

# Fundamentals of Electrical Circuits - Chapter 1

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- 1S. A high-resolution computer display monitor has 1280x1024 picture elements, or pixels. Each picture element contains 24 bits of information. If a byte is defined as 8 bits, how many megabytes (MB) are required per display?

Given:  $1280 \times 1024$  pixels per display  
 $1 \text{ pixel} = 24 \text{ bits}$   
 $8 \text{ bits} = 1 \text{ byte}$

**Solution:**

$$(1280 \times 1024) \text{ pixels} \left( \frac{24 \text{ bits}}{1 \text{ pixel}} \right) \left( \frac{1 \text{ byte}}{8 \text{ bits}} \right) = 3.93 \times 10^6 \text{ bytes}$$

$$3.93 \times 10^6 \text{ bytes} \left( \frac{1 \text{ MB}}{1 \times 10^6 \text{ bytes}} \right) = 3.93 \text{ MB}$$

There are 3.93 MB per display.

Note: 1K bits is equal 1024 bits but here 1K bits is approximated to 1,000.

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- 1U. A high-resolution computer display monitor has 2560x2044 picture elements, or pixels. Each picture element contains 24 bits of information. If a byte is defined as 8 bits, how many megabytes (MB) are required per display?

**Solution:**

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- 2S. Some species of bamboo can grow 250 mm/day. Assuming the individual cells in the plant are 10  $\mu\text{m}$  long, how long, on average, does it take a bamboo stalk to grow a 1-cell length?

Given: grow 250 mm/day  
 $1 \text{ cell} = 10 \mu\text{m}$

**Solution:**

$$250 \frac{\text{mm}}{\text{day}} \left( \frac{1 \text{ m}}{1000 \text{ mm}} \right) \left( \frac{1 \text{ cell}}{10 \mu\text{m}} \right) \left( \frac{10^6 \mu\text{m}}{1 \text{ m}} \right) \left( \frac{1 \text{ day}}{24 \text{ hours}} \right) \left( \frac{1 \text{ hour}}{3600 \text{ s}} \right) = 0.28935 \frac{\text{cells}}{\text{s}}$$

$$\left( 0.28935 \frac{\text{cells}}{\text{s}} \right) t = 1 \text{ cell}$$

$$t = \frac{1}{0.28935} \text{ s}$$

$$t = 3.5 \text{ s}$$

It will take about 3.5s to grow 1 cell

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- 2U. Colorado Spruce Trees grow an average of 8.5 inches per year. How long does it take for this type of tree to grown 10  $\mu\text{m}$ .

**Solution:**

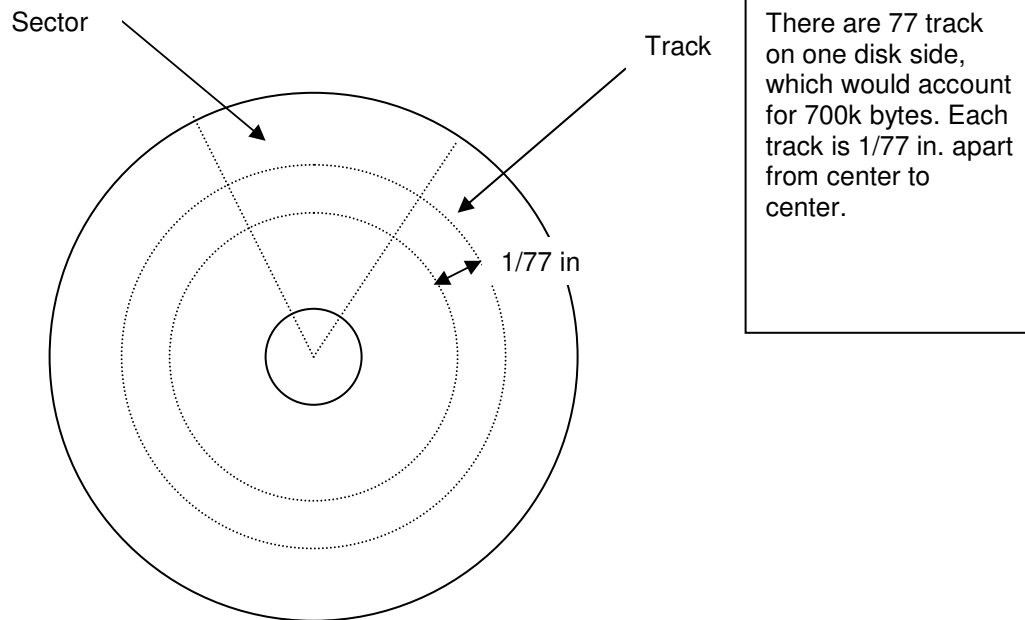
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- 3S. A double-sided 3 1/2 " floppy disk holds 1.4 MB. The bits of data are stored on circular tracks, with 77 tracks per side. The radius of the innermost track is 1/2 ", while the radius of the outermost track is 1 1/2 ". The number of bits per track is the same, and there are eight bits in one byte. How much area does a bit stored on the innermost track occupy, in square micrometers?

Given: double-sided 1.4 MB

77 tracks per side  
 inner radius  $\frac{1}{2}$  "  
 outer radius  $1\frac{1}{2}$  "  
 8 bits = 1 byte

**Solution:**

77 tracks in 1 inch =  $77 \frac{\text{tracks}}{\text{in}}$ , meaning that every track is  $\frac{1}{77}$  in apart from center to center



$$\left( \frac{700000 \text{ bytes}}{77 \text{ tracks}} \right) = 9090.909091 \frac{\text{bytes}}{\text{track}}$$

- Each track length is like the circumference of a circle  $\text{circumference} = 2\pi r$

$$\frac{9091 \text{ bytes}}{2\pi(0.5) \text{ in}} \left( \frac{8 \text{ bits}}{\text{byte}} \right) = 23150 \frac{\text{bits}}{\text{in}}$$

$$\frac{1}{23150 \frac{\text{bits}}{\text{in}}} = 4.3196467 \times 10^{-5} \frac{\text{in}}{\text{bit}}, \text{ which is the length of each bit}$$

- We now know the length of each bit, and the width of each bit, so we can now find the area of each bit.

$$4.3196467 \times 10^{-5} \text{ in} * \frac{1}{77} \text{ in} = 5.60993 \times 10^{-7} \text{ in}^2$$

$$5.60993 \times 10^{-7} \text{ in}^2 \left( \frac{2.54 \text{ cm}}{1 \text{ in}} \right)^2 \left( \frac{10^4 \mu\text{m}}{1 \text{ cm}} \right)^2 = 362 \mu\text{m}^2 \text{ per bit}$$

There are  $362 \mu\text{m}^2$  per bit on a cd

- 3U. A Blu-ray disc holds about 128 Giga Bytes (8 bits / byte) of data on four layers. Assuming that each bit takes  $405 \text{ nm}^2$ . What is the minimum area required for each layer of Blu-ray?

**Solution:**

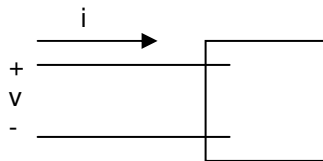
- 4S. The current entering the upper terminal of element shown below is  $i = 20\cos(5000t) \text{ A}$ .

Assume the charge at the upper terminal is zero at the instant the current is passing through its maximum value. Find the expression for  $q(t)$ .

**Solution:**

Given:  $i = 20\cos(5000t) \text{ A}$ .

Max current = zero charge ( $t=0$ )



Theory:

$$i = \frac{dq}{dt}$$

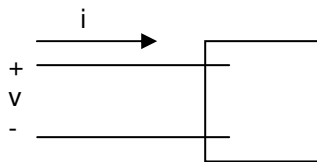
$$q(t) = \int i \, dt$$

$$i = \frac{dq}{dt} = 20\cos(5000t)$$

$$q = \int_0^t i \, dt = \int_0^t 20\cos(5000t) \, dt = \frac{20}{5000} \sin(5000t) \Big|_0^t = \frac{2}{500} \sin(5000t) \text{ C} = 4\sin(5000t) \text{ mC}$$

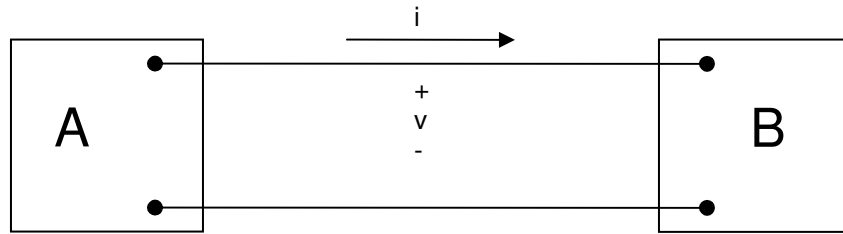
- 4U. The current entering the upper terminal of element shown below is  $i = 50\sin(2000t) \text{ A}$ .

Assume the charge at the upper terminal is zero at the instant the current is passing through its minimum value. Find the expression for  $q(t)$ .



**Solution:**

- 5S. Two electric circuits, represented by boxes A and B, are connected as shown below. The reference direction for the current  $I$  in the interconnection and the reference polarity for the voltage  $v$  across the interconnection are as shown in the figure. For each of the following sets of numerical values, calculate the power in the interconnection and state whether the power is flowing from A to B or visa versa.

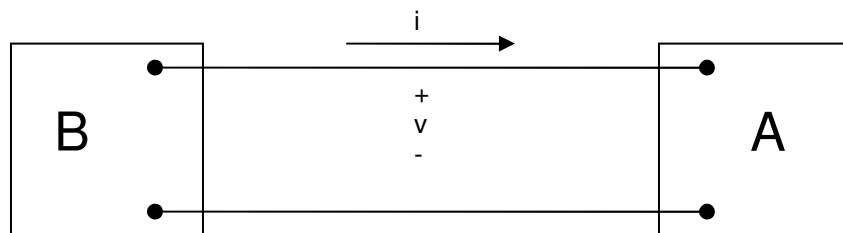


	<b><i>I</i>(A)</b>	<b><i>v</i>(V)</b>
<b>1</b>	15	20
<b>2</b>	-5	100
<b>3</b>	4	-50
<b>4</b>	-16	-25

**Solution:**

	<b><i>I</i>(A)</b>	<b><i>v</i>(V)</b>	<b><i>P</i>=<i>IV</i> (W)</b>	<b>Direction</b>
<b>1</b>	15	20	300	A to B
<b>2</b>	-5	100	-500	B to A
<b>3</b>	4	-50	-200	B to A
<b>4</b>	-16	-25	400	A to B

5U. Two electric circuits, represented by boxes A and B, are connected as shown below. The reference direction for the current *I* in the interconnection and the reference polarity for the voltage *v* across the interconnection are as shown in the figure. For each of the following sets of numerical values, calculate the power in the interconnection and state whether the power is flowing from A to B or visa versa.



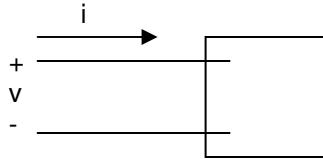
	<b><i>I</i>(A)</b>	<b><i>v</i>(V)</b>
<b>1</b>	10	29
<b>2</b>	-5	-5
<b>3</b>	8	10
<b>4</b>	16	-25

**Solution:**

6S. The voltage and current at the terminals of the circuit element shown below are zero for  $t < 0$ . For  $t \geq 0$  they are:

$$v = 50e^{-1600t} - 50e^{-400t} \quad V$$

$$i = 5e^{-1600t} - 5e^{-400t} \quad mA$$



- Find the power at  $t = 625 \text{ usec}$
- How much energy is delivered to the circuit element between 0 and 625 usec?
- Find the total energy delivered to the element.

**Solutions:**

**a) Find the power at  $t = 625 \text{ usec}$**

*Theory :*

$$P = IV$$

$$P(t) = I(t)V(t)$$

*Solution :*

$$\begin{aligned} P(t) &= (5e^{-1600t} - 5e^{-400t}) \left( \frac{1}{1000} \right) (50e^{-1600t} - 50e^{-400t}) \\ &= \left( \frac{1}{1000} \right) [250e^{-3200t} - 250e^{-2000t} - 250e^{-2000t} + 250e^{-800t}] \\ &= \frac{1}{4} (e^{-3200t} - 2e^{-2000t} + e^{-800t}) \\ P(625 \times 10^{-6}) &= \frac{1}{4} (e^{-3200 \times 625 \times 10^{-6}} - 2e^{-2000 \times 625 \times 10^{-6}} + e^{-800 \times 625 \times 10^{-6}}) = 42.21 \text{ mW} \\ P(625 \times 10^{-6}) &= 42.21 \text{ mW} \end{aligned}$$

**b) How much energy is delivered to the circuit element between 0 and 625 usec?**

*Theory :*

$$P = \frac{dw}{dt} = \left( \frac{dw}{dq} \right) \left( \frac{dq}{dt} \right) = VI$$

*Solution :*

$$\begin{aligned} W_{0 \rightarrow 625 \mu s} &= \int_0^{625 \times 10^{-6}} \frac{1}{4} (e^{-3200t} - 2e^{-2000t} + e^{-800t}) dt \\ &= \frac{1}{4} \left( \frac{-1}{3200} e^{-3200t} + \frac{2}{2000} e^{-2000t} - \frac{1}{800} e^{-800t} \right) \bigg|_0^{625 \times 10^{-6}} \\ &= \frac{1}{4} \left[ \left( \frac{-1}{3200} e^{-3200 \times 625 \times 10^{-6}} + \frac{2}{2000} e^{-2000 \times 625 \times 10^{-6}} - \frac{1}{800} e^{-800 \times 625 \times 10^{-6}} \right) - \left( \frac{-1}{3200} + \frac{2}{2000} - \frac{1}{800} \right) \right] \\ &= 12.14 \mu J \end{aligned}$$

**c) Find the total energy delivered to the element.**

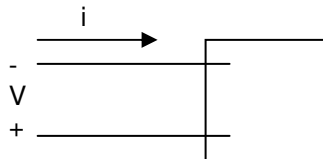
$$\begin{aligned}
 W_{0 \rightarrow \infty} &= \int_0^{\infty} \frac{1}{4} (e^{-3200t} - 2e^{-2000t} + e^{-800t}) dt = \lim_{L \rightarrow \infty} \frac{1}{4} \int_0^L (e^{-3200t} - 2e^{-2000t} + e^{-800t}) dt \\
 &= \lim_{L \rightarrow \infty} \frac{1}{4} \left( \frac{-1}{3200} e^{-3200t} + \frac{2}{2000} e^{-2000t} - \frac{1}{800} e^{-800t} \right) \bigg|_0^L = \frac{1}{4} \left[ 0 - \left( \frac{-1}{3200} + \frac{2}{2000} - \frac{1}{800} \right) \right] \\
 &= 140.625 \mu J
 \end{aligned}$$

The total energy is 140.625  $\mu J$ .

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6U. The voltage and current at the terminals of the circuit element shown below are zero for  $t < 0$ . For  $t \geq 0$  they are:

$$\begin{aligned}
 v &= 5e^{-200t} - 5e^{-400t} \quad mV \\
 i &= 10e^{-200t} - 20e^{-400t} \quad \mu A
 \end{aligned}$$



- Find the power at  $t = 225 \mu\text{sec}$
- How much energy is delivered to the circuit element between 0 and 225  $\mu\text{sec}$ ?
- Find the total energy delivered to the element.

**Solutions:**

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6Sb. The voltage and current for a passive element is given as:

$$\begin{aligned}
 v(t) &= 100 e^{-400t} \text{ V} & \text{for } t \geq 0 & \quad "v(t)=0 \text{ for } t < 0" \\
 i(t) &= 5t \text{ A} & \text{for } t \geq 0 & \quad "i(t)=0 \text{ for } t < 0"
 \end{aligned}$$

For the above condition, Find:

- Value of  $t > 0$  such that power delivered to the element is at maximum.
- The maximum power delivered to the element.

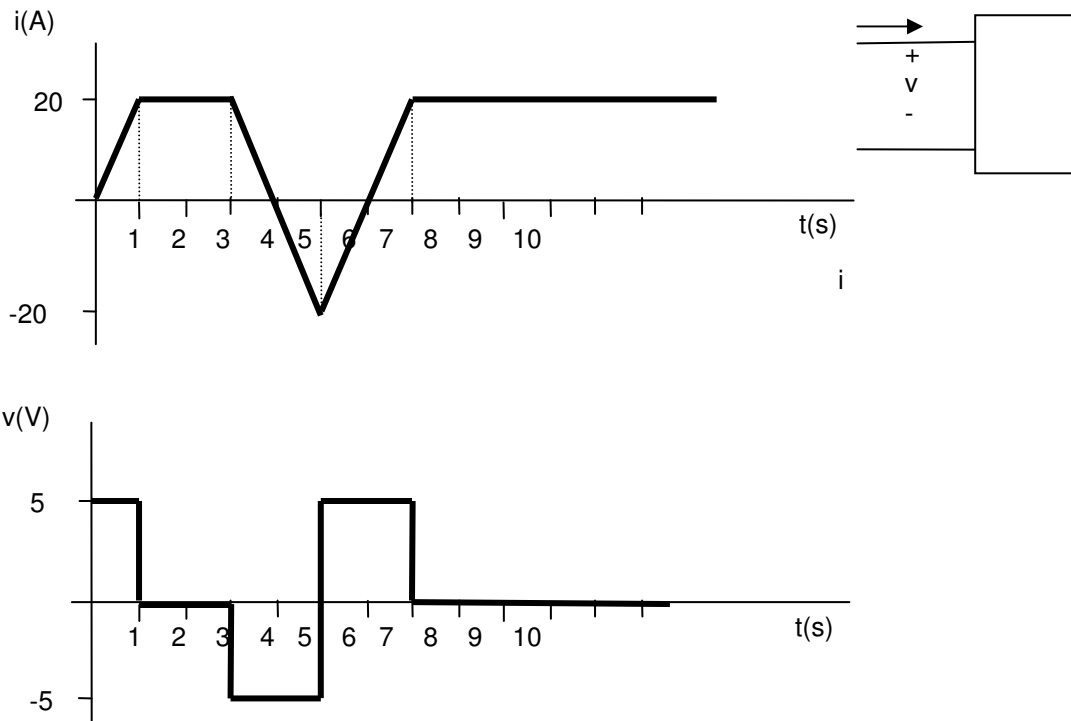
**Solution:**

$$\begin{aligned}
 \text{a) } p(t) &= v \cdot i = 500t e^{-400t} \\
 \frac{dp(t)}{dt} &= 500e^{-400t} - 200,000te^{-400t} = 0 \\
 (500 - 200,000t)e^{-400t} &= 0 \\
 t &= 1/400 \text{ Sec.} \\
 \text{b) } p(t) &= 500t e^{-400t} \\
 p(1/400) &= 1.25 e^{-1} = 0.459 \text{ W Maximum}
 \end{aligned}$$

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7S. For the voltage and current at the terminals of the circuit element is shown below:

- a) Sketch the power versus  $t$  plot for  $0 \leq t \leq 10$  s  
b) Calculate the energy delivered to the circuit element at  $t=1$ , 6, and 10 s.



**Solution:**

- a) Sketch the power vs.  $t$  plot

Apply  $p=v \cdot i$  to every  $t$  that either  $i$ ,  $v$  or both change.

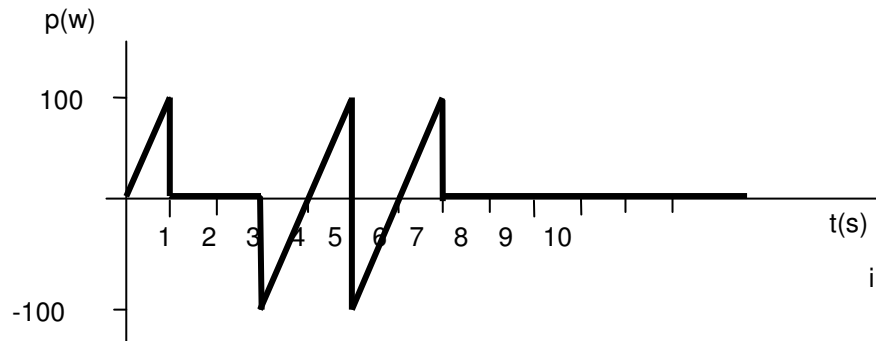
$$0 \leq t < 1 \rightarrow p=vi=5 \cdot (20t) = 100t$$

$$1 < t \leq 3 \rightarrow p=vi=0 \cdot (?) = 0$$

$$3 < t < 5 \rightarrow p=vi=(-5) \cdot (80-20t) = -400 + 100t$$

$$5 < t \leq 7 \rightarrow p=vi=5 \cdot (-120+20t) = -600 + 100t$$

$$t > 7 \rightarrow p=vi=(0) \cdot (20) = 0$$



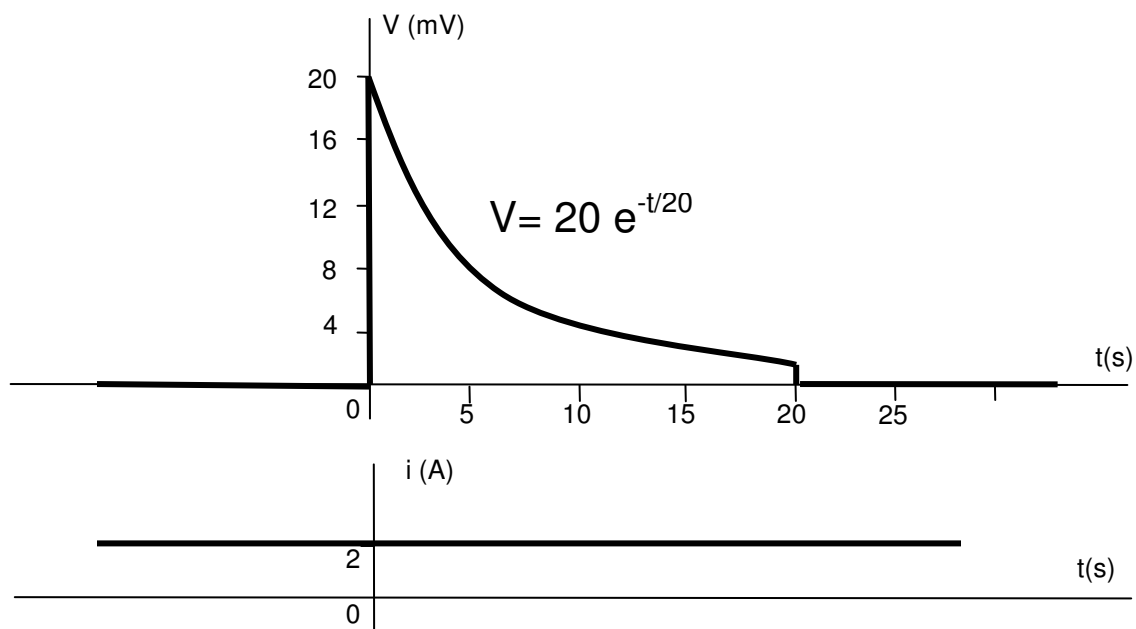
- b) Calculate the energy delivered to the circuit element at  $t=1$ , 6, and 10 s.  
To find the answer simply sum the area under the curve to the time stated:

$$W(1) = \frac{1}{2} (1)(100) = 50 \text{ Joules}$$

$$W(6) = \frac{1}{2} (1)(100) + \frac{1}{2} (-1)(100) + \frac{1}{2} (1)(100) + \frac{1}{2} (-1)(100) = 0$$

$$W(10) = \frac{1}{2} (1)(100) + \frac{1}{2} (-1)(100) + \frac{1}{2} (1)(100) + \frac{1}{2} (-1)(100) + \frac{1}{2} (1)(100) = 50$$

7U. The voltage and current at the terminals of an alternative energy source is shown below. Find the average power during the period  $0 < t < 20\text{s}$  and total energy delivered.

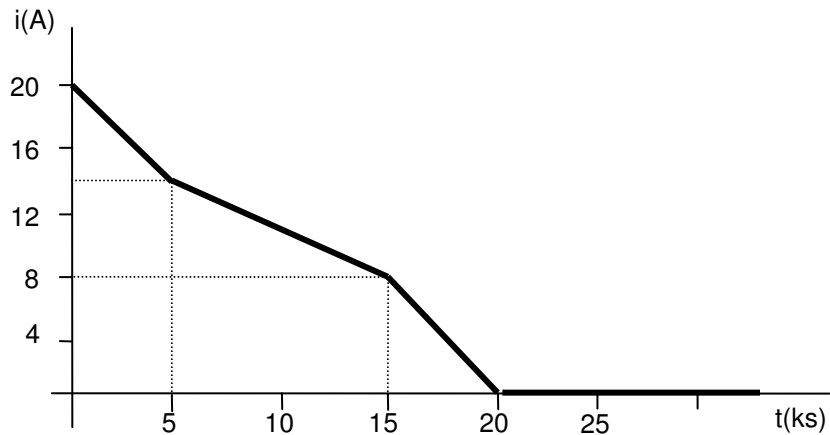
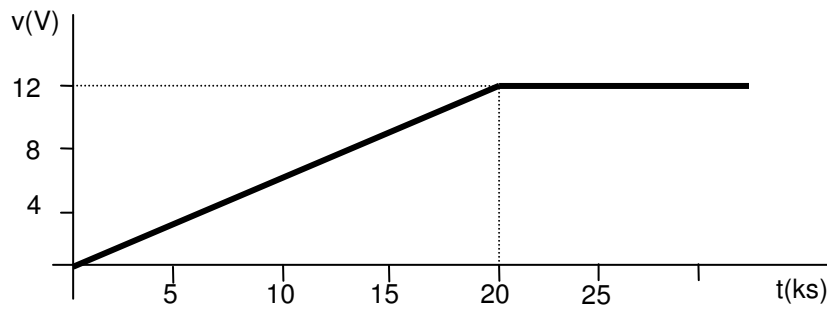


**Solution:**

7Sb. The voltage and current at the terminals of an automobile battery during a charge cycle are shown below:

- Calculate the total charge transferred to the battery
- Calculate the energy transferred to the battery.





**Solutions:**

a) Find  $Q_{\text{total}}$

$Q = \text{Area under the curve } i(t)$

$$= \frac{1}{2}(6 \cdot 5) + (14 \cdot 5) + \frac{1}{2}(6 \cdot 10) + (8 \cdot 10) + \frac{1}{2}(8 \cdot 5) = 215 \text{ k Coulomb(C)}$$

b) Find  $W_{\text{total}}$

$$w = \int p(t)dt = \int v(t)i(t)dt$$

$$v(t) = 12t / 20,000 \text{ for } 0 \leq t \leq 20\text{ks}$$

$$v(t) = 12 \text{ for } t \geq 20\text{ks}$$

$$i(t) = 20 - 6t / 5,000 \quad 0 \leq t \leq 5\text{ks}$$

$$i(t) = 17 - 6t / 10,000 \quad 5\text{ks} \leq t \leq 15\text{ks}$$

$$i(t) = 32 - 8t / 5,000 \quad 15\text{ks} \leq t \leq 20\text{ks}$$

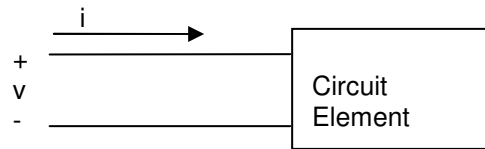
$$w = \int_0^{5,000} (12t / 20,000)(20 - 6t / 5,000)dt + \int_{5,000}^{15,000} (12t / 20,000)(17 - 6t / 10,000)dt + \int_{15,000}^{20,000} (12t / 20,000)(32 - 8t / 5,000)dt$$

$$w = 950 \text{ k Joules.}$$

7Sc. The voltage and current at the terminals of the circuit element are zero except for the time interval shown below with its corresponding voltage and current expressions:

$$v(t) = 2 - 0.25t \quad V, \quad 0 \leq t \leq 40s$$

$$i(t) = -5 + 0.5t \quad A, \quad 0 \leq t \leq 40s$$



When ( $t=?$ ) maximum power is being delivered to the circuit element? (prove that the  $t$  value found is the maximum)

**Solution:**

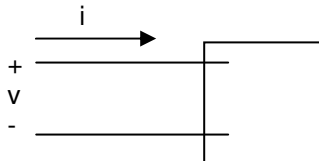
$$P(t) = V(t)I(t) = (2 - 0.25t)(-5 + 0.5t) = -10 + 2.25t - .125t^2$$

$$P'(t) = 2.25 - 0.25t$$

$$\text{Min \& Max} \rightarrow P'(t) \rightarrow 2.25 - 0.25t = 0 \rightarrow t = 9 \text{ Sec.}$$

$$\text{It is max since } P''(t) = -.25 < 0$$

8S. The voltage and current at the terminals of the circuit element shown here are zero for  $t < 0$  and  $t > 40$  s. In the interval between 0 and 40 s the expressions are



$$v = t(1 - 0.025t) \quad V, \quad 0 \leq t \leq 40s$$

$$i = 4 - 0.2t \quad A, \quad 0 \leq t \leq 40s$$

- At what instant of time is the power being delivered to the circuit element maximum?
- What is the power at the time found in part A?
- At what instant of time is the power being extracted from the circuit element maximum?
- What is the power at the time found in part c?
- Calculate the net energy delivered to the circuit at 0, 10, 20, 30, and 40s.

**Solution:**

**A) At what instant of time is the power being delivered to the circuit element maximum?**

Theory :

$$P = \frac{dw}{dt} = \left( \frac{dw}{dq} \right) \left( \frac{dq}{dt} \right) = VI$$

Solution :

$$\begin{aligned} P = VI &= t(1 - 0.025t)(4 - 0.2t) \\ &= t(4 - 0.2t - 0.1t + 0.005t^2) \\ &= t(0.005t^2 - 0.3t + 4) \\ &= 0.005t^3 - 0.3t^2 + 4t \end{aligned}$$

$$P'(t) = 0.015t^2 - 0.6t + 4$$

$$P''(t) = 0.03t - 0.6$$

**When  $P'(t) = 0$ , it is a max/min?**

$$0 = 0.015t^2 - .6t + 4$$

$$t = \frac{+0.6 \pm \sqrt{0.6^2 - 4 * 0.015 * 4}}{2 * 0.015} = \{8.453, 31.55\}$$

**Which is a maxima and minima?**

$$P''(8.453) = -0.34641 < 0 \rightarrow P(t = 8.453) \text{ is Maxima}$$

$$P''(31.55) = 0.3465 > 0 \rightarrow P(t = 31.55) \text{ is Minima}$$

*From the maxima and minima we could find which is extracted and delivered, but here is another way.*

$$P(8.453) = 15.396 \quad W, \text{ meaning Delivered / Consumed}$$

$$P(31.55) = -15.396 \quad W, \text{ meaning Extracted / Generated}$$

So, power delivered is at a maximum at 8.453s.

**B) What is the power at the time found in part A?**

$$P''(8.453) = -0.34641, \text{ Maxima}$$

$$P(8.453) = 15.396, \text{ Delivered}$$

The power is 15.396 W

**C) At what instant of time is the power being extracted from the circuit element maximum?**

$$P''(31.55) = 0.3465, \text{ meaning Minima}$$

$$P(31.55) = -15.396, \text{ meaning Extracted}$$

The time is 31.55 s

**D) What is the power at the time found in part c?**

15.396 W extracted

**E) Calculate the net energy delivered to the circuit at 0, 10, 20, 30, and 40s.**

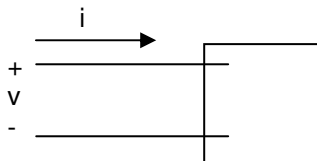
$$W = \int_0^t (0.005t^3 - 0.3t^2 + 4t) dt = 0.00125t^4 - 0.1t^3 + 2t^2 \Big|_0^t$$

$$W = 0.00125t^4 - 0.1t^3 + 2t^2$$

At  $P(0) = 0$  W, so assume  $W(0) = 0$  J

<b>W(0)</b> = 0	J
<b>W(10)</b> = 112.5	J
<b>W(20)</b> = 200	J
<b>W(30)</b> = 112.5	J
<b>W(40)</b> = 0	J

8U. The voltage and current at the terminals of the circuit element shown here are zero for  $t < 0$  and  $t > 20$  s. In the interval between 0 and 20 s the expressions are



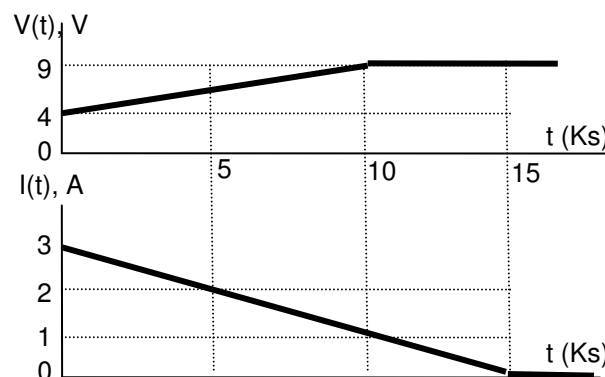
$$v = t(1 - 0.05t) \text{ V}, \quad 0 \leq t \leq 20\text{s}$$

$$i = 2 - 0.8t \text{ A}, \quad 0 \leq t \leq 20\text{s}$$

- At what instant of time is the power being delivered to the circuit element maximum?
- What is the power at the time found in part A?
- At what instant of time is the power being extracted from the circuit element maximum?
- What is the power at the time found in part c?
- Calculate the net energy delivered to the circuit at 0, 5, 10 and 20s.

**Solution:**

8Sb. Using the voltage and current charts of an electrical element shown below:



- Calculate the total charge transferred to the element ( $t > 0$ ).
- Calculate the total energy transferred to the element ( $t > 0$ ).

**Solution:**

c) Total Charge,  $Q = ?$   
 $Q = \text{area under } I = \frac{1}{2} (3A \times 15 \times 10^3 S) = 22,500 \text{ Coulomb}$

d) Energy,  $W = ?$   
 $W = P_{\text{ave}} \times \Delta t = (V \times I)_{\text{ave}} \times \Delta t$

Two distinct sections:

Part 1)  $0 \leq t \leq 10 \text{ KS}$

$$v(t) = 4 + 0.0005t$$

$$i(t) = 3 - 0.0002t$$

$$W1 = \int_0^{10,000} (4 + 0.0005t)(3 - 0.0002t) dt = \int_0^{10,000} (12 + 0.0007t - 10^{-7}t^2) dt = 12t + 0.00035t^2 - \left(\frac{10^{-7}}{3}\right)t^3 \Big|_0^{10,000}$$

$$W1 = 121,667 \text{ Joules}$$

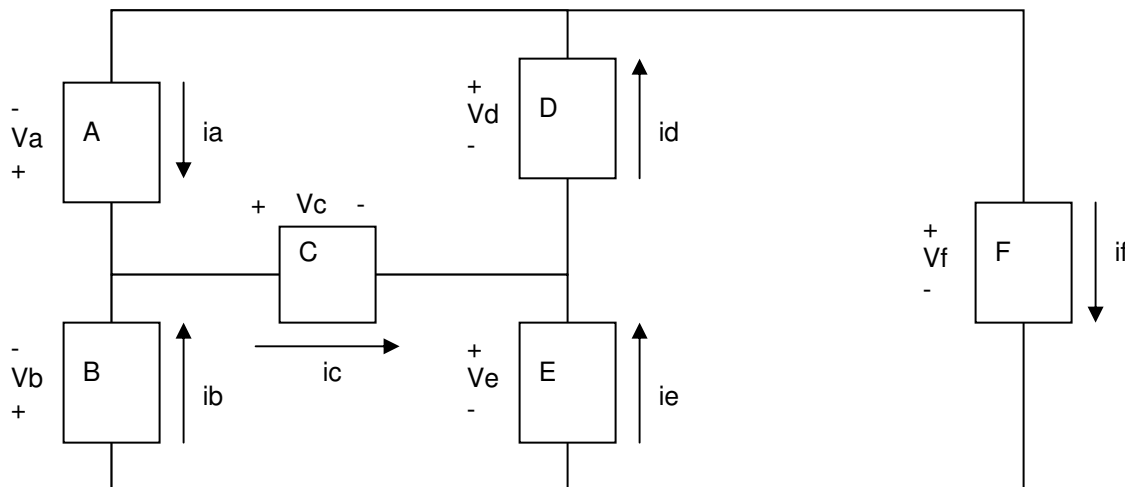
Part 2)  $10 \leq t \leq 15 \text{ KS}$

$$I_{\text{ave}} = (1+0) / 2 = 0.5 \text{ A}; \quad V_{\text{ave}} = 9 \text{ V} \rightarrow W = (V \times I)_{\text{ave}} \times \Delta t$$

$$W = (0.5 \times 9) \times (5 \times 10^3) = 22.5 \times 10^3 \text{ Joules}$$

$$\text{Total } W = 121.7 \times 10^3 + 22.5 \times 10^3 = 144.2 \times 10^3 \text{ Joules}$$

9S. The numerical values for the currents and voltages in the following circuit are given in Table. Find the total power developed in the circuit.



Element	Voltage (V)	Current (A)
A	-18	-51
B	-18	45
C	2	-6
D	20	-20
E	16	-14
F	36	31

**Solution:**

Element	IV (W)	Passive Conv. Sign	Power (W)	Direction
A	918	-	-918	Developed

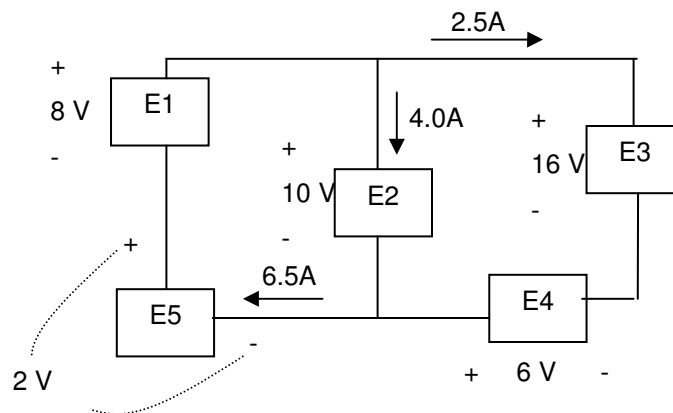
<b>B</b>	-810	+	-810	Developed
<b>C</b>	-12	+	-12	Developed
<b>D</b>	-400	-	400	Absorbed
<b>E</b>	-224	-	224	Absorbed
<b>F</b>	1116	+	1116	Absorbed

Net power =  $(-918-810-12+400+224+1116) \text{ w} = 0 \text{ w}$

Total power developed =  $(918+810+12) \text{ w} = 1740 \text{ w}$

The total power developed is 1740 w.

**9U.** For the following ideal circuit, find the total power delivered and consumed. Is this is a valid circuit?



**Solution:**