier shown below, compute
a) the differential mode gain
b) the common mode gain
c) the CMRR


## Solution:

a) Common Mode Gain $=\mathrm{A}_{\mathrm{cm}}=\frac{V_{o}}{V_{c m}}=\frac{R_{a} R_{d}-R_{b} R_{c}}{R_{a}\left(R c+R_{d}\right)}=\frac{24-25}{1 *(1+24)}=-\frac{1}{25}$
b) Difference Mode Gain

$$
\mathrm{A}_{\mathrm{dm}}=\frac{V_{o}}{V_{d m}}=\frac{R_{d}\left(R_{a}+R_{b}\right)+R_{b}\left(R_{c}+R_{d}\right)}{2 R_{a}\left(R c+R_{d}\right)}=\frac{24(1+25)+25(1+24)}{2 * 1 *(1+24)}=24.98
$$

c) Common Mode Rejection Ratio $=\mathrm{CMRR}=\left|\frac{A_{d m}}{A_{c m}}\right|=\left|\frac{24.98}{-\frac{1}{25}}\right|=624.5$

8U. In the difference amplifier shown below, compute
a) the differential mode gain
b) the common mode gain
c) the CMRR


Solution:

9S. Find $i_{0}$ in the following circuit using Op Amp DC Model where open loop gain, $A=10,000$, Input resistance, Ri of $200 \mathrm{k} \Omega$ and output resistance, Ro of $50 \Omega$.


Solution:
redraw the circuit with Op Amp DC Mode and use Node Voltage to analyze the circuit:


3 essential nodes $\mathrm{Vo}, \mathrm{Vn} \&$ reference node.
KCL at $\mathrm{Vn} \rightarrow(\mathrm{Vn}-0.5) / 2,000+\mathrm{Vn} / 200,000+(\mathrm{Vn}-\mathrm{Vo}) / 10,000=0$
KCL at $\mathrm{Vo} \rightarrow(\mathrm{Vo}-\mathrm{Vn}) / 10,000+\mathrm{Vo} / 5,000+\mathrm{Vo} / 2,500+\left(\mathrm{Vo}-10^{4}(-\mathrm{Vn})\right) / 50=0$
Simplify the equations:
$121 \mathrm{Vn}-20 \mathrm{Vo}=-50$
$\left(2 \times 10^{6}-1\right) V n-207 \mathrm{~V} 0=0$
Solve to find Vo \& lo

9U. Find $V_{o}$ in the following circuit using LM 324 Op Amp DC Model where open loop gain, $A=10^{5}$, Input resistance, Ri of $2 \mathrm{M} \Omega$ and output resistance, Ro of $75 \Omega$.


## Solution:

