## Fundamentals of Electrical Circuits - Chapter 2

1S. The terminal voltage and terminal current were measured on the device shown below. The values of $v$ and I are given in the table below. Use the values in the table to construct a circuit model for the device consisting of a single resistor.


## Solution:

OR

| $I(\mathrm{~mA})$ | $\mathrm{v}(\mathrm{V})$ |
| :---: | ---: |
| -20 | -160 |
| -10 | -80 |
| 10 | 80 |
| 20 | 160 |
| 30 | 240 |

Solution:


1U. The terminal voltage and terminal current were measured on the device shown below. The values of $v$ and I are given in the table below. Use the values in the following table to construct a circuit model for the device consisting of a single resistor.


Solution:
2S. Given the circuit below, use KCL method to find:
a) the value of $i_{a}$
b) the value of $i_{b}$
c) the value of $v_{0}$
d) the power dissipated in each resistor
e) the power delivered by the 200 V source.


## Solution:

a) $\mathrm{ia}=$ ?


KCL at $\mathrm{V} 1 \rightarrow(\mathrm{~V} 1-200) / 40+(\mathrm{V} 1-0) / 300+(\mathrm{V} 1-0) / 75=0$
Simplify equation $1 \rightarrow 25 \mathrm{~V} 1=3000 \rightarrow \mathrm{~V} 1=120 \mathrm{~V}$
$\mathrm{la}=(\mathrm{V} 1-0) / 300=120 / 300=0.4 \mathrm{~A}$
b) $i_{b}=$ ?
$\mathrm{i}_{\mathrm{b}}=(\mathrm{V} 1-0) / 75=120 / 75=1.6 \mathrm{~A}$
c) $\mathrm{v}_{\mathrm{o}}=$ ?
$\mathrm{V}_{\mathrm{o}}=\mathrm{V} 1=120 \mathrm{~V}$
e) Power at voltage source.
$\mathrm{i}=(\mathrm{V} 1-200) / 40=-2 \mathrm{~A}$
$P_{200 v}=v * i=200 *(-2)=-400 \mathrm{~W}$ Generated

2U. Given the circuit shown below find:
a) the value of $i_{a}$
b) the value of $i_{b}$
c) the value of $v_{x}$
d) the power dissipated in each element (resistors)
e) the power delivered by the 100 V source.


## Solution:

3S. For the following circuit use KCL method to:
a) Find the current ig and io in the circuit shown below.
b) Find the voltage vo
c) Verify that the total power developed equals the total power dissipated.


## Solution:


a) Find the current $i_{g}$ and $i_{0}$ ?

Given $\rightarrow(\mathrm{V} 1-0) / 60=4 \rightarrow \mathrm{~V} 1=240 \mathrm{~V}$
$\mathrm{I}_{\mathrm{o}}=(\mathrm{V} 1-0) / 120=2 \mathrm{~A}$
KCL at $\mathrm{V} 1 \rightarrow-\mathrm{ig}+\mathrm{V} 1 / 60+\mathrm{V} 1 / 120=0$
$\rightarrow \mathrm{ig}=6 \mathrm{~A}$
b) $\mathrm{v}_{0}=$ ?
$\mathrm{V}_{\mathrm{o}}=\mathrm{V} 1=240 \mathrm{~V}$
c) Total Power Developed ?= Total Power Dissipated
$P_{\text {developed }}=\mathrm{vo}$ *ig $=240$ * $6=1440 \mathrm{~W}$
$\mathrm{P}_{\text {dissipated }}=\mathrm{P}_{60 \Omega}+\mathrm{P}_{30 \Omega}+\mathrm{P}_{90 \Omega}=(60)^{*}(4)^{2}+(30)^{*}(2)^{2}+(90)^{*}(2)^{2}=1440 \mathrm{~W}$
Therefore: $P_{\text {developed }}=P_{\text {dissipated }}=1440 \mathrm{~W}$
3U. For the following circuit use KCL method to:
a) Find the current ig and io in the circuit shown below.
b) Find the voltage vo
c) Verify that the total power developed equals the total power dissipated.


## Solution:

4 S . The voltage and current were measured at the terminals of the device shown below along with the tabulated results.
a) Construct a circuit model for this device using an ideal voltage source and a resistor.

b) Use the model to predict the amount of power the device will deliver to a $40 \Omega$ resistor.

## Solution:

a) Construct a circuit model
when $\mathrm{L}_{\mathrm{L}}=0$ "Open Circuit" $\rightarrow$ Source Voltage Vs $=30 \mathrm{~V}$.
When $\mathrm{V}_{\mathrm{L}}=0$ "Open Circuit" $\rightarrow$ Source Resistance Rs $=30 / 2=15 \mathrm{~V}$.

b) Power delivered to 40 ohms.


$$
\begin{aligned}
& \mathrm{i}=31 /(15+40)=30 / 55=0.55 \mathrm{~A} \\
& \mathrm{P}_{40 \Omega}=\mathrm{r}^{\star} \mathrm{i}^{2}=40^{*}(0.55)^{2}=11.9 \mathrm{~W} \text { "Consumed" }
\end{aligned}
$$

4 U . The voltage and current were measured at the terminals of the device shown below along with the tabulated results
a) Construct a circuit model for this device using an ideal voltage source and a resistor.

b) Use the model to predict the amount of power the device will deliver to a $10 \mathrm{k} \Omega$ resistor.

## Solution:

5S. For the following circuit:
a) Find the power dissipated in each resistor.
b) Find the power supplied by the 240 V ideal voltage source
c) Verify that the power supplied equals the total power dissipated


## Solution:


a) Power dissipated in each resistor
$\mathrm{KVL} \mathrm{I}_{1} \rightarrow-240+4\left(\mathrm{I}_{1}-\mathrm{I}_{2}\right)+20\left(\mathrm{I}_{1}-\mathrm{I}_{3}\right)=0$
$\left.\mathrm{KVLI} I_{2} \rightarrow 4\left(I_{2}-I_{1}\right)+5\left(I_{2}-I_{3}\right)+22.5 I_{2}=0\right)$
$\left.\mathrm{KVLI}_{3} \rightarrow 20\left(\mathrm{I}_{3}-\mathrm{I}_{1}\right)+5\left(\mathrm{I}_{3}-\mathrm{I}_{2}\right)+15 \mathrm{I}_{3}=0\right)$
Simplify
$24 I_{1}-4 I_{2}+20 I_{3}=240$
$-4 I_{1}+31.5 I_{2}-5 I_{3}=0$
$-20 I_{1}-5 I_{2}+40 I_{3}=240$
solve:

$$
\rightarrow \mathrm{i}_{1}=19 \mathrm{~A}, \mathrm{i}_{2}=4 \mathrm{~A}, \mathrm{i}_{3}=10 \mathrm{~A},
$$

$\left.\mathrm{P}_{4 \Omega}=\left(\mathrm{I}_{1}-\mathrm{i}_{2}\right)\right)^{2} \mathrm{R}=(19-4)^{2}(4)=900 \mathrm{~W}$
$\left.P_{20 \Omega}=\left(l_{1}-i_{3}\right)\right)^{2} R=(19-10)^{2}(20)=1620 \mathrm{~W}$
$\left.P_{5 \Omega}=\left(I_{3}-i_{2}\right)\right)^{2} R=(10-4)^{2}(5)=180 \mathrm{~W}$
$\left.P_{15 \Omega}=\left(\mathrm{i}_{3}\right)\right)^{2} \mathrm{R}=(10)^{2}(15)=1500 \mathrm{~W}$
$\left.P_{22.5 \Omega}=\left(\mathrm{i}_{2}\right)\right)^{2} R=(4)^{2}(22.5)=360 \mathrm{~W}$
b) Power delivered by Voltage Source
$\mathrm{P}_{240 \mathrm{~V}}=\mathrm{VI}=(240)(19)=4560 \mathrm{~W}$
c) Verified power deliver is equal to power dissipated
$P_{\text {dissipated }}=P_{4 \Omega}+P_{20 \Omega}+P_{5 \Omega}+P_{15 \Omega}+P_{22.5 \Omega}=900+1620+180+1500+360=4560=P_{\text {delivered }}$
This fact means the system is valid.
5 U . For the following circuit, use KVL method to:
a) Find the power dissipated in each resistor.
b) Find the power supplied by the 120 V ideal voltage source
c) Verify that the power supplied equals the total power dissipated


## Solution:

5 Sb . The current $i_{a}$ and $i_{b}$ in the following circuit are 4 A and 2 A respectively. Apply KVL to find the power delivered by the current source.


## Solution

Loop1 $\rightarrow 8 I_{1}+12 I_{1}+4 I_{1}+24\left(I_{1}-I_{3}\right)+4\left(I_{1}-I_{2}\right)=0$ Loop2 $\rightarrow \mathrm{I}_{2}-\mathrm{I}_{3}=\mathrm{Ig}$ Other relationships:
$I_{z}=-I_{a}=-4 \mathrm{~A}$
$\mathrm{I}_{1}=\mathrm{I}_{\mathrm{b}}=2 \mathrm{~A}$
Plug $I_{1} \& I_{3}$ in Loop $1 \rightarrow$
$24(2)+24(2+4)+4\left(2-I_{2}\right)=0 \rightarrow I_{2}=50 \mathrm{~A}$
From Loop2 eq. $\rightarrow \mathrm{Ig}=50-(-4)=54 \mathrm{~A}$
From Loop $2 \mathrm{KVL} \rightarrow-80+4\left(\mathrm{I}_{2}-\mathrm{I}_{1}\right)+\mathrm{Vg}+12 \mathrm{I}_{2}=0$ $-80+4(50-2)+\mathrm{Vg}+12(50)=0 \rightarrow \mathrm{Vg}=712 \mathrm{~V}$
$P=\lg \mathrm{Vg}=(54)^{*}(712)=38,448 \mathrm{~W}$


6S. For the following circuit, use KVL Method to:
a) Find the voltage $v_{y}$.
b) Show that the power generated in the circuit equals the total power absorbed.


## Solution:

a) Find Vy

KVL at $\mathrm{I}_{1} \rightarrow-7.2+55 \times 10^{3} \mathrm{I}_{1}+0.7+\left(\mathrm{I}_{1}-\mathrm{I} 2\right) 200=0$
KVL at $\mathrm{I}_{2} \rightarrow \mathrm{I}_{2}=-49$
Therefore $\rightarrow-7.2+55 \times 10^{3} \mathrm{I}_{1}+0.7+\left(\mathrm{I}_{1}+49\right) 200=0 \rightarrow \mathrm{I}_{1}=-0.178 \mathrm{~A}$
KVL at $I_{2} \rightarrow 200\left(I_{2}-I_{1}\right)-V y+500\left(I_{2}\right)+9=0 \rightarrow 200(-49+0.178)-V y+500(-49)+9=0$ $\mathrm{V} y=-34255.4 \mathrm{~V}$
b) $\mathrm{P}_{\text {generated }}=\mathrm{P}_{\text {absorbed ? }}$

| Power | Absorbed (+) | Generated (-) |
| :---: | :---: | :---: |
| -(7.2)( $\mathrm{I}_{1}$ ) $=-(-7.2)(-0.178)$ | 1.28 |  |
| $(55000)^{*} \mathrm{I}_{1}{ }^{2}=(55000)^{*}(0.178)^{2}$ | 1,742.62 |  |
| $+(0.7)\left(\mathrm{I}_{1}\right)=(0.7)(-0.178)$ |  | -1.39 |
| $200(11-12)^{2}=200(-0.178+49)^{2}$ | 476,727.50 |  |
| $(\mathrm{Vy})(-\mathrm{l} 2)=(-34255.4)(49)$ |  | -1,678,515 |
| $500 * \mathrm{I}_{2}^{2}=500^{*}(49)^{2}$ | 1,200,500 |  |
| 9*(-49) |  | -441 |
| Total: | 1,678,971 W | -1,678,957 W |

6U. For the following circuit, use KVL Method to:
a) Find the voltage $v_{y}$.
b) Show that the power generated in the circuit equals the total power absorbed.


Solution:

6 Sb. For the following circuit, find value $R_{x}$ such that la $=0$ for all values of $R_{a}$.


## Solution



## Option A - Shot cut SOLUTION

For la to be zero, we must have V1 = V2 which means the two sets of resistors must be proportional:

$$
10 / 20=40 / \mathrm{Rx} \rightarrow \mathrm{Rx}=80 \mathrm{k} \Omega
$$

## Option B - FULL SOLUTION

Condition $\rightarrow I_{3}-I_{2}=I_{a}=0 \rightarrow I_{3}=i_{2}$
KVL Loop $1 \rightarrow-10+50,000 I_{1}+10,000\left(I_{1}-I_{2}\right)+20,000\left(I_{1}-I_{3}\right)=0 \rightarrow 80,000 I_{1}-30,000 I_{2}=10$
KVL Loop $\left.2 \rightarrow 10,000\left(I_{2}-I_{1}\right)+40,000 I_{2}\right)=0 \rightarrow 50,000 I_{2}-10,000 I_{1}=0 \rightarrow I_{1}=5 I_{2}$
KVL Loop $3 \rightarrow 20,000\left(I_{3}-I_{1}\right)+R_{x} I_{3}=0 \rightarrow\left(20,000+R_{x}\right) I_{2}-20,000 I_{1}=0$
Plug second equation into first loop equation $\rightarrow 400,000 \mathrm{i} 2-30,000 \mathrm{i} 2=0 \rightarrow \mathrm{i} 2=1 / 37,000$
Use third equation to find $\mathrm{Rx}=80 \mathrm{k} \Omega$
6 Sc. The current $i_{a}$ and $I_{b}$ in the following circuit are 4A and 2A respectively. Apply KVL to find the power delivered by the current source.


## Solution:

KVL Loop1 $\rightarrow(8+12+4) 2+24(\lg +6)+4(4+2)=0 \rightarrow \lg =-9 \mathrm{~A}$ KVL Loop2 $\rightarrow \mathrm{Vg}=-(6+10)(\mathrm{lg}+4)-24(\mathrm{ig}+6) \rightarrow \mathrm{Vg}=152 \mathrm{~V}$

Therefore
$P=\lg \mathrm{Vg}=(-9)^{*}(152)=-1368 \mathrm{~W}$


Or
Mesh $1 \rightarrow 8 l_{1}+12 I_{1}+4 I_{1}+24\left(I_{1}-I_{3}\right)+4\left(l_{1}-I_{2}\right)=0$
Mesh $2 \rightarrow \mathrm{I}_{2}-\mathrm{I}_{3}=\mathrm{Ig}$
Other relationships:

$$
I_{2}=-I_{a}=-4 \mathrm{~A}
$$

$$
I_{1}=I_{b}=2 \mathrm{~A}
$$

Plug $I_{1} \& I_{2}$ in Loop $1 \rightarrow$

$$
24(2)+24\left(2-I_{3}\right)+4(2+4)=0 \rightarrow I_{3}=5 \mathrm{~A}
$$

From Loop2 eq. $\rightarrow \lg =-4-(5)=-9 \mathrm{~A}$
From Loop $2 \mathrm{KVL} \rightarrow-80+4\left(\mathrm{I}_{2}-\mathrm{I}_{1}\right)+\mathrm{Vg}+12 \mathrm{I}_{2}=0$
$-80+4(-4-2)+V g+12(-4)=0 \rightarrow V g=152 \mathrm{~V}$
$P=\lg V g=(-9)^{*}(152)=-1368 \mathrm{~W}$


7S. Based on the following model, draw a circuit model of the path of current through the human body for a person touching a voltage source with both hands who has both feet at the same potential as negative
terminal of the voltage source


Physiological Reaction
Barley Perceptible
Extreme Pain
Muscle Paralysis
Heart Stoppage

Current
3-5 mA $35-50 \mathrm{~mA}$ 50-70 mA 500 mA

## Solution:



7 U. Suppose the power company installs some equipment that could provide a 250 V shock to a human being. Is the current that results dangerous enough to warrant posting a warning sign and take other precautions to prevent such a shock?
Assume that if the source is 250 V , the resistance of the arm is $400 \Omega$, the resistance of the trunk is $50 \Omega$ and the resistance of the leg is $200 \Omega$. Use the model given below.


Physiological Reaction
Barley Perceptible
Extreme Pain
Muscle Paralysis
Heart Stoppage

Current 3-5 mA $35-50 \mathrm{~mA}$ $50-70 \mathrm{~mA}$ 500 mA

## Solution:

8S. To understand why the voltage level is not the sole determinant of potential injury due to electrical shock, consider the case of a static electricity shock. When you shuffle your feet across a carpet, your body becomes charged. The effect of this charge is that your entire body represents a voltage potential. When you touch a metal door knob, a voltage different is created between you and the doorknob, and current flows -- but the conduction material is air, not your body!

Suppose the model of the space between your hand and the doorknob is a $1 \mathrm{M} \Omega$ resistance. What voltage potential exists between your hand and the doorknob to cause mild shock, if the current causing mild shock is 3 mA ?

## Solution:



$$
V=I^{*} R=\left(3^{*} 10^{-3}\right)\left(10^{6}\right)=3,000 \mathrm{~V}
$$

8 U . To understand why the voltage level is not the sole determinant of potential injury due to electrical shock, consider the case of a static electricity shock. When you shuffle your feet across a carpet, your body becomes charged. The effect of this charge is that your entire body represents a voltage potential. When
you touch a metal door knob, a voltage different is created between you and the doorknob, and current flows -- but the conduction material is air, not your body!

Suppose the model of the space between your hand and the doorknob is a $2 \mathrm{M} \Omega$ resistance. What voltage potential exists between your hand and the doorknob causes extreme pain, if the current causing extreme pain is 40 mA ?

## Solution:

8Sb. A person accidentally grabs one of the conductors of a 220 volts source with both bare hands while standing bare feet in a puddle of water where the other conductor lays. Given the resistance of each Arm is $400 \Omega$, resistance of each leg is $200 \Omega$, and resistance of torso is $50 \Omega$, draw the ideal circuit model of this configuration and calculate the current flow through the person's torso.

## Solution:

Req $=(400| | 400)+50+(200| | 200)=350 \Omega$
$\mathrm{I}=\mathrm{V} / \mathrm{R}=220 / 350=0.63 \mathrm{~A}$


8Sc.A bare foot person accidentally steps on a conductor to one terminal of a DC voltage source while holding to the other terminal. Draw the equivalent circuit diagram and calculate the minimum voltage from the source that would cause electric shock.

Note: Human torso, leg and arm have $50 \Omega, 200 \Omega$ and $400 \Omega$ resistance respectively. 35 mA current through human body is sufficient to shock.

## Solution:

$\mathrm{V}=\mathrm{IR}$
$\mathrm{V} \geq(35 \mathrm{~mA})(650 \Omega)=22750 \mathrm{mV}$


8Sd. A person accidentally grabs conductors connected to each end of a dc voltage source, one in each arm. Assuming each arm has a resistance of $400 \Omega$, Calculate the minimum voltage from the dc voltage source required to shock the person ( 35 mA is sufficient to shock). Draw the equivalent circuit diagram.

## Solution:

$$
\begin{aligned}
& V=I R \\
& V \geq(35 \mathrm{~mA})(800 \Omega)=28000 \mathrm{mV} \\
& V \geq 28 \mathrm{~V}
\end{aligned}
$$



