## Fundamentals of Electrical Circuits - Chapter 3

1S. For the circuits shown below,
a) identify the resistors connected in parallel
b) Simplify the circuit by replacing parallel connect resistors with equivalent resistor.


## Solution:

a) Resistors in Parallel
$9 \mathrm{k} \Omega||18 \mathrm{k} \Omega|| 6 \mathrm{k} \Omega$
b) Simplify

Req for parallel Resistors $=1 /(1 / 9+1 / 18+1 / 6)=3$


1U. For the circuits shown below,
a) identify the resistors connected in parallel
b) Simplify the circuit by replacing all resistors with equivalent resistor.


## Solution:

2S. Find the equivalent resistance seen by the source in the following circuit.


## Solution:

Simplify?
$\operatorname{Req}=(1200| | 600)+300+400+500=1 /(1 / 1200+1 / 600)+1200=1600 \Omega$


2 U . Find the equivalent resistance seen by the source in the following circuit.


## Solution:

2Ub. Find the equivalent resistance $\left(R_{a b}\right)$ in the following circuit with respect to terminals $a b$.


## Solution:

3S. For the following circuit,
a) Calculate the no-load voltage vo for the voltage-divider.
b) Calculate the power dissipated in R1 and R2.
c) Assuming that only $1 / 4 \mathrm{~W}$ resistor are available. The no-load voltage is to be the same as in (a). Specify the smallest value of R1 and R2.


Solution:
a) find no-load vo?

$$
\text { vo }=I R_{2}
$$

$v o=\left(50 /\left(R_{1}+R_{2}\right)\right)^{*} R_{2}=(50 /(2000+500)) * 500=10 \mathrm{~V}$
b) Powers?
$\mathrm{P}_{\mathrm{R} 1}=\mathrm{R}_{1} *\left(50 /\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)\right)^{2}=2000$ * $(50 / 2500)^{2}=0.8 \mathrm{~W}$
$P_{R 2}=R_{2}^{*}\left(50 /\left(R_{1}+R_{2}\right)\right)^{2}=500 *(50 / 2500)^{2}=0.2 \mathrm{~W}$
c) Given: $\mathrm{P}_{\mathrm{Max}} \leq 1 \mathrm{~W}$ \& vo $=10 \mathrm{~V}$; Find possible value of $R_{1} \& R_{2}$
vo $=10 \mathrm{~V} \rightarrow$ vo $=\left(50 /\left(R_{1}+R_{2}\right)\right)^{*} R_{2}=10 \rightarrow R_{1} / R_{2}=4$
$\mathrm{P}_{\mathrm{Max}} \leq 1 \mathrm{~W} \rightarrow \quad \mathrm{vo}^{2} / \mathrm{R}_{2} \leq 1 \mathrm{~W} \rightarrow 10^{2} / \mathrm{R}_{2} \leq 1 / 4 \mathrm{~W} \rightarrow \mathrm{R}_{2} \geq 400 \Omega$
$\mathrm{P}_{\text {Max }} \leq 1 \mathrm{~W} \rightarrow \quad \mathrm{vo}^{2} / \mathrm{R}_{1} \leq 1 \mathrm{~W} \rightarrow 40^{2} / \mathrm{R}_{1} \leq 1 / 4 \mathrm{~W} \rightarrow \mathrm{R}_{1} \geq 6400 \Omega$
The values of R1 \& R2 must satisfy 3 rules: $R_{1} \geq 6400 \Omega, \quad R_{2} \geq 400 \Omega \& R_{1} / R_{2}=4$
There fore the Minimum value $\rightarrow R_{2}=400 \Omega, R_{1}=6400 \Omega$

3U. For the following circuit,
a) Calculate the no-load voltage vo for the voltage-divider.
b) Calculate the power dissipated in R1 and R2.
c) Assuming that only 0.25 W resistor are available. The no-load voltage ( Vo ) is to be the same as in part (a). Specify the value of R1 and R2 that meets the limitations.


Solution: Not Provided
4S. Use voltage division or current division to find the specified voltages or currents:
a) Suppose the voltage drop across the $24 \Omega$ resistor in the following circuit A is 40 V , positive at the top of the resistor. What is the voltage drop across the $18 \Omega$ resistor?
b) Suppose the current in the $10 \Omega$ resistor in the following circuit $A$ is 60 mA , flowing from left to right. What is the current flowing in the $30 \Omega$ resistor?
c) Suppose the current in the $1.2 \mathrm{k} \Omega$ resistor in the following circuit $B$ is 9 mA , flowing from left to right.

What is the current in the $30 \mathrm{k} \Omega$ resistor?
d) Suppose the voltage drop across the $4 \mathrm{k} \Omega$ resistor in the following circuit B is 50 V , Positive at top of the resistor. What is the voltage drop across the $2.4 \mathrm{k} \Omega$ resistor?

(A)

(B)

## Solution:

a) Given $\mathrm{V}_{24 \Omega}=40 \mathrm{~V}$, Find $\mathrm{V}_{18 \Omega}$

$V_{18 \Omega}=I R=(40 /(30+18))^{*} 18=15 \mathrm{~V}$
b) Given $I_{10 \Omega}=60 \mathrm{~mA}$, Find $\mathrm{I}_{30 \Omega}=$ ?


Req $=24 \|(18+30)=16 \Omega \rightarrow V_{1}=16 * 60=960 \mathrm{mV}$
$I_{30 \Omega}=\mathrm{V} 1 /(30+18)=960 / 48=20 \mathrm{~mA}$
c) Given $I_{1.2 \mathrm{k} \Omega}=9 \mathrm{~mA}$, Find $\mathrm{I}_{30 \Omega}=$ ?


## Current Divider

$$
\begin{aligned}
& \operatorname{Req}=(1.2+7.2 \| 2.4+2)=5 \mathrm{k} \Omega \\
& \mathrm{~V} 1=(9 \mathrm{~mA})(5 \mathrm{k} \Omega)=45 \mathrm{~V} \\
& \mathrm{I}_{30 \Omega}=\mathrm{V} 1 / 30=45 / 30=1.5 \mathrm{~mA}
\end{aligned}
$$

d) Given $\mathrm{V}_{30 \mathrm{k} \Omega}=50 \mathrm{~V}$, Find $\mathrm{V}_{2.4 \Omega}=$ ?


$$
\begin{aligned}
& \operatorname{Req}=7.2 \| 2.4=1 /(1 / 7.2+1 / 2.4)=1.8 \mathrm{k} \Omega \\
& \mathrm{~V} 2=(\mathrm{V} 1 /(1.2+1.8+2))^{* 1.8=(50 / 5)^{*} 1.8=18 \mathrm{~V}}
\end{aligned}
$$

4 U . Use voltage division or current division to find the specified voltages or currents:
a) Suppose the voltage drop across the $40 \Omega$ resistor in the following circuit A is 40 V , positive at the top of the resistor. What is the voltage drop across the $10 \Omega$ resistor?
b) Suppose the current in the $10 \Omega$ resistor in the following circuit A is 60 mA , flowing from left to right. What is the current flowing in the $30 \Omega$ resistor?
c) Suppose the current in the $2 \mathrm{k} \Omega$ resistor in the following circuit B is 10 mA , flowing from left to right.

What is the current in the $60 \mathrm{k} \Omega$ resistor?
d) Suppose the voltage drop across the $4 \mathrm{k} \Omega$ resistor in the following circuit B is 50 V , Positive at top of the resistor. What is the voltage drop across the $2.4 \mathrm{k} \Omega$ resistor?


## Solution:

5S. The Current in the $9 \Omega$ resistor in the circuit shown below is 1 A .
a) Find $v_{g}$.
b) Find the power dissipated in the $20 \Omega$ resistor.


## Solution:

a) $\mathrm{Vg}=$ ?
$\mathrm{I}_{9 \Omega}=1 \mathrm{~A} \rightarrow \mathrm{~V}_{9 \Omega}=9 * 1=9 \mathrm{~V}$
Req1 $=(1+2) \| 9=2.25 \Omega$

$\mathrm{I} 1=\mathrm{V} 1 / 2.25=9 / 2.25=4 \mathrm{~A}$
$\mathrm{V} 2=11^{*}(4+2.25)=25 \mathrm{~V}$
Req2 $=(4+2.25)| | 25=5 \Omega$
Req3 $=(20| | 5)=4 \Omega$


$$
\begin{aligned}
& I 2=V 2 / 5=25 / 5=5 A \\
& V 3=12 *(5+3)=40 V
\end{aligned}
$$

Req4 $=((5+3) \| 40)=6.67 \Omega$

$\mathrm{I} 3=\mathrm{V} 3 / 6.67=40 / 6.67=6 \mathrm{~A}$
$\mathrm{V} 4=13$ * $(4+6.67)=6$ * $10.67=64 \mathrm{~V}$
$14=(\mathrm{V} 4) / 32=2 \mathrm{~A}$
$\mathrm{KCL} \rightarrow-\mathrm{Ig}+\mathrm{I} 4+\mathrm{I} 3=0 \rightarrow \mathrm{Ig}=8 \mathrm{~A}$
$\mathrm{KVL} \rightarrow-\mathrm{Vg}+\left(10^{*} 8\right)+(32 * 2)=0 \rightarrow \mathrm{Vg}=144 \mathrm{~V}$
b) $\mathrm{P}_{20 \Omega}=$ ?

Using I3 $\rightarrow \mathrm{V}_{4 \Omega}=6{ }^{*} 4=24 \mathrm{~V}$ which is the same as $\mathrm{V}_{20 \Omega}$
$\mathrm{P}_{20 \Omega}=\left(\mathrm{V}_{20 \Omega}\right)^{2} / 20=(24)^{2} / 20=28.8 \mathrm{~W}$
5 U . The Current in the $5 \Omega$ resistor in the circuit shown below is 100 mA .
a) Find $v_{s}$.
b) Find the power dissipated in the $40 \Omega$ resistor.


## Solution:

5Ub. Find the power delivered by the source (Vs = 10 Volts) in the following circuit.


## Solution:

6 . a) For the following ammeter circuit, show that the current in d'Arsonval movement is always $1 / 100^{\text {th }}$ of the current being measured.
b) What would the fraction be if the $100 \mathrm{uV}, 10 \mathrm{uA}$ movement were used in a 1 A ammeter?
c) Would you expect a uniform scale on a d'Arsonval Ammeter?


## Solution:

a) Internal Resistanct of d'Arsonval $=R_{D A}=100 u \mathrm{~V} / 10 \mathrm{uA}=10 \Omega$ so we can redraw a ohmic equivalent:


Apply the current divider $\rightarrow \mathrm{i}_{\mathrm{m}}=\mathrm{i}_{\text {meas }}(10 / 99 /(10 / 99+10)) \rightarrow \mathrm{i}_{\mathrm{m}} / \mathrm{i}_{\text {meas }}=1 / 100$ We
b) Ratio $=i_{m} / i_{\text {meas }}=10 u A / 1 \mathrm{~A}=1 / 100,000$
c) Yes, if d'Arsonval is assumed to be ideal.

6U. a) For the following ammeter circuit, what is the ratio of current through d'Arsonval movement $\left(\mathrm{i}_{\mathrm{m}}\right)$ to the current being measured ( $\mathrm{i}_{\text {meas }}$ ).

b) What would the fraction be if the 100 uV , 10 uA movement were used in a 1 A ammeter?
c) Would you expect a uniform scale on a d'Arsonval Ammeter?

Solution:

7S. A d'Arsonval voltmeter is shown below. Find the value of Rv for each of the following full Scale Readings:
a) 50 V
b) 5 V
c) 250 mV
d) 25 mV


## Solution:

* We know that d'Arsonval movement has Ram $=20 \mathrm{mV} / 1 \mathrm{~mA}=20 \Omega$
* In general we have $\mathrm{Vv}=\mathrm{Iv}(\mathrm{Rv}+20)$ Since $\operatorname{Max} \mathrm{V}$ is given in the question, we should use $\mathrm{I}=1 \mathrm{~mA}$ which is causes maximum movement as the Iv
a) $\mathrm{Vv}=50 \quad \mathrm{~V} \rightarrow 50=.001(\mathrm{Rv}+20)-\rightarrow \mathrm{Rv}=49,980 \Omega$
b) $\mathrm{Vv}=5 \quad \mathrm{~V} \rightarrow 5=.001(\mathrm{Rv}+20)-\rightarrow \mathrm{Rv}=4,980 \Omega$
c) $\mathrm{Vv}=250 \mathrm{mV} \rightarrow .250=.001(\mathrm{Rv}+20)-\rightarrow \mathrm{Rv}=230 \Omega$
d) $\mathrm{Vv}=25 \mathrm{mV} \rightarrow .025=.001(\mathrm{Rv}+20)-\mathrm{Rv}=5 \Omega$

7U. A d'Arsonval voltmeter is shown below. Find the value of Rv for each of the following full Scale Readings:
a) 100 V
b) 25 V
c) 500 mV
d) 5 mV


## Solution:

8 S . The bridge circuit shown below is energized from a 6 V dc source. The bridge is balanced when $\mathrm{R}_{1}=200 \Omega$ , $\mathrm{R}_{2}=500 \Omega$ and $\mathrm{R}_{3}=800 \Omega$.
a) What is the value $R_{x}$.
b) How much current (in milliamperes) does the dc source supply?
c) Which resistor in the circuit absorbs the most power? How much power does it absorb?
d) Which resistor absorbs the least power? How much power does it absorb?


## Solution:

a) Balanced Bridge $\rightarrow$ lo $=0$ and $V_{1}=V_{2}$

Currents through R1 and R3 are the same I1
Current through R2 and Rx are the same 12
$V=6=11^{*}(200+800) \rightarrow I 1=6 \mathrm{~mA}$
$\mathrm{V} 1=800$ * $6 \mathrm{~mA}=4.8 \mathrm{~V}=\mathrm{V} 2$
$\mathrm{V}_{\mathrm{R} 1}=\mathrm{V}_{\mathrm{R} 2} \rightarrow 200 * 11=500 * 12 \rightarrow 200 * 6=500^{*} 12 \rightarrow \mathrm{I} 2=2.4 \mathrm{~mA}$
$\mathrm{V}_{\mathrm{R} 3}=\mathrm{V}_{\mathrm{Rx}} \rightarrow 800 * 11=\mathrm{R}_{\mathrm{x}}{ }^{*} 12 \rightarrow 800 * 6=\mathrm{R}_{\mathrm{x}}{ }^{*} 2.4 \rightarrow \mathrm{R}_{\mathrm{x}}=2,000 \Omega$
b) I = current of dc source = ?
$\mathrm{I}=\mathrm{I} 1+\mathrm{I} 2=6+2.4=8.4 \mathrm{~mA}$
c) Which resistor absorbs most power?
$\mathrm{P}_{\mathrm{R} 1}=(200)^{*}\left(6^{*} 10^{-3}\right)^{2}=7.2 \mathrm{~mW}$
$\mathrm{P}_{\mathrm{R} 2}=(500)^{*}\left(2.4^{*} 10^{-3}\right)^{2}=2.88 \mathrm{~mW}$
$\mathrm{P}_{\mathrm{R} 3}=(800)^{*}\left(6^{*} 10^{-3}\right)^{2}=28.8 \mathrm{~mW}$
$P_{R x}=(2000)^{*}\left(2.4^{*} 10^{-3}\right)^{2}=11.5 \mathrm{~mW}$
R3 $=800 \Omega$ absorbs the most power equal to 28.8 mW
d) Which resistor absorbs least power?
$\mathrm{R} 2=500 \Omega$ absorbs the most power equal to 2.88 mW
8 U . The bridge circuit shown below is energized from a 10 V dc source. The bridge is balanced when $\mathrm{R}_{1}=100 \Omega$ ,$R_{2}=200 \Omega$ and $R_{3}=1200 \Omega$.
a) What is the value $R_{x}$.
b) How much current (in milliamperes) does the dc source supply?
c) Which resistor in the circuit absorbs the most power? How much power does it absorb?
d) Which resistor absorbs the least power? How much power does it absorb?


## Solution:

9S. a) Find the equivalent resistance Rab in the circuit shown below by using $\Delta$-to- $Y$ transformation involving the resistors R2,, R3 and R4.
b) Repeat (a) using a Y-to- $\Delta$ transformation involving resistors R2, R4 and R5.
c) Give two additional $\Delta$-to-Y or Y-to- $\Delta$ transformations that could be used to find Rab.


## Solution:

a) $\Delta$-to- Y transformation with R2, R3 and R4.


$$
\begin{aligned}
& R 1 t=\frac{R b t R c t}{R a t+R b t+R c t}=\frac{10 * 50}{40+10+50}=5 \Omega \\
& R 2 t=\frac{R a t R c t}{R a t+R b t+R c t}=\frac{40 * 10}{40+10+50}=4 \Omega \\
& R 3 t=\frac{R a t R b t}{R a t+R b t+R c t}=\frac{40 * 50}{40+10+50}=20 \Omega
\end{aligned}
$$

New Circuit with Transformation

$\operatorname{Req}(a b)=13+5+((8+4) \|(20+4))+7=33 \Omega$
b) Y-to- $\Delta$ transformation with R2, R4 and R5.


$$
\begin{aligned}
& R a t=\frac{R 1 t R 2 t+R 2 t R 3 t+R 3 t R 1 t}{R 1 t}=\frac{10 * 8+8 * 40+40 * 10}{10}=80 \Omega \\
& R b t=\frac{R 1 t R 2 t+R 2 t R 3 t+R 3 t R 1 t}{R 2 t}=\frac{10 * 8+8 * 40+40 * 10}{8}=100 \Omega \\
& R c t=\frac{R 1 t R 2 t+R 2 t R 3 t+R 3 t R 1 t}{R 3 t}=\frac{10 * 8+8 * 40+40 * 10}{40}=20 \Omega
\end{aligned}
$$

New Circuit with Transformation
$\operatorname{Req}(\mathrm{ab})=13+20 \|((50| | 100)+(4| | 80))+7=33 \Omega$
c) Identify at least a couple of transforms


9U. a) Find the equivalent resistance Rab in the circuit shown below by using $\Delta$-to- $Y$ transformation involving the resistors R2,, R3 and R4.
b) Repeat (a) using a Y-to- $\Delta$ transformation involving resistors R2, R4 and R5.
c) Give two additional $\Delta$-to- Y or Y -to- $\Delta$ transformations that could be used to find Rab.


Solution:

